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Brief Relational Mathematics Counseling
as an Approach to Mathematics Academic Support
of College Students taking Introductory Courses

A DISSERTATION

submitted by

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In partial fulfillment of the requirements
for degree of
Doctor of Philosophy

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Abstract

Traditional approaches to college mathematics support focus on cognitive aspects of the student's approach and only incidentally address affective problems such as mathematics and testing anxiety. Because such affective conditions may be symptoms of underlying relational problems rooted in a student's learning history, I proposed a brief relational mathematics counseling approach that integrates cognitive constructivist mathematics tutoring into a brief relational conflict counseling framework (Mitchell, 1988; Windschitl, 2002).

I hypothesized that using this approach, professional tutors who also took on a role of mathematics counselor, could help underachieving students, during an introductory level mathematics college course, improve their approach to mathematics and avoid failure. This pilot study was conducted with an introductory statistics class at a small urban commuter university in the Northeast United States. Ten of the 13 students in the class volunteered for counseling; each participant had an average of 5 sessions. The counselor helped students explore their relationships with their mathematics selves, internalized presences, and interpersonal attachments while sessions focused on mathematics and course management. This new approach to mathematics support helped the counselor and the student become aware of mathematics relational patterns impeding the success of the student, and allowed both to develop constructive ways to change counterproductive behaviors.

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CHAPTER I

THE NEED FOR A NEW MATHEMATICS COUNSELING APPROACH TO SUPPORT COLLEGE STUDENTS

This research grew out of my experience as a mathematics learning specialist in academic support centers at two- and four-year colleges and universities in the Northeast United States. Over the years, certain students who came to me for help puzzled me. Some whose skills seemed inadequate or whose experience of mathematics seemed too damaging went on to succeed, while others succeeded at the introductory level but could not continue and eventually changed to a major that had fewer mathematics requirements or none at all. Too many of my students—typical students with no diagnosable learning disabilities—withdrawn or failed. Again and again I noticed that academic proficiency alone could not explain my students' success or failure. I became convinced that there was another way of understanding how college students learn mathematics—one that my training and experience did not give me tools to address at that point.

A young woman I will call Janet was one such puzzling student. A freshman who was taking a business precalculus course, Janet was in the practice of coming once or twice a week to the Mathematics Support Center during drop-in time to work on her homework. The peer tutors or I would check on her and sit down with her if she was struggling. One morning there were few students and no peer tutors present, just me, the professional tutor. Janet was sitting close to the table with her notebook and mathematics text on the table in front of her. As I went over to check on how she was doing, she pulled her hands out from under the table. Without thinking, I blurted out, "What were you doing?" Shamefaced, Janet replied, "I was counting on my fingers." She was working on factoring a quadratic equation and was trying to work out what factors of 24 summed to 11 by tapping her fingers on the

underside of the table. I asked why *under* the table—and not above, where she could see her fingers—and she told me that when she was six years old, in first grade at a parochial school, her teacher had rapped students' knuckles whenever she caught them using their fingers to help with arithmetic problems. The teacher had forbidden the use of fingers or other counting materials, taking the need for them to mean a student had not done her homework or memorized her addition facts. So Janet had learned to keep her counting hidden and had never committed her addition facts reliably to memory.

Janet grasped quadratic equations—clearly she had the aptitude to memorize these arithmetic facts. The question was why she hadn't. At the age of 19, Janet was stuck in mathematical behavior that was now neither appropriate nor necessary, although it was sensible in the early grades. She was managing in her course, although it always seemed to be a quiet struggle and she never seemed confident of her outcomes. I began to wonder about how Janet's first grade teacher's treatment and her ongoing need to rely on hidden counting had affected Janet's view of herself as a mathematics learner. I wondered whether she was now aware that it is considered developmentally appropriate for first graders to count using physical objects. Or did she still believe, as she seemed to have then, that the teacher was justified in her knuckle rapping and that she was bad at mathematics, as the teacher implied?

Janet was confident and doing well in her other courses; it was only in mathematics that she was struggling. Even within mathematics there seemed to be discrepancies in her confidence and achievement. She grasped difficult precalculus concepts such as the composition of functions, but her tentative grasp of underlying arithmetic facts often seemed to undermine her confidence in her understanding of such advanced concepts. Despite her

difficulties and unrealistic underconfidence, Janet did persist and earned a B⁻ in her course—not as high a grade as I felt she *could* have earned, however. Her persistence also puzzled me. I saw other low-confidence students, with no early trauma and with a sound grasp of the underlying arithmetic facts and concepts, who came intermittently to the mathematics support center, seemed to feel helpless to change their gloomy expected outcome, and ended up withdrawing or failing.

Janet was the kind of student whom I often found myself wanting to help but not knowing how. I could recommend the upcoming mathematics anxiety workshop. Although anxiety did not seem to be at the core, Janet did have affective problems with mathematics that included anxiety. Likewise, I could help her master her precalculus content. But from past experience I knew that these interventions were unlikely to affect Janet's overall approach to herself learning mathematics. And yes, I could (and did) tell her about research and good practice in elementary education that showed that her teacher had been wrong and that Janet's use of fingers in first grade did *not* mean that she was bad at mathematics. But I suspected that simply communicating this information would not be enough to convince her that *she* had always been and was now able to understand and master mathematics. Her progress continued to be achieved at what appeared to be considerable emotional cost and little sense of personal ownership; she attributed her relative success to the peer tutors and me.

The struggling students who inspired me to undertake the research described in this dissertation are in many ways typical of American college students. Regardless of their major, in most U.S. colleges or universities, students are required to take a mathematics course at an introductory college level for a liberal arts degree; some must

go further for their major. More students withdraw from or fail these courses than any other college courses (Dembner, 1996a, 1996b). Students who do not fulfill their college's mathematics requirements often abandon or change their academic and career goals. Students like Janet may persist in mathematics for their major, but fail to develop the confidence to apply it to related courses or in the workforce.

Colleges and universities have attempted to address the problem of failing and withdrawing mathematics students and the alarming attrition rateⁱ of students from mathematics and mathematics-related majors in college (Madison, 2001; National Research Council, 1991). The most prevalent assumption on the part of colleges and universities is that this failure and attrition can be attributed to students' deficient high school mathematics backgrounds. Perhaps they are taking classes for which they do not have the prerequisite knowledge.

Increasingly since the mid-1980s, academic institutions have attempted to support struggling students and to encourage those who are more confident to continue studying mathematics by instituting placement testing; developing short courses to teach prerequisite mathematics and study skills; establishing learning resource centers that usually provide peer and professional tutoring for individuals and groups, instituting behavioral or cognitive counseling programs, and offering workshops focusing on study skills or testing anxiety (Boylan, 1999; Hadwin & Winne, 1996). Many also offer pre-college level developmental mathematics courses.ⁱⁱ All of these are efforts by the institution to reduce failure and enhance retention. Much of it may be seen to fall under the umbrella of what used to be called remedial but is currently called developmental education.ⁱⁱⁱ

From the perspective of the individual student, the outcomes of such efforts are uncertain, however. In my position as a mathematics specialist in the learning support center, a central piece of the university's failure reduction and retention effort, I see that while students who make strategic use of such resources can achieve at a higher level in mathematics coursework, many who need help are not strategic in accessing it. Furthermore, the predominantly cognitive, skills-based approaches that the learning support center offers are ineffective for understanding and addressing the problems faced by underconfident, anxious, or avoidant students. The measures currently in place fail to fully address the problems of college students like Janet—those with what I have come to understand as *poor mathematics mental health*.^{iv} The number of students I encounter who need help of a kind not provided by current approaches leads me to believe that poor mathematics mental health may be central to our national failings in mathematics.

The Learning Support Center Context

The research that is described in this dissertation grows directly out of the questions that plagued me over the course of a 15-year career as a learning center mathematics specialist. My objective was to find new approaches to helping students struggling with mathematics in the specific context of a college or university learning support center. I believe that academic support center personnel are well positioned to apply a new, more holistic approach to helping students struggling with poor mathematics mental health because of the opportunities for professional tutors to work one-on-one with students, the separation from regular classroom dynamics, and the semi-autonomy of typical learning support centers that makes organizational changes and the

piloting of innovative approaches easier. However, this study was designed with a lively awareness of the practical challenges of working with students in this setting.

In academic support centers like the one where I work, mathematics learning specialists overwhelmingly focus on mathematical skills and concepts. The pressures of everyday practice in an academic support center, the urgency the students feel because of the limited time available, and the importance of mathematics as an academic gatekeeper combine to create among academic support personnel a tendency toward unreflective pragmatism (Lundell & Collins, 1999). This pragmatism is characterized by only incidental assessment of affective issues as well as limited mathematics assessments. This short-term view leads to a default tendency to focus only on the course mathematics, especially on procedures and skills rather than understanding. Mathematics tutors are under great time pressure: Although students may make ongoing weekly appointments, we see the typical student only when he^v chooses to come in. The incidental nature of our contact with the student exacerbates the problem. For too many students, this approach is not working adequately.

The problem is not that mathematics learning specialists do not know about cognitive and affective factors significant for achievement, but rather that we know them abstractly and as separate factors, and lack an approach for gaining, prioritizing, and using this knowledge effectively. In addition, I increasingly had the troublesome sense that the cognitive and affective expressions that we see (e.g., Janet's finger counting and underconfidence) may be *symptoms* rather than *causes* of a student's real difficulties. We deal daily with the interaction of these overt and hidden factors and their meaning for a particular student, but this meaning often eludes us. The quick diagnosis of a student's

central problem, whether overtly cognitive or affective, underlying, or an interaction of these, is a particular challenge in the learning center context.

For all of these reasons, it is unusual in mathematics academic support to find out about students' mathematics learning histories and the understanding, beliefs, attitudes, and habits that they developed as a result. My frustration with my limited ability to help Janet and others like her grew at the same time as I began to recognize clues to the puzzle. More and more, I became certain that the cognitive approach predominant in my field was not enough to understand how and why college students undertake to learn mathematics. How could I address the root source of Janet's arithmetic problem, her approach to coping with the problem over many years, and her evidently low mathematics self-esteem? I began to see that I needed a fuller understanding of Janet's history and its effects on her present mathematics experience in order to understand what I, as a mathematics learning specialist, could do to help her change her mind about herself as a mathematics learner.

I began to wonder if affective issues and learning histories might be important determiners of achievement in mathematics among typical college students. I suspected that cognitive outcomes were related not only to academic preparation, but to relational dynamics and affective experiences: an elementary school teacher who humiliated a student for asking a question or a parent who told a student that she inherited the family "we-cannot-do-math gene." Experiences like these may lead to otherwise inexplicable gaps in basic number facts and number sense or hazy understanding of the algebraic variable. Why did certain students find themselves unable to think, interact, or connect with the instructor? Could it be that students with otherwise adequate mathematics skills and aptitudes are limited by unconscious forces linked to earlier mathematics learning experiences that cause

them to repeat counterproductive practices? Might one defining negative experience with a teacher in grade school or in high school affect a student's lifelong learning of mathematics? How might poor preparation interact with a student's mathematics identity to affect his approach in the current course?

I wanted to find out what would happen if a mathematics learning specialist did have the opportunity to delve into these questions in the learning center context. What would be the result on a student's mathematics achievement in the current and future semesters if I were able to offer support based on a more holistic picture of that student as a mathematics learner? If mathematics learning specialists could find ways to understand and help the whole person rather than dealing with his parts—ways to address the mathematics mental health of their students—we might be able resolve these problems and more effectively and reliably help him go on to achieve long-term mathematics goals.

The Study

In my capacity both as a mathematics learning specialist and a doctoral student, I have searched for ways to understand students' mathematics mental health, diagnose their difficulties, and help them holistically and effectively. Through the study that is described in this dissertation, I have sought to create and test a more holistic approach of academic support that would help the many students I encountered with academic mathematics problems rooted deeply in relational conflict and other traumas that thwarted the development of their mathematical identities.

This research was based on the hypothesis that an adequate knowledge of the student as a whole person doing mathematics may be a pivotal part of academic support personnel's plan for understanding and supporting him through his mathematics course. This hypothesis

led to four research questions that are informed by the set of challenges particular to an academic support center setting:

1. What does a mathematics learning specialist need to know about a student in order to understand him as a whole person doing mathematics?
2. What processes can be used to gain this understanding quickly while he is taking a mathematics course?
3. How can a mathematics learning specialist use this fuller understanding of the student to help him in the mathematics course he is taking?
4. What does a mathematics learning specialist need to understand about himself as a counselor and tutor in order to help the student succeed?

The search to answer these questions and thus understand and effectively intervene in each student's complex interactions between his mathematics affect and cognition led me outside the narrow boundaries of the field of mathematics academic support. The field of counseling psychology—in particular, relational psychotherapy—emerged as providing the most perceptive ways to understand the effects of students' mathematics learning histories on their current learning challenges. In chapter 2, I discuss the work of scholars I have drawn from. By adapting theories and practices of relational psychotherapy to mathematics learning, and then combining these new methods with the cognitive approaches that I had been practicing for years, I arrived at a *brief relational mathematics counseling* approach. I describe the development of this approach in chapter 3. To investigate the approach, I piloted it with students taking a summer introductory-level statistics course taught at a small, urban, commuter state university in the Northeast. The remainder of the dissertation describes and discusses the study itself—the use of case study methodology and the criteria I

used in my choice of particular cases to present in chapter 4; the presentation of the class as the case that creates the context for the individual cases in chapter 5; the individual cases in chapter 6; analysis of results and developing theory in chapter 7; and evaluation, limitations and implications of the study, as well as recommendations for further research in chapter 8.

The goal of this study was to develop, pilot, and evaluate a mathematics counseling approach based on brief relational therapy approaches (with cognitive therapy and developmental psychology contributions) designed to help individuals attain sound mathematics mental health and success in reaching their own mathematics goals. This involved identifying, adapting, and developing instruments and approaches that explore students' mathematics learning; their history, beliefs, and attitudes about learning; and their relational patterns as they participate in an introductory level college mathematics course. Students engaged in a brief course of mathematics relational counseling with me as the mathematics counselor using these instruments and approaches.

This study contributes an approach to the field of mathematics academic support that combines aspects of mathematics and personal therapy approaches drawn from cognitive, affective, and relational theory. It is designed to help college academic support staff understand and help the student as a whole person doing mathematics. It combines what are typically considered to be quite unrelated, disparate elements of mathematics learners and those who help them, that is, mathematics cognition and affect expressed in distinctive relational patterns (his, mine, and ours). The results provide some preliminary data to establish groundwork for the development and use of this individual counseling approach to improve students' mathematics mental health and success in required college mathematics courses.

My goals can be further summarized thus:

1. To identify, adapt, and develop instruments and approaches that explore students' mathematics learning, their history, feelings, attitudes, and beliefs about learning, and their relational patterns as they may affect progress in an introductory-level college mathematics course;
2. To pilot a mathematics counseling approach based on brief relational therapy approaches (with cognitive therapy and developmental psychology contributions) with the goal of helping individuals attain good-enough^{vi} mathematics mental health and success; and
3. To evaluate assessment and treatment instruments and approaches, and more importantly, the brief relational mathematics counseling approach itself.

Over the past few years, my colleagues have looked at me quizzically when I tell them that my research explores how relational therapy that is rooted in Freud can help college students achieve in mathematics. Admittedly, my approach is quite unconventional. On the surface, the teaching and learning of mathematics seem to have little to do with the murky realm of unconscious motivations and relational conflicts. But when I observed my students' behavior, addressed their achievement problems as symptoms, and asked them to talk to me about how they felt about their teachers, their peers, themselves, and the subject of mathematics itself, the results were rife with conscious and unconscious motivations that were often in conflict, and counterproductive relational patterns in which students seemed stuck.

While there are many tools to assess how affect effects achievement in mathematics and cognitive and behavioral treatments to address problems, to my

knowledge the only practitioner who has attempted to understand how mathematics issues can be addressed using a holistic individual approach based in Freudian psychotherapy is Lusiane Weyl-Kailey (1985), a Parisian psychotherapist who had been a mathematics teacher. Her work was conducted in a clinical setting with school children whose psychological and emotional disturbances she found to be connected with their mathematics learning problems. She used psychopedagogy—an integration of Freudian therapeutic and pedagogical approaches—to understand the psychological effects of mathematics on her clients in order to improve both their mathematics learning and their psychological health (Tahta, 1993; Weyl-Kailey, 1985). While she is a psychotherapist who brings her understanding of mathematics pedagogy into her therapy with disturbed students who had mathematics learning issues, I am a mathematics educator who proposes to bring Freudian-related relational conflict therapy as a new approach into the learning support of average mathematics students who have affective and relational barriers to their mathematics learning.

In this study I show that close psychological attention to unconscious motivations and conflicts is applicable not only for those whose mathematics learning problems may be related to personal emotional disturbances but more generally for ordinary college students whose psychological functioning is within the range of “normal,” and this counseling approach may be appropriately delivered in the educational setting. In the following pages, I will define a *mathematics self* that we all have, no matter how deeply neglected, damaged, or denied. I will explain how a teacher or tutor can be like a parent in the psychological development of this mathematical self. It is my hope that the theories

I have explored and the approach I have piloted will open the door to a new way of thinking about academic support that nurtures and heals students' mathematics selves.

When I begin to describe my work and my dissertation project, many people (university colleagues, students, friends, acquaintances, fellow partygoers or fellow church members) want to tell me *their* mathematics story. Each one wished that when they were struggling with the mathematics course that ended or changed their career aspirations, they had had someone knowledgeable in mathematics, mathematics pedagogical research findings, and relational counseling approaches who had been able to help them understand and get over their fears and low confidence so that they could proceed with their mathematics learning. For others the topic is so painful that they have to change the subject or walk away. And there are some who have a story of struggle and triumph and a few who never or rarely struggled, almost always “getting it” and succeeding. It is for the many who, for want of someone who could listen knowingly and intervene strategically, performed poorly or avoided or failed in the mathematics they needed, that I pursued this dissertation research.

ⁱ Of the 3.6 million U.S. mathematics students in ninth grade in 1972, only 294,000 persisted to *at-level* mathematics courses as freshmen in college in 1976. Only 11,000 continued to graduate with a bachelor's degree in mathematical sciences in 1980, and 2,700 succeeded in graduating with a master's degree in 1982. See National Research Council, 1991, p. 19, Figure 5. These figures are relatively dated but the current progression appears to be similar.

ⁱⁱ Currently, almost all community colleges and more than 60 percent of other colleges and universities in the United States offer developmental courses in mathematics, writing, study skills, and in some cases reading (Bibb, 1999; Dembner, 1996), mathematics developmental courses being the ones most enrolled in by freshmen, however (Phipps, 1998; Madison, 1990).

ⁱⁱⁱ In recent discussion of the evolution of developmental education in colleges and universities, Payne and Lyman note that the preference for the term "developmental" over "remedial and developmental" was formalized in 1976 when the name of the professional journal was changed to reflect that. They point out, however, that the field has been known by many other names in its long history (Payne, 1996). Higbee (1996) sees the essential difference between "remedial" and "developmental" as the difference between "to correct a previous wrong" and "to promote the growth of students to their highest potential" (p. 63), that is, the difference between a deficit and a growth orientation.

^{iv} Sheila Tobias (1993) uses the term "math mental health" to refer to a person's "willingness to learn the mathematics [he] needs when [he] needs it" (p. 12), using it as the criteria to assess a student's mathematics functioning beyond the cognitive. In contrast, in adopting her term I include under it all aspects of a student's mathematics functioning including cognitive factors.

^v In odd numbered chapters I use the masculine, "he," "him," and "his" for the third person singular generic pronoun. In even numbered chapters I use the feminine, "she," "her," and "hers."

^{vi} I have adapted the use of Winnicott's (1965) term "good-enough" for this study. A full discussion of his use of it and my adaptation comes in chapter 2.

CHAPTER II

THE THEORETICAL CONTEXT FOR A RELATIONAL COUNSELING APPROACH

I identified in chapter I the central problem that learning specialists face when we try to help students achieve their potential in college-level mathematics. We focus narrowly on course-related mathematics skills and concepts; we may help the student improve her grade but fail to understand and help herⁱ as a whole person doing mathematics. The focus is so much on helping her pass her course that we do not stop long enough to listen and understand what is really preventing the success she aspires to. What if I had the opportunity to hear her story and understand how certain people or experiences might have affected how she is doing mathematics now? What if I knew how to help her unravel herself from beliefs and behaviors that seemed to be standing in the way of her success, beliefs and behaviors that had developed over the years as the result of those people and experiences? I determined that if there were a way to use an individual counseling approach that could be incorporated into regular mathematics support offered through the learning support center, the problem I had identified might be resolved.

I was then faced with the task of finding and/or developing a counseling approach or approaches adaptable to the central mathematics learning task, compatible with the educational setting, and, most importantly, perceptive of underlying causes. In this chapter, I describe my search for such a counseling approach and demonstrate how my research into existing theories in the fields of education and counseling psychology provided the insight I needed to help the whole person doing mathematics.

CONTRIBUTIONS FROM THE FIELD OF MATHEMATICS EDUCATION

First I asked if researchers and practitioners in the field of mathematics education had also perceived the problem I had identified and, if so, what they had done about it. I found that there is a large body of research on cognitive (cf. Hiebert & Lefevre, 1986; Piaget, 1969) and affective (cf. McLeod, 1989, 1992, 1997; McLeod & Ortega, 1993) factors of mathematics functioning and on the relationship between cognition and affect (cf. Boaler, 1997; Buxton, 1991; Skemp, 1987). Pragmatic approaches to improving students' mathematics functioning problems range from those that focus primarily on cognitive problems (changing mathematics pedagogy or curricula), through those that focus primarily on affective problems (chiefly alleviating emotional symptoms such as anxiety), to those that focus simultaneously on both cognitive and affective problems (some dealing with affect and cognition separately, cf. Nolting, 1990), others dealing with them as interconnected factors (cf. Carter & Yackel, 1989; Tobias, 1993).

Researchers and practitioners of mathematics education concur that a student's mathematics functioning involves both cognitive and affective factors, although there is little clarity on how these factors interact (cf. McLeod, 1992; Schoenfeld, 1992). As a minimum, they suggest in order to understand how a student is functioning mathematically, a mathematics learning specialist needs to know what the student understands of the prerequisite mathematics, how well she can apply that background understanding in learning new mathematics concepts and procedures, and any affective orientations she has developed that might affect that learning process.

Cognitive Factors

To know what the student understands of the prerequisite mathematics, a college usually attempts to gauge her current level of competence using high school records and course-taking history, a college-devised placement test, Scholastic Aptitude Test (SAT) or American College Test (ACT) mathematics score, an interview, or some combination of these. If course placement is mandated by this process, the student and the mathematics learning specialist have some assurance that the level of difficulty of the current course is within range of her capabilities. Other aspects of the student's cognitive processing known to have affected her mathematics learning and present achievement such as her preferred mathematics learning style,ⁱⁱ concept developmental levels, and long- and short-term memory are generally not assessed, so little is known except what a learning specialist observes in tutoring. A student's awareness of her own learning processes and her strategic study skills, when developed in relation to current coursework, have also been found to be significant cognitive factors that are often linked with achievement (Hadwin & Winne, 1996).

The cognitive effects of the mathematics teaching approaches the student has experienced may be even more significant. Students who have experienced predominantly procedural rather than conceptualⁱⁱⁱ teaching approaches are likely to see mathematics learning as memorization of procedures rather than understanding of concepts and their connections, making security in the mathematics they know tenuous and new learning more difficult (Boaler, 1997; Skemp, 1987). Students who have experienced a teacher transmission and textbook exercise approach rather than a student-

centered, problem-solving approach are not likely to have developed effective strategies for approaching new mathematics learning (cf. Schoenfeld, 1985, 1992).

While it seemed that in my approach I would need to be mindful of all these cognitive factors as potentially significant in a student's success, I was concerned about the challenge of identifying aspects of mathematics affect that might be just as significant and understanding how these factors interacted.

Mathematics Functioning: Affective Factors are Crucial

In academic support, I had found that research on affect—beliefs, attitudes, and feelings—and its effects on students' mathematics learning and achievement is even more difficult than research on cognition to translate into understanding an individual's beliefs and feelings about her mathematics learning. It also seems more difficult to apply this understanding to developing a plan to help her succeed in her course. Mathematics and testing anxiety, locus of control, issues of learned helplessness, attribution, as well as achievement motivation are all affective factors that have been demonstrated to be factors in mathematics achievement (Dweck, 1975, 1986; Hembree, 1990; X. Ma, 1999).

Mathematics cognitive psychologists like Skemp (1987) who look at students from the perspective of mathematics cognition have identified negative affective orientations and outcomes linked with teaching and learning approaches. Others like Buxton (1991) who have looked at students from the perspective of their affective difficulties with mathematics have identified problems in their cognition and cognitive learning approaches. To make this even more complex, demographic characteristics (gender, socioeconomic status, age, first language, and race or ethnicity) may interact to magnify

or minimize individual effects of a student's past experience on her achievement (Secada, 1992).

Existing Approaches that Attend to both Affect and Cognition

Mathematics support personnel and researchers have struggled to understand interactions among students' cognition and affect on their mathematics resilience and achievement. Much work has been done in the attempt to develop ways of helping. From this research, four major approaches have emerged. Each is a pragmatic attempt to help adults overcome their underachievement, aversion, and fear of mathematics. The approaches yield important information for my work, although I found that their usefulness is limited by the fact that they are either not directly applicable to the setting I am investigating or they do not provide an adequate framework for holistic understanding and counseling.

The First Approach: Freestanding Anxiety Reduction Workshops or Short Courses

The most typical approach is a freestanding^{iv} course or workshop where the participants do mathematics as they tell their mathematics stories. Through this they become conscious of their own affect, habitual reactions, and beliefs about mathematics and the effects on their mathematics identities (cf. Buxton, 1991; Kogelman & Warren, 1978; Tobias, 1993).

To this list Carter and Yackel (1989) added another: adults' enculturation in and orientation to mathematics learning. They used Skemp's (1987) categories, distinguishing between an "instrumental"^v mathematics orientation (characterized by a "just teach me how to do it—I don't want to understand it" procedural approach) and a "relational"^{vi}

mathematics orientation (characterized by an “I want to understand why it is so and how this relates to what I already know” conceptual approach). They found that an instrumental approach was generally linked to heightened anxiety and passive behaviors, while students taking a relational approach used active problem-solving strategies and make positive attempts to construct mathematical understandings. Carter and Yackel used journal writing, cognitive constructivist problem-solving approaches, and cognitive behavior therapy techniques such as cognitive restructuring to help participants move from an instrumental (procedural) to relational (conceptual) orientation to mathematics. They found that students who made this change also experienced a significant reduction in mathematics anxiety.^{vii}

One important limitation of Tobias’s or Carter and Yackel’s approach for my work, however, is the fact that it is freestanding and thus not linked with a college course. Although participants tend to become less anxious and gain confidence, there may be little positive effect on their achievement in a college courses taken concurrently (E. Yackel, personal communication, January 21, 2000). Notwithstanding, these researchers do contribute some significant elements to the design of my approach: Practitioners like Tobias or Carter and Yackel stress group work and focus on identifying (and challenging) counterproductive thoughts and behaviors at the conscious level. Their successful use of cognitive counseling techniques such as hypothesis testing of faulty beliefs^{viii} and cognitive restructuring^{ix} prompted me to investigate cognitive counseling further for its possible contributions to my approach. Most importantly this process integrates focus on research-supported conceptual mathematics approaches with linked affective outcomes rather than treating cognition and affect separately. I determined to investigate how I

might incorporate Carter and Yackel's successful use of constructivist problem-solving mathematics pedagogy to change counterproductive mathematics orientation and affect into my approach in a college learning assistance context.

*Second Approach: Study Skill and Anxiety Reduction Co-Courses
Linked to a College Mathematics Course*

The second approach noted in the literature consists of a second course or lab linked to a college mathematics course. Addressing counterproductive beliefs and habits, these co-courses focus on developing skills for mastering the mathematics content of the college course and are typically effective in improving students' achievement (cf. Stratton, 1996; Nolting, 1990). Even further improvement in achievement resulted for students diagnosed with high external locus of control^x when a brief course of individual cognitive counseling aimed at internalizing locus of control and reducing helplessness in the mathematics learning situation was provided (Nolting, 1990). This success encouraged me in my pursuit of an individual counseling approach, but my experience told me that it is not only students with high external locus of control who could benefit from individual counseling; students like Janet (see chapter 1) have other forms of emotional impediments to achieving their mathematics potential.

I noted the consistent findings of Stratton, Nolting, and other researchers (Hadwin & Winne, 1996) that students benefit more from study skills and negative affect reduction courses or workshops that are linked to a particular academic course they are taking simultaneously than from freestanding offerings that are not specific to a particular course. This finding was a key incentive for me in pursuing an approach that could be tailored to a particular course the student was taking and delivered simultaneously.

*A Third Approach: Mathematics Instructors Addressing
Affect in the Classroom*

The third approach involves the mathematics instructor herself incorporating mathematics journaling and/or history taking, open discussion of feelings, and conceptual understanding and problem-solving development into the course curriculum (cf. Rosamond, cited in Tobias, 1993, pp. 232-236). This type of self-contained situation where instruction and support to overcome affective and cognitive challenges are combined in the classroom is unusual. Its feasibility depends on the availability of instructors who understand not only the importance of affect in mathematics learning but also how to incorporate such understanding into classroom instruction of adults while also covering the material mandated by the college mathematics department. I speculated that if I were supporting students of such an instructor in the learning support center, I might still find individuals for whom the whole class treatment of affect was not sufficient. Importantly however, helping these students individually access and address the core of their difficulties would almost certainly be facilitated by the significance placed on affective issues by the instructor. This understanding made me conscious of the importance of attending in my design to the effect of the current instructor and classroom approaches on a student I was helping.

Approach Four: Individual Counseling Approaches

The fourth approach focuses on work with individuals who experience psychological disturbances triggered by mathematics learning or directly impacting their mathematics learning. There has been a long tradition of the use of behavior and cognitive behavior counseling with individuals and groups adversely affected by mathematics anxiety,

using such techniques as desensitization, guided imagery, and relaxation training (Nolting, 1990; Richardson & Suinn, 1972).

Integrating Attention to Affect with Attention to Cognition: Summarizing Mathematics Practitioners' Contributions—Implications for my Approach

There are particular mathematics education researchers and practitioners who have studied concepts important to me as I developed this approach to college students' mathematics mental health. Some were pivotal. Especially important is Carter and Yackel's (1989) and Tobias's (1993) finding that participants must engage and succeed in conceptual mathematics in order to improve their view of themselves as mathematics learners and their mathematics mental health.

Factors related to mathematics functioning are expressed differently and lead to different outcomes in different learning contexts. The helplessness that Nolting (1990) noted in students with external locus of control as well as the passivity that Carter and Yackel (1989) observed in instrumental (procedural) mathematics learners have been linked with *performance* motivation^{xi} in achievement situations (Dweck, 1986). However, I noted that these results must be sensitively interpreted, since helplessness has also been linked with *learning* motivation in high-achieving girls subjected to over-procedural teaching. Understanding these factors within the current mathematics classroom context and listening to the student helps avoid thoughtless direct application of large group experimental findings to the individual (cf. Boaler, 1997).

The need for sensitivity again emphasizes the need for a whole-person approach that may be conceptualized in terms of a students' mathematics mental health (see chapter 1, endnote iv). Whether the designers of these approaches whom I have cited state it explicitly as a goal or not, their workshops and co-courses that included in-class

journaling and discussions of emotion and in some cases narrowly specific individual cognitive or behavioral counseling, helped participants to varying extents to become aware of their mathematics mental health challenges. To the extent that participants were supported in addressing these challenges, their mathematics mental health often improved. Some even became willing (and able) to learn the mathematics they needed when they needed it (Tobias, 1993). In none of the approaches examined here, however, was there the opportunity for an individual to explore her unique mathematics mental health challenges with a suitably qualified professional while she was engaged in a college course.

My focus on mathematics mental health as a way of conceptualizing students' overall mathematics functioning had become clear from examination of these approaches, and affirmed for me the need in the field for an individual approach to helping a student while she was taking a course. A logical next step was to identify or develop an individual counseling approach that could provide a framework for simultaneously providing the mathematics cognitive support and acknowledging and addressing students' affective problems.

CONTRIBUTIONS DRAWN FROM COGNITIVE THERAPY (CT)

When I explored the wide range of counseling psychologies that might be applicable, the approach that first drew my attention was cognitive therapy (CT), which was developed by Aaron T. Beck in the 1970s. I knew of CT's links with cognitive psychology and had already noted the use of a number of its techniques in alleviating mathematical affective problems. I found that techniques of cognitive behavior therapy (CBT) which developed from Bandura's (1986) use of social-cognitive theory to merge

behavior therapy and cognitive therapy, had also been used effectively in educational settings. I resolved to explore how CT theory (and CBT, where applicable) and techniques might be adapted for my use as a basis for mathematics counseling.

Exploring Cognitive Therapy (CT) Theory as a Framework for Mathematics Counseling

I wanted to help students become aware of how their past experiences and current beliefs about themselves might be affecting their mathematics functioning; I also sought ways to help students modify the underlying orientation and overt behaviors that were preventing their success. I wondered if cognitive therapy (CT) and cognitive behavioral therapy (CBT) approaches with their focus on helping clients change their counterproductive ways of thinking and behaving might offer what I was looking for.

CT conceives of awareness as a continuum rather than a dichotomy separating conscious from unconscious experience. Beck (1976), the founder of CT, proposed that “**Man has the key to understanding and solving his psychological disturbance within the scope of his own awareness**” (p.3). Beck argued that his CT approach would change the person’s view of herself from “a helpless creature of [her] own biochemical reactions, or of blind impulses, or of automatic reflexes [as he contended that Freudian theorists claimed]” to a person “capable of unlearning or correcting” the “erroneous, self-defeating notions” she had previously learned such as, in this context, her supposedly genetic inability to do mathematics (p.4). CT focuses more on *how* the patient distorts reality than on *why*. In therapy, “the therapist helps a patient to unravel his distortions in thinking and to learn alternative, more realistic ways to formulate his experiences” (p. 3).

Cognitive Therapy (CT) and Mathematics Depression

Students' emotional difficulties with mathematics often seemed to me to be different from traditionally recognized mathematics anxieties or phobias. When I examined CT's conceptualization of depression, I realized that much of what I had observed could be seen as a type of situational mathematics depression. I have seen in students' expressed negative views of their mathematics selves, mathematics worlds, and mathematics futures, a more local or situational counterpart of negative views of one's self, one's world, and one's future that, according to Beck (1977), characterize a depressed person's orientation to life. I had also noticed that (as Beck, 1977, and Seligman, 1975, did in clients with generalized depression) this mathematics depression was almost invariably linked with helpless beliefs and behaviors in the mathematics context. The promise of being able to differentiate depression from anxieties in the mathematics learning setting added an important piece to my approach.

Other Cognitive Therapy (CT) Contributions

Dweck (1986), Beck (1977), and others have emphasized the importance of and techniques for identifying and verbalizing erroneous and negative automatic thoughts in order to test their veracity and defuse their power. Nolting (1990), Buxton (1991), and others suggest the importance of students becoming consciously aware of their own affect. Buxton (1991), Tobias (1993), Carter and Yackel (1989) and Stratton (1996) observe the therapeutic value of recognizing one's already existing mathematics aptitude and finding oneself capable of doing mathematics. Cognitive and cognitive behavior therapy (CT and CBT) and counseling techniques have been used effectively and extensively in educational settings. As noted above, Tobias (1993), Carter and Yackel

(1989), and others use CBT techniques such as cognitive restructuring, hypothesis testing of faulty beliefs, assigning affective homework, and desensitization in their mathematics anxiety reduction workshops. Nolting (1990) also demonstrates the efficacy of a limited CBT cognitive restructuring approach (see endnote ix) to reduce the external locus of control of certain beginning algebra students.

I determined that each of these CBT techniques might become part of my toolbox to help students. CT in theory (though not always in practice) takes a constructivist, problem-centered approach in that the client is seen to be the author of her own cure and the counselor becomes a coach as they collaboratively identify key problems that the client works to solve. This is the stance I chose as a mathematics learning specialist, to take with my students. I saw an important advantage of CT/CBT's brief therapy mode in college mathematics counseling. A course of CT/CBT therapy ranges from as few as three to as many as thirty sessions, but is typically conducted in ten to twenty sessions, a promising match for a college semester timeframe.

*Limitations of Cognitive Therapy (CT) as a Framework for
Addressing Mathematics Mental Health Issues*

CT still left unaddressed, however, how a student's present patterns of mathematics functioning may have been influenced by her past experiences, which I had identified as crucial for understanding and helping mathematics students. CT does not consider the present role of the unconscious in sabotaging conscious motivations. I have found that students are not dealing only with erroneous automatic thoughts that can be identified and reasoned with; they often seem influenced by unconscious motivations out of their awareness that stem from their past experiences and that are in conflict with their conscious desires. As a mathematics tutor relating with the student, I also find myself

reacting and behaving in ways that puzzle me. In CT I did not find a way of understanding these aspects of the student or myself or our interaction.

CT and CBT theorists contend that understanding the origins of a psychological problem is not essential for producing behavior change (Wilson, 1995). The CT approach thus helps identify and deal with symptoms but does not provide a way to unearth the root of the problem. Perhaps, though, I reasoned, more than behavior change might be needed for a student to succeed in mathematics. When I am confronted with a student's puzzling behaviors she may be unaware of and contradictory automatic thoughts that *she* does not even understand, it may be difficult to find ways to refute them even with good present evidence or research or logic. With some, resolving the puzzle may require an understanding of its beginnings and its developmental history.

CT/CBT counselors do not see a need to investigate unconscious motivations and internalized relationships, nor do they examine present relationships to find clues to the person's difficulties. It is precisely these motivations and relationships that I hypothesized were key contributors to understanding a student's mathematics mental health challenges. Although CT/CBT provided invaluable elements, I concluded that CT could not supply the overarching framework for a holistic appraisal of a student's mathematics mental health.

CONTRIBUTIONS FROM RELATIONAL CONFLICT PSYCHOANALYTIC THEORY

In recognizing the need to address root causes of mathematics affective problems, I returned to theorists of mathematics affect such as McLeod (1992) and looked more closely this time at their endorsement of classical Freudian-type analysis and counseling approaches albeit for cases of extreme mathematics emotionality (see McLeod, 1992, citing Tahta,

1993). In cases of severe disturbance some mathematics educators and therapists have looked at or advocate looking at the role of individual students' unconscious in their mathematics learning difficulties (cf. Buxton, 1991; McLeod, 1992, 1997). As I noted in chapter 1, Weyl-Kailey (1985) uses Freudian psychoanalytic techniques in a clinical setting to probe and remediate puzzling mathematical behaviors as she uncovers and treats related psychological disturbances.^{xii} Weyl-Kailey and others (see endnote xii) found that attention to students' unconscious motivations gives insights that other approaches do not. These researchers did not, however, use such approaches to understand and help "normal," struggling college students in the educational setting succeed in their current course, and it is these "normal" students I planned to help.

Because my interest was in the mathematics mental health of ordinary students, not just those with extreme difficulties, I had earlier rejected the utility of psychoanalytic theory. I found no critical tradition in mathematics education that understood mathematics affective and cognitive problems as symptoms of underlying causes rooted in each student's learning history and expressed in her current patterns of behavior and relationships. But I now saw the promise of psychoanalysis in its attention to the unconscious and the present effects of the past on *everyone*. Indeed McLeod (1997) noted with interest Buxton's (1991) suggestion that some struggles of such ordinary students with mathematics might well be understood in terms of Freud's concept of the superego. I resolved to explore Freud's theory and the theories that evolved from it.

The work of Stephen A. Mitchell (1988) emerged as highly relevant to my research because it used a form of relational conflict psychotherapy derived from Freudian psychoanalysis to help ordinary adults who had goals but were so embedded in

relational patterns with themselves and their significant others (both internal and external) that those goals were not being fulfilled. Rather than seeing people through a classical Freudian lens as largely driven by instinctual pleasure-seeking and aggression drives that continually engender internal conflict along a largely predetermined developmental path, Mitchell's (2000) relational conflict theory recognizes that people are hardwired for human relationships and that their drives, motivations, and conflicts are focused around developing and maintaining those relationships with others and with themselves. In 1988 Mitchell integrated the three major relational strands of psychotherapy that emerged from Freud's classical psychoanalysis: self psychology, object relations, and interpersonal psychology. Each of these strands emphasized one dimension of what Mitchell termed as a person's *relationality* or her current behavior that are the outcome of the development of her self, her external and internalized objects, and her interpersonal attachments (Mitchell, 2000). When I considered these dimensions in the context of a student's mathematics learning experience, I interpreted them as follows:

1. Mathematics self or selves;
2. Internalized mathematics presences or objects;^{xiii} and
3. Interpersonal mathematics relational or attachment patterns.

Understanding a student's mathematics relational dimensions, how they are positioned in relation to each other, and how they interact with one another to express her relationality might provide the insight into the origin and development of her puzzling behaviors and conflicts that I was seeking.

Relational Conflict Theory as a Framework

Relational psychotherapies rest on the premise that repetitive relationship patterns derive from the human tendency to preserve the continuity, connections, and familiarity of a personal interactional world.^{xiv} They recognize that the task of understanding the person and helping her disembed from counterproductive interactional patterns may be more complex and indirect than cognitive therapy concedes. Like cognitive therapists and unlike classical Freudian psychoanalysts, relational theorists regard the person as able to consciously choose to change her patterns of thinking and behavior (Mitchell & Black, 1995).^{xv}

Relational Theory, Development, and the Past

These insights from relational conflict theory promised to explain much of what had puzzled me in the learning assistance center. Relational theory acknowledges that human beings may proceed as if straightforwardly pursuing conscious goals but asserts that, at the subconscious level, they seek to maintain an established sense of self and patterns of relationship. In the learning center, I often found students who consciously avowed a determination to succeed while they simultaneously behaved in ways that jeopardized that success. The self is not a static entity, however; it simultaneously affects and is affected by internal and external realities. As Mitchell notes, the dialectic between self-definition and maintaining connection with others is complex and intricate. He theorizes that humans “develop in relational matrices and psychopathology is a product of disturbances in both past and present relationships and their interactions” (Mitchell, 1988, p. 35). Similarly, students’ mathematics difficulties may be the product of their mathematics learning experiences and relationships interacting with current situations.

Relational theory does not consider people developmentally arrested by early failures (as object relations theorists believe), but rather that they have constricted relational patterns that have developed in distorted ways in response to initial and subsequent environmental and personal failures. This seems an apt depiction of both the beginning and the outcomes of many students' mathematics learning histories.

These earliest experiences affect subsequent development. Understanding the past is crucial... [because] the past provides clues to deciphering how and why the present is being approached and shaped the way it is. ... [T]he residues of the past do not close out the present; they provide blueprints for negotiating the present. (Mitchell, 1988, p.149, 150)

My puzzling students' normal mathematical development may have been *constricted* by these negative experiences, and, as a result, subsequent relationships with teachers, mathematics, and self became distorted. Their mathematics development had also been affected by the effects of their own good and bad choices. The ways they relate now to mathematics, to the instructor, and to me, the tutor, provide clues to their past and to how to alter their present course.

Relational theory was offering me a way to understand the development of a student's mathematics identity or what I came to call her *mathematics self*. This theory offered me a way to understand how certain experiences and people might have been internalized and might affect students' current perceptions of teachers. It also offered me a way to understand how loss or change in mathematics and teacher relationships might have affected their current relationships to the subject and to teacher.

Relational Theory and the Student-Tutor/Counselor Relationship

Mitchell's theory also challenged me with the prospect that a tutor would have to take a stance toward the student quite different from the traditional stance. The tutor must

be prepared to see herself as an integral part of a current relationship with the student and be willing and able to use her own feelings and reactions along with the student's reactions to her as clues to understanding the student's past. These clues could be used to work out with her what to do differently now so as not to reproduce counterproductive relational patterns likely to hinder student success.

Following Freudian psychoanalysis, relational therapists observe and analyze this relationship between the counselor and the client to collect key data germane to the client's relational patterns. In this framework, a mathematics counselor would also observe and analyze this relationship between herself and the student to provide key data on the student's relational patterns. Relational therapy is not the same as mathematics relational counseling, however. In relational therapy, the interpretation of a client's transference of her past relationships into the relationship with the counselor and the counselor's countertransference in reaction in her relationship with the client are central to the psychoanalytic process. By contrast, although the mathematics counselor's conscious awareness and examination of this transference-countertransference dynamic will be key to her relational understanding, there is not likely to be time for lengthy discussion of this dynamic, nor would the student's need for immediate mathematics help or the educational setting make lengthy discussion appropriate. The admittance of transference-countertransference as key to diagnosis in mathematics counseling will, however, radically change the orientation to the student and her need for mathematics support. Relational mathematics support is not only about the student but it is also about how the mathematics tutor or counselor experiences the relationship with the student. The

ways the tutor feels free or constrained in the tutoring relationship become important elements in understanding the student.

Limitations of Relational Conflict Psychoanalytic Theory for this Setting

In embracing relational conflict psychoanalytic theory as the basis of a new approach to improving students' mathematics mental health, I had to consider appropriate boundaries. It is important to caution myself and the field that adapting relational psychotherapies to an educational setting without proper training is problematic. Even given what I now saw to be the appropriate relational emphasis, the sphere of relational history exploration needed to be kept limited to mathematics learning settings. Should the tutor become aware of connections with more generalized mental health problems during that exploration, referral to an appropriate mental health professional would be indicated. Exploration of the present tutor-student relationship would also have to be bounded by the educational setting.

Further, any history exploration would need to be conducted *while* they were working on the mathematics. The traditional psychoanalytic leisure to explore at length the person's relational past as well as the present therapist-client relationship would not be possible or appropriate. Nevertheless bounded strategic engagement of the student in the task of exploring and connecting present mathematical behaviors and relationships with past experiences for the purpose of freeing her to change these behaviors and relationships, does seem appropriate and is what this relational approach requires.

ADAPTING RELATIONAL CONFLICT THEORY TO HELP STUDENTS DO MATHEMATICS

In order to explore the commonalities I saw between my own puzzling math students and the adults for whom treatment with Mitchell's relational conflict therapy was applicable, I needed to understand what a bounded and strategic exploration of a student's mathematics learning history should entail from a relational perspective. In particular, I had to investigate what the findings of the three major relational theories that Mitchell integrated into his theory about relationality (self psychology, object relations (internalized presences), and attachment theory) could tell me about how a student's mathematics relationality might have developed and be expressed in the present. I also needed to know about impediments to healthy development along the way, about what a student's presenting symptoms tell about that development and current unconscious relational conflicts that may impede her mathematical progress. I also needed to know ways to improve her mathematics mental health.

For his conflict relational theory, Mitchell (1988, 2000) drew on (among others) key theorists, Kohut (1977) for the self dimension, Fairbairn (1952) for the object relations (internalized presences) dimension, and Bowlby (1973) for the interpersonal attachment dimension, to explain how each of these relational dimensions differ from and complement each other in understanding and helping clients. So these are the principle theorists I chose as the basis for my approach.

In the following sections, I show how each of the three dimensions of a student's relationality around mathematics learning, explained by the Kohut's theory of self, Fairbairn's theory of internalized presences, and Bowlby's theory of interpersonal attachments, yields a distinctive picture of one aspect of her mathematics identity and

how she likely developed in relation to the mathematical parenting she received. I show how these distinctive pictures complement each other. When taken together, they yield a useful picture of her relationality and the mathematics relational conflicts that now challenge her, as I illustrate by applying the theories to Janet (see chapter 1) following the discussion of each dimension.

The First Dimension: The Self and Mathematics Mental Health

Self psychology (Kohut, 1977; Mitchell, 1988) looks at adults' relational difficulties to discover how their self development might have proceeded and what their current self needs are. This perspective provides me a way of understanding the mathematics self of an adult student, that is, the core of her mathematics identity. The other dimensions then elaborate on interactions with that self. The mathematics self may be seen as part of a person's academic self, in turn situated in the person's nuclear self.

According to Kohut (1977), to develop a healthy self the child must experience mirroring: unqualified recognition, delight, and admiration from a parent or primary caretaker^{xvi}. She also needs the opportunity and indeed the invitation to idealize and incorporate into her self a parent image,^{xvii} first as part of herself (selfobject) and eventually as ideals and values for the self (cf., the superego; Kohut, 1977, p. 185; St. Clair, 1990, p.157).

If we consider early elementary teaching to be analogous to early parenting, the development of a healthy mathematical self requires the teacher to initially mirror the child's developing mathematical identity, to recognize it, and to delight in it, much as Piaget^{xviii} (1973) and many cognitive constructivist theorists urge (Windschitl, 2002). Simultaneously the teacher provides herself as the mathematical teacher image for the

student to idealize and to incorporate as part of herself. If early classroom conditions facilitate this learning process the student's mathematics self development will likely proceed in a healthy manner.

The elementary teacher's roles in nurturing and facilitating the growth of the student's self, in particular her academic self, corresponds in a very real sense to the roles of each parent; the mother provides the mirroring and the father provides the parent image to be incorporated (see endnotes xvi and xvii). She must reflect back (mirror) to the student her mathematics ability, she must allow the student to idealize and internalize her mathematics values, and she must provide developmentally appropriate experiences (both triumphs and disappointments). The teacher mediates between the formal subject matter required by the mathematics curriculum and the informal mathematics the child has already developed.^{xix} As the child learns, interactions and connections are made among her normal cognitive development, innate curiosity and exploration, and the environment (Ginsburg & Oppen, 1979; Piaget, 1967; Vygotsky, 1986).

For growth to proceed, she must then experience tolerable reality. The self's development needs the teacher to occasionally delay or fail to respond immediately to the student's demands, thus forcing the self to develop abilities to meet her own demands. The student needs to realize that she is not, after all, all-powerful or all-knowing^{xx} (even in her teacher's or parent's eyes) nor is her idealized teacher or parent perfectly able to meet all her needs. The idealized teacher can no longer be the epitome of rectitude, wisdom, or love she initially experienced. She becomes frustrated with the teacher's imperfect mirroring and experiences tolerable disappointments with the idealized teacher, along with broadening experience that supports her own ability to learn and grow. These

conditions contribute to the development of a self that integrates a realistic assessment of the limits to her own prowess and value with a realistic assessment of the capacity and limitations of the idealized teacher or parent.

The internalized teacher's mathematical values and ideals are integrated as the student's own. These internalized values and ideals then provide structure and boundaries as the child's own competence develops. When this process proceeds appropriately the internal self-structure is consolidated and provides what Kohut (1977) calls "a storehouse of self confidence and basic self-esteem that sustains a person throughout life" (p. 188, footnote 8). This is the hallmark of a person who exhibits what Kohut refers to as healthy narcissism. However, the need for mirroring and permission to idealize continues into adulthood. This is a key understanding for a college mathematics counselor to consider.

If the teacher or parent responds to every demand or fails to respond at all, it hinders healthy growth of the nuclear self because the student's own competence does not develop in a healthy manner. If a teacher's failure to respond appropriately takes the form of overindulgence (e.g., providing too easy tasks and unwarranted praise, having high expectations with little pressure for the student to meet them) the student's grandiosity is not appropriately challenged by reality and she develops what Winnicott might call a false mathematics self (cf. St. Clair, 1990). Her self-esteem remains low because her competence does not develop appropriately but a defense is likely in the form of unrealistic *overconfidence*. She "knows" she can achieve if she wants/tries to. On the other hand if the teacher's response is in the form of chronic neglect^{xxi} (e.g., expecting little when a student falters or seems slow to grasp concepts and subsequently ignoring her need for challenge, tracking into low level tracks) she fails to see herself mirrored in

the teacher and her mathematics self fails to develop. In the extreme this may result in what almost feels like the absence of a mathematics self (cf. Cara in Knowles, 2001). Her competence and therefore her self-esteem remain low as is true for the overindulged student, but the neglected student's defense is likely to be different, in the form of unrealistic *underconfidence*. She is sure that she cannot succeed.

This study of self development allows me to see that a student whose mathematics self is vulnerable because it is underdeveloped or undermined has likely developed defenses (typically under or overconfidence and accompanying avoidance behaviors) to protect this self from further damage. Although her conscious goal is success in her mathematics course, she likely acts in ways that jeopardize that goal. Her self-esteem is compromised or low and she may have little underlying belief that she can succeed. Her unconscious goals are in conflict with her conscious ones and she remains embedded in her familiar patterns of relationship with self (cf. Mitchell, 1988).

*The Second Dimension: Internalized Presences—Objects Relations and
Mathematics Mental Health*

Object relations theory principally focuses on the person's interior relational world. This world is conceptualized as the person's self in relationship with internalized and altered others (objects of the persons' feeling and drives), with split-off parts of self, and with external others (objects). Whereas the focus of self psychology is on the development of structures of the self, the focus of object relations is more on how early interpersonal relationships are internalized and on how the inner images of the self and the other (object) are formed and shape perceptions and ongoing relationships with real and internalized others (Fairbairn, 1952; St. Clair, 1990). From this perspective, a student's internal reality is peopled by objects and selfobjects that affect her mathematics

self and the way she perceives external reality, in this context, the current mathematics instructor and course.

If parenting is experienced as threatening enough to the self, bad internalized presences are formed, creating internal conflict that distorts the person's perceptions of present reality. Fairbairn (1952) contends that "internalized bad objects are present in the minds of us all at deeper levels" (p. 65) and the degree to which they negatively affect us in the present depends in part on how bad we experienced the original external other (object) to be.^{xxii}

In the elementary classroom, a student cannot get away from the teacher and, in fact, needs her. If the teacher humiliates the student or those around her, abuses her verbally and or even physically,^{xxiii} or otherwise creates a classroom environment that the child experiences as unsafe, the child may cope with what feels like an intolerably unsafe situation by holding the teacher to be good (right) and internalizing the bad part of the teacher in order to feel safe, at least externally. She may then handle her now intolerably unsafe internal situation by the defense of repressing the bad internalized object (the teacher) or by a defense that Fairbairn (1952) calls the "defense of guilt" or "the moral defense" (p.66). That defense is accomplished thusly: The student or child is in a situation where she feels surrounded by bad objects. Because this is intolerably frightening, she converts this into a new situation where her objects (parents, caregivers, teachers) are good and she herself is bad. A student or child who has suffered abuse or neglect typically refuses to characterize the parent as bad, but is quick to admit that she herself is bad.

It is not only students who have been abused who see themselves as bad and feel shame and guilt; neglected students also feel shame for their deficiencies. The shame of both abused and neglected students seems related to a sense of nakedness or sin, as if their internalized mathematics object world is dominated by mathematics in the form of a judgmental superego^{xxiv} or by a bad mathematics teacher, threatening to unveil the deficiencies of the vulnerable trying-to-hide mathematics self^{xxv} and the result is a fearful, beleaguered mathematics self (cf. Buxton, 1991).

A bad teacher presence (or object) assaults or conflicts with the student's developing mathematics self and sabotages future relationships with teachers, even good ones. What is pertinent for understanding the adult is not so much what *actually* occurred between the teacher and child^{xxvi} but how the child *experienced* the mathematics teacher and mathematics, how she internalized them, and how she as an adult now experiences them. The student's initial transference relationship with the mathematics counselor and the instructor is likely to reveal much about such presences. If her internalized good^{xxvii} or bad presences (especially internalized bad mathematics teacher-objects) are not brought to consciousness and released, they may continue to control the present-day learning relationships in a negative way.

Whether the student's efforts to deal with internalized bad teacher presences have involved repression of bad teacher presences, moral conversion into herself being bad, or another defense, when she enters the current classroom these unconscious forces are activated and internal conflict develops between resignation to her mathematical badness and her motivations to succeed in the class. Internalized presences may be so prominent that they take precedence over current reality; the student may relate to the present

teacher as if she were in the original classroom. Conflicts arise when this mismatch between her internal and external reality negatively affects her progress in the course. If these conflicts are not resolved satisfactorily her desire to succeed or even survive in the course may be thwarted.

*The Third Dimension: Interpersonal Relational Attachments and
Mathematics Mental Health*

The exploration of object relations gave me insights into how a student's *internal* relational world might be configured and might now be affecting her. Attachment theory promised to give me insight into the development, significance, and challenges of her *external* interpersonal relationship dimension of relationality. In particular, attachment theory examines the ways the person forms ongoing relationships with significant persons in her life and work (Bowlby, 1965, 1982). Often her tendency towards dependent, detached, ambivalent, or self-reliant relationships will provide clues to the security of her early relationships and her subsequent experiences of loss or change in those relationships. The extent to which a college student seeks the help she needs when she needs it from her instructor, learning assistance personnel, or other suitably knowledgeable person has been found to be an important factor in her success (cf. Downing, 2002, Zimmerman & Martinez-Pons, 1990). The student's established attachment relational patterns may determine whether she is likely to make contact at all with those who could help her, and if she does, how she proceeds to do so.

Attachment theorist John Bowlby (1973, 1982) and his colleagues found strong evidence of a child's instinctive need for secure attachment to a particular parent figure. The attachment-caregiving bond developed between child and mother figure^{xxviii} is seen as crucial to child's survival and forms the basis for any future attachment relationships

the child develops. The type of attachment achieved by the child varies according to the type of caregiving the mother figure provides the child. Most important factors in mother's caregiving are her responsiveness to the child's signals (e.g., crying) and the extent to which she initiates social interactions with her baby (Bowlby, 1982, pp.312-318, referring to studies by Schaffer & Emerson, 1964, and a study by Ainsworth, Blehar, Walters, & Wall, 1978). *Secure* attachment is achieved when the caregiving by the mother figure is characterized by being sufficiently available and responsive. The mother figure becomes the *secure base* from which the child can move out and explore her world, but return to for comfort and reassurance in times of distress.

Researchers have found that a child's *insecure* attachments can be explained by the caregiver's behaviors towards the child. The caregiving that *detached* insecure children receive is consistently detached, with the mother figure rarely responding to the child's expressed needs and rarely herself initiating positive interaction with the child. Children whose insecure attachments are *ambivalent*, alternating between demanding contact with their mother figure and resisting, receive inconsistent or conflicted caregiving that the child finds unpredictable in its quantity or quality or both (Ainsworth, Blehar, Walters, & Wall, 1978). Another insecure pattern, *disorganized* attachment, is characterized by fear of the caregiver or of her leaving or loss (Jacobsen & Hofmann, 1997).^{xxix}

The peculiar mark of a securely attached child is her exploratory, adventurous behavior, as long as she is assured of the availability of her attachment figure if needed. By contrast, the insecurely attached child is preoccupied by frequently thwarted attempts to avoid further separations from her attachment figure; she stays close and is afraid to

explore lest she be abandoned or punished, or she tries to meet her own needs, distancing herself from her detached attachment figure. The secure person's behaviors lead to learning; those of the insecure person's tend to inhibit it. Students' academic competence through adolescence is also likely to be positively related to the security of their attachments (Jacobsen & Hofmann, 1997). These outcomes are not unexpected. Many educational researchers have demonstrated that the student's learning is dependent on her investigating and interacting with her environment (cf. Dewey, 1903; Piaget, 1973; and others).

The subsequent ability of a person who has developed insecure attachments to form relationships with others will be negatively affected and may be permanently marred. By analogy, early experiences in a mathematics classroom where the teacher does not understand or respond to the child's need for cognitive and emotional support, challenge, and latitude for exploration may lead to a sense of insecurity and difficulty with trusting the next teacher and subsequent mathematics material. Her beliefs and behaviors may resemble anxious learned helplessness on the one hand or mistrusting independence on the other.

People whose primary secure attachment relationships have not been unduly disrupted usually develop into adults who form secure attachments. They are what Bowlby (1973) calls truly self-reliant, "able to rely trustingly on others when occasion demands and to know on whom it is appropriate to rely" (p. 359).^{xxx} Because these people are confident that an attachment figure will be available to them when they need it (a secure base), they are much less predisposed to intense or chronic fear than a person who does not have that confidence (Bowlby, 1973; Sable, 1992; Weiss, 1991). They are

more resilient and able to negotiate difficult circumstances more successfully than those whose early attachment bonds were insecure (see also Werner and Smith, 1982). In contrast, adults whose attachment bonds were insecure or whose secure attachment bonds were traumatized are likely to establish insecure attachments and have difficulty in withstanding life's or the mathematics classroom's difficulties in a healthy manner. Attachment bonds they form as adults are likely to be anxious, ambivalent, detached, disorganized, or a combination of these.^{xxxi}

The teacher-student relationship, especially in the early years (generally through third grade), is a type of attachment/caregiving relationship more than a relationship of community^{xxxii} although the teacher is not a substitute parent for her students. Even in the early grades, there are important distinctions between parent figure roles and the teacher roles. In particular, the teacher's relationship with the child should be characterized by appropriate responsiveness and caregiving without the intense emotional involvement of parental attachment (Katz, 2000).^{xxxiii} As the student gets older, the focus of the teacher's "detached concern" care becomes a narrower one with more emphasis on providing an academic secure base and less on emotional involvement (cf. endnote xxxiii). A tutor or learning counselor role is perhaps an intermediate one, with more emotional involvement and partiality than is generally appropriate for a teacher. In a small college, for older adult students, as well as for adolescent/young adult students, the power differential in the 20- to 30-student classroom between the instructor and students and its similarities to classrooms of the past can activate established teacher relational patterns that are more akin to adult attachment than community relationships.

An unsafe or unsupportive classroom environment can certainly cause or contribute to the development of insecure attachments to teacher or mathematics or both (Dodd, 1992; Fiore, 1999; Jackson & Leffingwell, 1999; Knowles, 1996; Mau, 1995; Tobias, 1993). Students' subsequent avoidance of mathematics has been linked with ambiguous and unsupportive classroom environments (Patrick, Turner, Meyer, & Midgley, 2003).

It is not only student-teacher attachments that are affected by the way the teacher manages the learning environment. Student-mathematics attachments are also affected. U.S. elementary teachers are likely to lack a secure base^{xxxiv} in the arithmetic they teach (L. Ma, 1999), and those with insecure attachments are less able to provide secure attachments to those in their care (Ainsworth, 1989; Bowlby, 1980). By extension we may assume that in these mathematics classrooms, students' attachments to the mathematics itself are vulnerable. Classrooms where the instructor provides either too much or too little conceptual mathematics structure may inhibit students from making healthy attachments to the mathematics. Likewise, teacher-as-authority mathematics classrooms may also hinder healthy student attachments to the mathematics. Instead students may develop an anxious attachment to mathematics that undermines their confidence in feedback they get from working with the mathematics, and may keep them unhealthily dependent on the teacher for decisions about whether they are proceeding correctly. Confirming this, Skemp (1987) considers unhealthy dependence on the teacher to be one of the chief drawbacks of an overly procedural approach to teaching mathematics.

When the teacher's attachment to the mathematics is insecure, she is likely to cling anxiously to procedures, not daring to explore or question, fearful that her procedural grasp of the mathematics may be lost. She is less able to entertain students' queries (much less, encourage their exploration) and is likely to respond with censure to correct or logical approaches that differ from her grasp of the mathematics (cf. Corwin, 1989; L. Ma, 1999). But if the procedural teacher has a secure mathematics base, the prognosis for students' secure attachment to the mathematics is better even if it is hampered by lack of encouragement to explore the mathematics for herself and construct her own understandings with the teacher as guide. It is not only procedural transmission pedagogical approaches that may jeopardize students' attachments to mathematics. Students in laissez-fair classrooms are likely to lack a mathematical secure base^{xxxv} and even those in constructivist problem-solving classrooms may feel anxious and abandoned unless they are oriented to expect uncertainty as part of the problem-solving process and appreciate the real availability of a mathematical secure base.^{xxxvi}

Students' well-developed secure attachments to teachers and to mathematics can be disrupted by a negative experience with a teacher or encountering a type of mathematics or teaching style that result in a poor grade or failure. How well a person of any age negotiates loss and avoids distortion of psychological development depends on three factors.^{xxxvii} The third factor: the continuity and quality of her relationship with other primary attachment figure/s after loss or separation (Bowlby, 1980) is of particular importance to a mathematics counselor working with students. It seems to me that a counselor would find it easier to help students who had at some time experienced secure

attachment to mathematics to reattach to it than those who had never felt securely attached to mathematics.

Change can also disrupt mathematics and teacher attachment relationships and without support to negotiate the change students may remain stuck in a natural resistance that could jeopardize their future success. Even when changes can be seen by outsiders to be for the good, people are likely to resist or even reject^{xxxviii} those that cause disruptions to their attachments to relationships and circumstances. This may help to explain why a student repeating a course taught by a different instructor may resist approaches that are different (and often preferable), even though the student initially failed with the approaches she clings to. When students find themselves in a classroom whose approach is different from the ones they are used to they are likely to experience what Marris calls a “conservative impulse” to resist changes that call into question their familiar ways of doing mathematics (cf. Bookman & Friedman, 1998). I realized that helping these students recognize and work through their resistance might free them to benefit from the new course situation, but that would only be possible if I or the instructor or both provided a secure base and the students could attach to it in the new situation. In order to successfully resolve the effects of loss or change experienced as loss, a person must work through a grief process^{xxxix} to “retrieve the meaning of the experience and restore a sense of the lost attachment that still gives meaning to the present” (Marris, 1974, p.147, 149).

If a student has developed attachment patterns to mathematics teachers or tutors that are characterized by investing either too little or too much reliance in the teacher or tutor, their success or at least growth in mathematics learning may be compromised. They face a likely conflict between maintaining their familiar but counterproductive attachment

patterns and their willingness to risk trusting a relative stranger enough (e.g., the learning counselor who is a mathematics “teacher”) in order to attain a healthier balance between their responsibilities and getting the appropriate help they need.

Application of Relational Conflict Theories to the Case of Janet

When I look again at Janet (see chapter 1) through the lens of relational conflict theory, it seems likely that her first grade teacher’s failure to mirror her already existing mathematics ability and the teacher’s developmentally inappropriate prohibition of the use of concrete models to build understanding and provide transitions to internalized knowledge had impeded the development of her mathematics self-esteem which is the basis for a sound sense of mathematics self. This teacher had pushed underground her use of fingers as a transitional object^{xj} so that she had never developed beyond needing them (at least emotionally) and still used them for security in an insecure world, despite the risk of embarrassment, or worse, shame. This seems to have resulted in her seeing herself as bad (at mathematics) because the teacher had to be good (or at least correct) in her judgment of Janet and in her actions. It had thus distorted her sense of her mathematics self.

Despite this inauspicious start it was now apparent that Janet’s mathematics competence had developed though it remained undermined and her self-esteem remained low. She expressed her low self-esteem in an underconfident, resigned (perhaps depressed) determination to proceed, with little hope of feeling secure in her grasp of the material. It seemed that Janet had failed to develop initial secure attachments to mathematics or to mathematics teachers and now her relationships with those from whom

she sought help seemed wary; she hid from them her shameful and illegitimate techniques, expecting ridicule. Traditional understandings of Janet's affective problems couched in terms of mathematics or testing anxiety and counterproductive beliefs related to helplessness and her other unhelpful approaches, may now be seen as clues to her underlying relational issues. Thus these affective problems could now be seen as *symptoms* rather than *causes* of her difficulties. What was sound and healthy about her affective orientation to mathematics learning could likewise be seen as symptomatic of aspects of sound mathematics self-esteem.

Janet's mathematics cognitive knowledge, conceptions, and approaches can be seen in context of and as outcomes of her mathematics relational history. Her current patterns of mathematics learning and production can be understood as symptomatic of her underlying sense of mathematics self.

Mathematics Parenting of Janet from the Three Perspectives

Each of the three relational perspectives gave me insight into parenting and analogously into teaching as parenting. On reflection, two considerations stood out.

First, although it might appear that each says basically the same things about the essentials and processes of early teacher-parenting, in fact that is not the case. Each perspective *does* give different insights. Taken as a group of theories they are, as Mitchell (1988, 2000) has shown, complementary with intersecting areas of interest.

Understanding the different related conflicts an adult might be experiencing, depending on the dimension, promised to yield much in effectively diagnosing a student's mathematical challenges. Second, I realized that Winnicott's (1965) concept of good-

enough mothering or parenting is a unifying concept that applies in each perspective and could be especially useful in my work with college students.

Different perspectives on teacher-parenting. Looking again at Janet, self psychology's perspective would prompt me to examine her confidence level in relation to her mathematics achievement to gauge the state of her mathematics self-esteem. When I found that she expressed unrealistic underconfidence given her achievements I would speculate that her early (and subsequent) teacher-parents failed to adequately nurture her developing mathematics self. As her counselor I would explore this speculation with her and look for ways to re-parent her mathematics self now. I would find and help her recognize and receive as her own her existing competencies and understandings (through mirroring). I would expect and push the development of further competencies and understandings by initially allowing her to idealize and rely on me but progressively challenging, frustrating, and disappointing her so that she would become more and more reliant on her own competent self.

An object relations perspective would lead me to clues to Janet's internal mathematics relational life. I would now look for evidence of internalized teacher presences, her use of repression as a defense, her moral conversion of herself as bad to keep her bad teacher good, or other unconscious defenses in the face of her experiences of trauma in relation to teacher-parents (or parents as teacher/tutors). The teacher-parenting central to this perspective is what the child experienced as traumatic. The discrepancy between how she now relates to her current teacher and/or tutor, and how they are in reality, is a clue to the influence of internalized realities. Applicable counseling interventions would involve the counselor's providing herself as an especially

“good” teacher-parent and helping the student to become consciously aware of the “goodness” of the current classroom teacher so that she can safely let go of detrimental internalized teacher presences and incorporate instead the “good” teacher and the “good” counselor.

From the perspective of attachment theory, I would notice Janet’s occasional wariness, and her intermittent dependence on me. She had little apparent relationship with her classroom teacher, and lacked confidence in how she did mathematics even in the face of good results. Taken together, these seem likely indicators of insecure attachments to mathematics teachers and to mathematics. These attachment patterns point to failure of past teacher-parents to provide a teacher secure base and a secure base in mathematics.

Early teachers may have provided a secure enough base only to have that disrupted by later teachers. In the case of Janet it seems that she had certainly not had an early mathematics teacher who offered her the cognitive and emotional support, challenge, and latitude for exploration that she needed to develop secure attachments to teachers or mathematics. As a consequence, she had developed ambivalent patterns of relationship. As a counselor I would provide myself as a consistent, safe secure base, nevertheless challenging Janet, and pushing her to move away and explore and make mistakes so she could experience returning to the base to find it secure and accepting.

While all three dimensions of a student’s relationality should be the objects of a counselor’s curiosity, it is likely that any particular student’s mathematics mental health problems might be based more firmly in one of the areas than in the others at the time of the brief counseling. Mitchell (2000) shows that as longer-term relational conflict therapy

proceeds and difficulties in one dimension are resolved, difficulties that emanate from other dimensions will likely emerge to be dealt with.

Good-enough teacher-parenting. A good-enough mother, like a good-enough teacher, provides sufficiently for the child to get a good start in life by adapting adequately to the child (or student) and her needs (St. Clair, 1990). This is an empowering acknowledgement of the inevitable imperfections in parenting or teaching that are nevertheless tolerable (or even necessary, within appropriate limits), for the healthy development of the student's self. Even if a student had experienced mathematics classrooms as bad, had low mathematics self-esteem, and viewed her prospects as bleak in the current class, I believed it was likely that we could find instances of good-enough teaching and understanding so that some of the bad could be appropriately reinterpreted and re-experienced as good-enough, providing bases for hope and progress. I use good-enough to refer to the present, not only to current teaching and tutoring/counseling conditions for the student but also to her process, progress, and outcomes. If the student and I can let go of a perfect-or-nothing requirement and instead embrace good-enough for ourselves, each other, and the teacher, we could perhaps make good-enough progress and the student could achieve good-enough success.

Janet's Relationality Summary

If I had offered Janet mathematics relational counseling, it would have involved the kind of mathematics tutoring designed to help her recognize, draw on, and develop her mathematical understandings and strengths while simultaneously attending to, processing and dealing with her affective and cognitive symptoms of difficulty. A newly developing self-esteem would likely have led to changes in her ways of seeing her mathematics self, improvement in the way her internal mathematics world was

configured and in repaired attachments to the tutor, the teacher and mathematics. This new freedom from formerly constricting relational patterns could lead to progress in alleviating her negative symptoms, maximizing her mathematics potential, and achieving good-enough success.

CONCLUSION: RELATIONAL CONFLICT THEORY AS A BASIS FOR MATHEMATICS COUNSELING

Relational conflict theory had given me a way to explore how a student's self-esteem and her beliefs, habits, ways of relating, and behaviors may be related to each other. How the three dimensions of her relationality interact (her self, her internalized presences, and her interpersonal attachment patterns), and the relational patterns she employs to express that interaction give me the understanding I sought. Relational theories point to some ways to identify and resolve her central conflict (Luborsky, 1976; Luborsky & Luborsky, 1995) and free her from the counterproductive relational patterns limiting her progress. My adaptation of Mitchell's (1988, 2000) relational conflict theory had given me a new way of looking at the student and at our relationship. I determined that it was an approach that could include the insights and best practices of traditional mathematics tutoring within a broader and deeper relational counseling framework (see Appendix A for a chart summary of the proposed mathematics relational counseling approach).

In the next chapter I show how a relational conflict counseling approach could be used appropriately and integrated with best practice mathematics tutoring in the setting of the learning assistance center.

ⁱ In even numbered chapters, I use “she,” “her,” and “hers” for the third person generic singular.

ⁱⁱ Davidson (1983) found strong links to hemispheric preference and clearly defined analytic (her Mathematics Learning Style I—left brain) and global (her Mathematics Learning Style II—right brain) learning styles in terms of students’ mathematical behaviors and approaches. Although these learning styles have not been found to be directly related to mathematical achievement, Krutetskii (1976) found that students who had a strong learning style preference and a relative inability with their other mode, found it difficult, if not impossible, to begin a problem using the other mode’s approach. Thus being forced to approach problems using another’s preferred approach greatly disadvantages these students. Learning flexibility, however, can strengthen performance.

Students with an analytic/Mathematics Learning Style I use predominantly verbal-logical methods to solve problems, use deductive approaches, and prefer to follow step by step procedures. Krutetski labels them analytic. Students with a global/Mathematics Learning Style II use predominantly visual-pictorial, inductive reasoning methods to solve problems and may know the answer to problems without being able to explain how they arrived at it. Krutetskii labels them geometric. I hesitate to use Krutetskii’s term geometric because although he and others have generally found some relationship between success in geometry and this visual processing right-brain preference learning style, analytic learners also achieve success in geometry courses. Success in geometry is therefore not a clear indicator of a global learning style.

ⁱⁱⁱ To explain the difference between procedural and conceptual teaching, I offer the following example. Teaching the factoring of a trinomial $X^2 + 7X + 12$ by finding factors of 12 that sum to 7 and putting those into $(X + \quad)(X + \quad)$ gives a procedure that may be memorized but probably neither linked with prior learning nor generalized to a more complex factoring problem such as $3X^2 + 16X - 12$. Thus a new procedure must be learned for this one, such as breaking apart $16X$ in a way that the coefficients multiply to equal -36 (i.e., 3×-12); then factor by grouping. Conceptual mathematics, as the term infers, is taught and learned as concept-based processes that put less load on rote memorization and are more easily generalizable to new more complex though related problems. For example the factoring of the trinomial $X^2 + 7X + 12$ using a conceptual approach might be linked with the earlier process of multiplying binomials $X + 3$ and $X + 4$ (and the geometric relationship of multiplying the length and width to get the area of a rectangle), and still earlier distributive explorations of operations on number using two digit by two digit multiplication (also area of a rectangle). The relationship between multiplication and division would be explored and equivalence of division with factoring made clear. Finding then that $(X + 3)(X + 4) = X^2 + 7X + 12$, and relating this with the idea that $23 \times 24 = 400 + 60 + 80 + 12$, that is, $20^2 + 3 \times 20 + 4 \times 20 + 3 \times 4$, leads students to explore the relationship between the 3 and 4 in the factors and the 7 and 12 in the product and to further explorations and discoveries that are applicable to other problems.

^{iv} That is, not specifically related to a mathematics course the student is currently taking.

^v Equivalent to what Hiebert (1986) and others refer to as “procedural” when they discuss mathematical understanding.

^{vi} Equivalent to what Hiebert (1986) and others refer to as balanced “conceptual” understanding with the requisite procedural knowledge.

^{vii} Measured on Richardson and Suinn’s (1972) *Mathematics Anxiety Rating Scale* (MARS).

^{viii} Hypothesis testing of a faulty belief might involve having a student who believes that people who do well in math just see it immediately and do not need to work, interview some high achieving math students who *do* have to work hard to understand and achieve.

^{ix} A cognitive restructuring exercise might involve having a student who sees herself a “bad” at math and points out as evidence the errors on her quizzes and tests and any overall poor grades, develop the practice of noticing instead not only the questions she did correctly but also her sound thinking even in the

questions she got wrong. If she combines this new practice with seeing the link between insufficient or inefficient preparation and her poor results, if she changes her preparation, and if she begins to see a change in her results, her overall approach should change and her perception of herself doing math should also improve. She has experienced cognitive restructuring.

^x A person whose locus of control in a mathematics learning setting is external is likely to attribute her achievement outcomes to factors that she feels she cannot change or control such as luck, the teacher, the tutor, the weather, her health at the time, her lack of intelligence in mathematics (that she believes is a fixed trait), etc. On the other hand a person whose locus of control in a mathematics learning setting is internal is likely to attribute her achievement outcomes to factors that she feels can change or control such as her own effort, her intelligence in mathematics that she believes can improve, getting the support she needs, strategic planning for tests, etc.

^{xi} A student whose motivation for achievement is primarily performance is focused on passing or getting a particular grade rather than on understanding the material. In contrast a student with learning achievement motivation is primarily focused on understanding the material.

^{xii} Others have looked into the unconscious and psychoanalytic symbols found in mathematics and discovered there the roots and explanations for panic, aversion, and defenses against mathematics (cf., Nimier, 1993; Tahta, 1993).

^{xiii} Object is used here in contrast with subject. In other words the object is the “other” in contrast with the subject, which is the “self.” According to object relations theory, early significant others become internalized in various healthy and unhealthy ways as internalized presences that influence how the person relates to others subsequently. I prefer to use the terms “other” or “presence” rather than “object” because “object” now has somewhat negative connotations implying a sense of persons as things.

^{xiv} [P]sychopathology, in its infinite variations, reflects our unconscious commitment to stasis, to embeddedness in and deep loyalty to the familiar... we experience our lives as directional and linear, but like Penelope... we unconsciously counterbalance our efforts, complicate our intended goals; seek out and construct the very restraints and obstacles we struggle against. (Mitchell, 1988, p.273)

^{xv} Freud’s view is that a person’s choices are largely determined by unconscious instinctual drives and forces outside of her conscious control. Recognizing that the person, in contrast, is responsible for her choices and actions, implies that helping her become conscious of her hidden motives should provide both more insight into puzzling behaviors and also the possibility of modifying hidden motives in light of conscious goals. The consistent relationship between academic/mathematics achievement and locus of control (see Nolting, 1990, McLeod, 1992) is pertinent here. Students who fail to see their own responsibilities in achieving success in a course, holding others or external factors responsible instead, consistently achieve less well than those who own that responsibility (internal locus of control).

^{xvi} Usually seen as the mother although the role rather than the gender is the central factor. Winnicott (1965) conceives of the function of mother as providing experiences to make possible a sense of authenticity and reality; that is, to provide “good-enough” mothering that leads to “maturity and the capacity to be alone ... [and] a belief in a benign environment”(p.32). Kohut (1977), in expanding Winnicott’s findings, also sees that the child’s nuclear or core self arises as the result of the interplay between her innate potentials and the responsiveness of the adult selves which the child internalizes as parts of herself.

^{xvii} Usually seen as the father although the role rather than the gender is the central factor.

^{xviii} Piaget (1973) contends that children’s intellect develops primarily through self-directed activity, both physical and mental. He asserts that all learning is “of a constructivist nature ... affirms a continuous surpassing of successive stages... leads to placing all educational stress on the spontaneous aspects of the child’s activity... The basic principle of active methods will have to draw its inspiration from the history of

science and may be expressed as follows: to understand is to discover, or reconstruct by rediscovery, and may be complied with if in the future individuals are to be formed who are capable of production and creativity and not simply repetition" (p.10).

^{xix} When a student tells a learning counselor that she has never been "good" at mathematics, even in first or second grade, we must question her early experience of teacher-parenting. Research findings assure us that barring a severe specific learning disability, developmental delay, emotional disturbance, or physical or emotional abuse or deprivation, the average intelligent child is mathematically capable when she enters school (Caufield, 2000; Hawkins, 1974; Kamii & DeClark, 1985; Kunzig, 1997). She has all she needs to explore and learn developmentally appropriate number and operation concepts and their symbolic representations, along with applications in solving problems based in her real world. If she does not remember experiencing success it suggests many possibilities but most likely is that her early teachers did not mirror her developing ability to do mathematics in a her own way or provide appropriate challenge and frustration to promote her competence.

^{xx} That is, her grandiose (to her, all-knowing and all-powerful) self is challenged and modified by reality.

^{xxi} Kohut (1977) maintains that it need not be specific traumatic events, but rather the chronic absence of the parent's empathic responses to the child's need to be mirrored and to idealize that may lead to pathology of self in the adult (p.187).

^{xxii} Fairbairn (1952) further asserts that

Whether any given individual becomes delinquent, psychoneurotic, psychotic, or simply 'normal' would appear to depend, in the main, on the operation of three factors: (1) the extent to which bad objects have been installed in the unconscious and the degree of badness by which they are characterized, (2) the extent to which the ego is identified with the internalized bad objects, and (3) the nature and strength of the defenses which protect the ego from these objects. (p.65)

^{xxiii} For example, rapping knuckles, pinching, hair pulling. Although these activities are illegal in the U.S., they continue to be practiced, particularly in poorer communities where parents may feel less empowered to challenge school practices.

^{xxiv} Or what Fairbairn (1952) calls the "internal saboteur."

^{xxv} This is a plausible explanation for what I have found to be the puzzling phenomenon of adult students apologizing to me when they find or I point out an error in their work (especially an error in arithmetic) saying, "I'm sorry," almost as if they have committed a sin and deserved punishment. Evidence of their badness has been revealed and the effect is invariably shame. Are they ashamed because their early teachers shamed them when they made such mistakes? Or are they ashamed that as adults they have revealed incompetence at something a young child *should* be able to do?

^{xxvi} It is important here to distinguish between what actually happened, that is, what the teacher did in the classroom, and how the adult student now remembers cognitively, affectively, and overall relationally experiencing it. The former is impossible to verify and is not as relevant as the latter which is what is affecting her now.

^{xxvii} Fairbairn (1952) referred to the internalization of good objects only in terms of the super-ego and the development of principles and values much as Kohut (1977) saw the healthy modification of the parent image. Bad objects on the other hand were internalized and interacted with the ego (operating part of the self) causing conflict and splitting, that is, trouble when they were repressed or otherwise dealt with internally.

^{xxviii} Although this person is most often the child's biological mother, others, including the father or other relative or unrelated person may be the mother figure for the child (Bowlby, 1982).

^{xxxix} There is some evidence that children of mothers who themselves suffer from unresolved attachment trauma or loss are likely to develop this disorganized attachment (Main & Hesse, 1990). It seems that many of the attachments formerly identified as ambivalent may be more accurately identified as disorganized.

^{xxx} Bowlby distinguishes the concept of “self-reliant” from that of “independent,” pointing out the cultural stereotype of an independent person as one who relies only on self and repudiating or not needing the help of others (Bowlby, 1973). Bowlby’s concept of self-reliance is closely linked with Werner and Smith’s (1982) concept of resilience and Lillian Rubin’s (1996) concept of transcendence. Werner and Smith found that a key to a child’s resilience under difficult circumstances was her significant relationship with an accepting, approving, and challenging adult. Likewise, Rubin found that adults she studied who had transcended abusive childhoods had all had such a relationship with an adult as a child, that had enabled them to survive emotionally and become self-reliant adults themselves.

^{xxxi} Disordered adult attachment behavior patterns linked with early insecure or interrupted attachment relationships include

1. Anxious attachment, characterized by over-dependence or clinging and severe separation anxiety, thought to be linked to threats of abandonment by the childhood mother figure or to her forcing the child to take on the caregiving role,
2. Insistent self-reliance, characterized by an apparent lack of any need for relationship or assistance, thought to be connected with early rejection or prohibitions on expressing emotions or needs as a child,
3. Insistent or anxious caregiving, typified by exclusive formation of one-sided relationships in which she is always the caregiver, thought to have developed from the experience of the mother figure’s expecting the child to mother her.
4. Detachment, characterized by emotional detachment and an inability to form stable bonds, stemming from separations from the mother figure that were severe or prolonged (Bowlby, 1980; Sable, 1992).

^{xxxii} Not all important relationships are attachment relationships. Attachment relationships, even for adults, are characterized by “proximity seeking [to the attachment figure], secure base effect, and separation protest” (Weiss, 1991, p.66). They contrast with community relationships “that link individuals to networks of fellow workers, friends, or kin” (p.68) that also likely characterize peer relationships in a college classroom.

^{xxxiii} Katz (2000) describes seven role dimensions where there are important distinctions between parenting and teaching. They are: scope of function, intensity of affect, attachment of adult to child, rationality, spontaneity, partiality, and scope of responsibility (p. 11). Of particular interest here are the dimensions of attachment and partiality. Katz proposes that whereas parenting should be characterized by optimal attachment with the child (essentially, secure attachment, appropriate caregiving), teaching should be characterized by optimal detachment, or “detached concern,” to use Maslach and Pines’ (1977) term, characterized by appropriate responsiveness and caregiving without the intense emotional involvement of parental attachment (whether and how this optimal detachment is to be achieved may be related to the teacher’s own attachment history and to resolution of transference and countertransference issues). With regard to the partiality dimension, the parent’s role is to be partial, biased towards her child; the teacher’s role is to be impartial, unbiased in relation to any one child but biased in her relationship with the class as a whole.

These teacher and mother roles work best for the child if they are age-appropriate and complementary. The teacher “is seen as wiser [academically] and stronger [in relation to classroom management] and therefore able to be protective at times when the self seems inadequate” (Weiss, 1991, p.68).

^{xxxiv} What Ma (1999) calls “a profound understanding of fundamental arithmetic.”

^{xxxv} The effects of a laissez-faire classroom may be detrimental. Poorly planned discovery learning situations where student are expected only to explore without knowledgeable and strategic teacher guidance

and support are unlikely to result in much mathematical learning (cf. G. Hein, personal communication, September 1994). They are very likely to result in knowledge base gaps and insecure attachment to mathematics.

^{xxvii} A cognitive constructivist problem-solving situation is likely to increase emotionality and jeopardize a student's relational attachments to teachers and to mathematics if not managed explicitly by the teacher (McLeod, 1992; Szetela, 1997; Windschitl, 2002). In such a situation where the student is expected to struggle over time with problems (with the teacher as guide or coach) she is likely to experience a range of emotions that includes frustration and anxiety. If the teacher helps her to expect this emotionality as a normal part of real problem-solving and to interpret and use it as a positive force in her process, her attachments to mathematics and mathematics teachers should strengthen, especially if the instructor provides herself as a reliable secure mathematics base whom the student can consult.

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1. The honesty and openness with which the person is prepared for or informed of the loss or separation, is included in the mourning, and is allowed to mourn and to express her mourning over time;
 2. The quality of attachment to the mother (or attachment) figure before the loss or separation; and
 3. The continuity and quality of her relationship with other primary attachment figure/s after the loss or separation (Bowlby, 1980).

^{xxix} Marris (1974) has observed what he calls the "conservative impulse" universally at work in people's responses to loss, separation, and change. He proposes that this conservative impulse is based on the fact that people develop meaning and purpose in the context of cumulative and long developed attachments in relationships and circumstances. He notes that the cognitive process of assimilation of new understandings into a person's existing cognitive schema, observed by Piaget, is similarly conservative. Changes that cause disruptions to these attachments and that do not allow a person's engagement in the struggle to develop new purpose and forge new attachments or assimilate the changes into former attachments, are likely to be met with resistance and rejection, even when the changes can be seen by outsiders to be for the good (Ginsburg & Oppen, 1979; Marris, 1974; Piaget, 1967).

^{xxx} Reactions to separation or loss of attachment figure, or change impacting attachment bonds, have been found to follow a common bereavement process, beginning with

1. *protest*, involving confusion and searching for the lost object, sadness, yearning, anxiety, and anger towards the lost attachment figure or agent of change, then
2. *despair*, depression, and disorganization, to
3. *detachment* from the attachment figure as defense, and finally to
4. *acceptance* of the loss (with ongoing sadness) if it is permanent, or to repaired attachment (typically accompanied by anger, distrust, and anxiety) if the attachment figure returns and resumes caregiving (Bowlby, 1980).

^{xi} A child's transitional object was typically a physical object such as a soft blanket used to smooth the sometimes painful transition from complete dependence on her caretaker to her own autonomy (Winnicott, 1989). I speculate that the emotional role of fingers, counters, and other manipulatives or physical mathematical models, may be to function as transitional mathematical objects. These objects often smooth the transition from externally verifiable to internally known mathematical understandings and they may be comparable to a young child's transitional object (or "blankie").

CHAPTER III

A NEW APPROACH: BRIEF RELATIONAL MATHEMATICS COUNSELING

In chapter 1 I describe the problem of mathematics support center professionals not having what is needed to adequately help many typical college students to succeed. In chapter 2, I discuss the scholarship that led me to a hypothesis that relational counseling in conjunction with best-practice mathematics tutoring might address this problem. In this chapter, I show how I generated a counseling approach by adapting the theories I had studied to the realities of my practice as a learning center tutor. I describe how I used relational conflict psychoanalytic theory as a basis for understanding best-practice traditional mathematics support insights, as an approach to the student-tutor/counselor relationship, and as a remedy to difficulties standing in the way of student success in a learning support center context. I explain how the most important theoretical underpinning of my approach—relational counseling—can be applied in a mathematics academic support context to give a new way of looking at a student and at the tutor-student relationship.

Drawing on my understanding of mathematics affective research findings and cognitive therapy I describe the development of tools designed to facilitate my understanding of students' affective and cognitive mathematics difficulties. I use the key terms from relational counseling that I redefined in chapter 2 in the context of mathematics learning to show how relational counseling approaches may be used to elucidate how symptoms are related, their underlying causes, and possible treatments. Finally, I summarize ways mathematics tutoring and relational counseling can be integrated in practice by describing roles of key participants.

THE CHALLENGES OF ADAPTING RELATIONAL MATHEMATICS COUNSELING TO THE LEARNING CENTER CONTEXT

The Therapy Approach and the Problem of Time Constraints

How could a relational counseling approach be offered appropriately and effectively in the college setting? Practical consideration led me first to consider time and institutional constraints: For my purposes a major limitation of the relational conflict approach is the necessary long-term nature of the therapy. A typical college semester is usually 15 or 16 weeks long. Realistically, potential contact time with a student is likely to be considerably less unless the student begins the semester conscious of hisⁱ need for assistance. Typically students recognize a need for support after the first quiz or exam which may be several weeks into the semester. I wondered if the short time available would be sufficient for a tutor to gain the in-depth understanding of the student that a relational approach promised. I was also concerned about the appropriateness of a therapeutic approach in an educational setting.

Given the educational setting, counseling, with its problem-centered approach and counselor teaching/talking emphasis, seemed on the surface more appropriate than therapy, which has a person-centered approach and relatively long-term investigative emphasis based on close listening (Corsini, 1995) . I considered the dual focus of relational mathematics counseling: mathematics tutoring and counseling. On the one hand mathematics tutoring is more problem-centered like counseling, since the focal problem is the student's understanding and ways of doing mathematics. Here the tutor is an expert in mathematics and takes on a coaching role as the student constructs new understandings from his already existing knowledge. On the other hand, relational conflict theory has generally been seen to involve client-centered therapy rather than

counseling. An approach to mathematics relationality then should be like therapy. It is person-centered, with the student considered more expert than the counselor in his own experiences, his personality, and his relationships. Here in contrast, the counselor's expertise is in investigating, listening, and interpreting how these explain the student's central mathematics relational conflict that needs resolution. While I call my developing approach relational mathematics *counseling*, it could perhaps be more accurately described as an integration of relational therapy into mathematics counseling.

I was aware of brief therapies, but most were problem- rather than person-centered, like cognitive therapy, and I wanted an adaptation of relational therapy that was both problem- *and* person-centered. This adaptation exists in Stadter's (1996) brief object relations approach. Although some relational (psychodynamic) psychoanalysts resist shorter courses of therapy for all but narrowly specified problems (cf. Sifneos, 1987), brief therapy models such as Stadter's (1996) do apply relational (object relations) counseling to time-limited settings. Brief therapy incorporates cognitive counseling techniques and differentiates between the ongoing relational focus and the more immediate symptomatic focus. A brief relational mathematics therapy approach needs to incorporate the three relational dimensions, integrate pertinent CT/CBT approaches, and allow the immediate focus to be on the learning of mathematics. Such an overarching mathematics counseling framework could yield a nuanced understanding of students' mathematics mental health that could lead to treatment in the limited time available in college settings. It contains all the elements of an explanatory framework that can be used to understand and support mathematics cognition and affect in the context of students'

mathematics relationships. Such an approach could appropriately be offered through the academic support center.

The Use and Misuse of Assessment Instruments

As with best-practice traditional tutoring, the initial task in relational mathematics counseling is to understand the student, his understanding, and his approach to mathematics well enough to formulate an effective course of action. Understanding must be followed quickly by effective and flexible implementation of the course of action making constant adjustments in response to new insights and feedback from the student, the results of assessments, and effectiveness of approaches. A relational counseling approach differs from traditional practice, however, in how it changes the support professional's ways of looking at himself, at the student, and at their relationship during the tutoring process, as well as how it expands the scope of inquiry when investigating and intervening in the student's mathematics learning.

A traditional approach to assessing or diagnosing a student's mathematics functioning is to use formal and informal paper-and-pencil assessments. These are generally used to identify the student's level on pertinent factors such as his mathematics affect and his aptitude, achievement, and/or developmental level on the mathematics to be attempted in the course.

Cognitive assessments I had used included in-house mathematics placement instruments, Scholastic Aptitude Test (SAT) quantitative scores, and in-class tests and quizzes. To assess affective orientation and identify possible affective symptoms of mathematics difficulty I had previously used a number of diagnostic past-experience questions, mathematics affect, and orientation surveys that explored students' beliefs,

attitudes and feelings. In researching for this study, I became familiar with other instruments. My first inclination on lighting upon relational conflict theory as my framework was to abandon these instruments and surveys, principally because of my frustration with not knowing how to prioritize, understand, and use the data they gathered. I quickly realized however, that given the short time available in a semester worked against the relatively time-consuming relational therapy approach to data-gathering so efficient data-gathering instruments would be necessary. Importantly, I realized that the relational conflict framework was my key to prioritizing, understanding, and using the data gathered by these instruments: Far from abandoning them, it seemed that my new approach *required* their use.

I looked for assessments to help students become conscious of their present condition with respect to their mathematics learning, both affective and cognitive, and become aware of what that revealed about their established relational patterns. I was aware that a new approach might deeply challenge not only the traditional uses of assessments but also students' conceptions of what assessments could and could not say about them.

Stephen J. Gould (1981) writes:

Few tragedies can be more extensive than the stunting of life, few injustices deeper than the denial of an opportunity to strive or even to hope, by a limit imposed from without, but falsely identified as lying within. (pp. 28-29)

Here Gould refers to the historical use of psychometric "biological labeling" to define and limit the intelligence or abilities of groups or individuals in the U.S. I expected that in introductory college mathematics-related classes in the U.S., there would be students

who had been subjected formallyⁱⁱ or informallyⁱⁱⁱ to such a denial of opportunity based in a limit imposed by inappropriate interpretation of testing results in mathematics. I had seen the effects of this denial to be affective, cognitive, and also relational. It negatively impacted a student's overall mathematics functioning, that is, his mathematics mental health. Accordingly I looked for assessment tools that could help the student become consciously aware of his mathematics limits, of his beliefs about those limits, and of his attitudes, emotions, and relationships related to his limits so that we could explore and detoxify the source of deceptive limits and constructively deal with real ones. I determined that in any use I made of assessments I would keep central the possibility—indeed, the expectation—of changes over time in the assessments for each student.

WHAT THE RELATIONAL MATHEMATICS COUNSELOR NEEDS TO KNOW ABOUT THE STUDENT

Three avenues of inquiry emerged as important when I considered what information I needed early in the process to begin to understand a student's mathematics relational patterns and provide a way of discussing those with him. First, I wanted to capture the student's sense of where he had come from mathematically, where he was now, and what he thought were his key issues. Second, I needed to know how the student was actually dealing with this mathematics class, the course instructor, and the content, both in the classroom and out. Third, I hoped to find ways to see myself in relationship with the student, and him in relationship with me to inform my interpretation of the first two.

Determining How the Student Sees Himself

In order to explore the student's sense of his mathematical progression, his current placement, and what he considers his key mathematical issues, a two-pronged

approach seemed feasible: (a) First, during counseling sessions, I would use direct and indirect questioning to analyze his mathematical orientation, approaches, and background, and (b) second, outside of the counseling session (e.g., in class, for homework), I would use strategic self-report surveying of factors I considered pertinent to a student's mathematics relationality, such as his beliefs, attitudes, and feelings around mathematics that I thought might be difficult to systematically gauge during counseling sessions. I could use his survey responses in counseling as a vehicle to focus on issues that might not otherwise arise.

During the Counseling Session

Mathematics background and experiences. In mathematics relational counseling, taking personal history that focuses on the person's experiences with significant mathematical others is likely the first essential to establishing a suitable relational focus and to a proper understanding of how to deal with his particular mathematics learning needs (for relational therapists' use of history taking, cf. Luborsky & Luborsky, 1995; Stadter, 1996). In mathematics anxiety reduction clinics (cf. Tobias, 1991), in some academic support settings, and in research studies (cf. Mau, 1995), it has been standard practice to invite adult students to tell or write their mathematics learning histories or autobiographies to explore their present negative affect in the mathematics learning situation, but is rare in the context of a college mathematics course, either in class or in tutoring because of time pressures to focus on course content. Since my new approach required it, however, I developed a *Mathematics History Interview Protocol* (see Appendix A) based on findings from a qualitative research study I conducted into college students' mathematics identity development and from my subsequent mathematics

academic support work with college students (Knowles, 1998). Important areas of inquiry include not only relational experiences with teachers, parents, peers, and others, but also which completed high school and college mathematics and areas of self-perceived mathematics competence and incompetence. I expected that because of time pressure and the urgency of the current mathematics course focus, this history will probably need to be gathered over several sessions, and history gathering would need to be integrated into the ongoing mathematics tutoring process so that students can see its connection and relevance to their current mathematics objectives.

Metaphor. “Metaphors are concrete images that require us to find the threads of continuity and congruence between the metaphor and the primary subject” (Deshler, 1990). The primary foci for students in mathematics counseling should be mathematics and themselves as mathematics learners. In my previous practice, I had asked students write metaphors for their experience of mathematics but I had not known how to explore beyond the obvious “threads of continuity and congruence” with students’ mathematics learning such as personal affective orientation to mathematics or beliefs about what mathematics is. Now I realized that metaphor might also give students access to their underlying *relationships* within their mathematics learning in an open-ended, indirect, imaginative way. The relational perspective gave me a way to explore a metaphor with a student, noticing clues to his sense of mathematics self, his internalized mathematics presences, and his mathematics or mathematics teacher attachments. I could see how a student’s metaphor might provide a unifier or common thread to piece together other data to understand the student’s central relational conflict patterns. In counseling, I needed to express my assumptions about his intended meaning in order to have the student clarify

or amend my perception. Such joint exploration seemed likely to unearth underlying and possibly unconscious relational connections. During and at the end of the course of counseling, students could reconsider their initial metaphors to see whether and how they had changed and what, if any, changes might signify with regard to outcomes of the course of counseling.

Mathematics negativity. In the mathematics learning situation, students with negative beliefs about their mathematics world, their mathematics selves, and their mathematics futures^{iv} tend to exhibit symptoms more like those of situational depression than the more commonly assumed anxiety. This mathematics “depression” can be debilitating in the learning situation, and students thus afflicted seem quite likely to give up quickly, withdraw, or fail. The severity of the negative outlook may change from week to week and, with that, the student’s energy to struggle with the coursework, in an inverse relationship between energy and severity of negative outlook. Dweck (1982), Beck (1977), and others have found that having a person articulate her negative self-statements may be the first step recognizing their irrationality and changing them. Having clients respond to questions about their world, themselves, and their future *each time* they met with a counselor has been found to help them and the counselor tackle negative self-statements in an ongoing and timely manner (Beck, 1976, 1977). Therapists using this method were also able to gauge the severity of the client’s negativity/depression and sometimes to prevent him from harming himself (Al-Musawi, 2001; Simon, 2002; Sprinkle et al., 2002). Analogous to this self harming in the mathematics learning situation is a student’s sabotaging his chances of success by avoiding work or even the mathematics class when his negativity and hopelessness become overwhelming.

In order to help my students become conscious of their thinking so they could consciously deal with it rather than withdraw, for this study I developed a set of line scales, each of which allows a range from positive to negative responses about the student's current mathematics course, self, and future that week (*JMK Mathematics Affect Scales*, see Appendix A). At *each* session the student will fill in the scales, and we could compare his responses to previous ones and discuss changes in relation to external circumstances, his progress with the course, and thoughts about himself. We might look for connections with his relational challenges and use this as feedback to help clarify the focal relational conflict he is working to resolve. We might discuss changes in routines and in his thoughts about himself that he might try to implement over the following week in response to the current evidence.

Outside the Counseling Session

The traditional means for finding out how a student sees himself as a mathematics learner has been the self-report affect survey. Surveys requiring responses on a five- or seven-point Lickert scale can be administered quickly in class or as a homework assignment. I wondered if I might collect such data on affect that could help provide a fuller picture of the student that could be missed if I relied solely on conversation in the counseling session.

Researchers have found two major areas of affect that interact directly with mathematics cognition (albeit in complex and not always explicable ways) (cf. McLeod, 1992). They are mathematics feelings (specifically, anxiety) and mathematics beliefs/attitudes (and attributions based on these beliefs/attitudes). I determined that I needed to find ways of observing or measuring students' levels of anxiety and

helplessness in new learning and testing situations (possibly indicating a damaged or underdeveloped mathematics self) as well as curiosity and mastery orientation (possibly indicating a healthy mathematics self) (Carter & Yackel, 1989; Skemp, 1987). I thought a survey of each of these two areas—feelings and beliefs—that investigated key factors linked with mathematics understanding and achievement might provide important points of discussion and clarification in counseling. In addition, if used as a pre- and posttest, it seemed possible that such surveys might reveal movement or change over the course of counseling. I weighed the limitations of such surveys (e.g., closed questioning, insensitivity to precision or depth or range of actual student feelings or beliefs) against their benefits (e.g., quick assessment [using small constellations of items] of research-confirmed key factors, and links with a student's underlying and overt relational patterns) to assess what and whether surveys of affective issues could be helpful in the counseling process.

Finding Out How the Student Does Mathematics Now

Mathematics diagnostics. Because I conjectured that students' mathematics relational challenges (especially their sense of mathematics self) might be closely linked with poor attachment to mathematics, I looked for diagnostics that could be administered in class or during a counseling session that could discern between perception and reality *and* that were linked closely enough with current course content to be useful guides to appropriate relational conversation.

Whatever the emphasis of an introductory college mathematics course, arithmetical prowess in number (small and large) and operation sense and the student's understanding of the algebraic variable seem to be pivotal areas to be explored. I

surmised that strategic use of an arithmetic and/or an algebra diagnostic could help both student and counselor better understand the affective and cognitive impact of the student's mathematics learning history. Once we had that information the student and tutor could jointly plan strategic mathematics interventions for this course (see Appendix B for the assessments I devised or adopted : *Arithmetic for Statistics Assessment*, Knowles, 2000; the *Algebra Test*, Sokolowski's, 1997, adaptation of Brown, Hart and Kuchemann's, 1985, *Chelsea Diagnostic Algebra Test*)

If the course had a specific applied emphasis (e.g., statistics) I wondered if a specific diagnostic of that application could also be helpful. For an example see Appendix B for Garfield's *Statistical Reasoning Assessment* used in my pilot study.

Mathematics course achievement. Mathematical tasks required in the course are naturally central in counseling. Students react to the grades they receive on course assessments—exams, homework assignments, projects—differently, I surmise, because of differences in their background experiences and relational challenges, and they also react differently to these grades. A pivotal challenge in counseling is to analyze a student's products with him in a way that helps him interpret his grades constructively. The counselor must try to understand his reactions and to help modify them if necessary in order that the student will approach the next assessment with a sense of responsibility and with a developing sense of his mathematics self. In this testing situation, the student feels most acutely that his mathematics self is being judged. He may evidence conflicting motivations and behaviors (e.g., wanting to succeed but also wanting to protect a vulnerable sense of mathematics self by not trying, so as to avoid judgment of ability).

These heightened conflicts are likely to become clearest during exam analysis discussions, so these discussions create special opportunities for relational counseling.

Mathematics practices and behaviors. How the student actually does mathematics may differ from how he perceives himself doing it. He may do it differently in different settings, and the counselor's observations and exploration of discrepancies should make the student aware of approaches that he may need to modify. The settings where the student does mathematics include the classroom, his home or dorm, and the learning support center. Typically the mathematics counselor can observe the student directly only in the learning support center, although he may be able to arrange classroom visits and/or receive instructor observation reports (with student permission). It would seem however that counselor observation of the student doing mathematics in different settings, particularly in the classroom could be crucial for a clear understanding of the student's mathematics relational issues.

The Student-Counselor Relationship

As I envisaged relational mathematics counseling, I realized that my relationship with the student and his with me could be vital to understanding his core challenges, but only if I purposely made our relationship a central object of inquiry and even, at times, a topic of discussion. I noted the pivotal place relational therapy gives to the client's transference of past analogous relationships to his relationship with the counselor and the counselor's countertransference responses to the client, acknowledging that much of the client's relationality is discernable through understanding and interpretation of this interchange. When I considered how I might integrate this observation and analysis of our relationship into what the student understands to be essentially a mathematics tutor-

tutee relationship, I realized that if I self-disclosed when I became aware of my own countertransferential impulses and asked about the student's sense of what was going on and who they thought I should be and what they thought I should be doing when I became aware of being other than who I was, we might establish a place for exploring what it might signify about their mathematics relationality. This approach seemed appropriate in the learning support setting, but I was aware making countertransference and transference issues explicit and be explored would likely differ markedly from student to student. However, I could now admit my own countertransference and my experience of the student's transference as data regardless of whether explicit discussion with the student felt appropriate. As I considered the importance of transference to a relational mathematics counseling approach, it also became clear to me that I needed to arrange supervision meetings in order to review and assess my transference-countertransference interpretations with a person knowledgeable in counseling psychology.

Understanding the Student's Mathematics Mental Health Conditions

My interpretation of a relational view of mathematics mental health holds that a student's relational patterns are adaptive. That is, he has developed ways of relating to mathematics, instructors, and required mathematics courses that serve his sense of mathematics self. His adaptations to mathematical circumstances may be conducive to growth and positive development; they may be detrimental and skew or stunt his development; or they may be somewhere in between. A student's state of mathematics mental health may range from sound to poor, depending on the sense of mathematics self he is attempting to maintain and the extent of conflict between contradictory goals he is experiencing.

I had noted certain conditions or sets of indicators (or a syndrome) that could be used to describe a student's state of mathematics mental health. These conditions could be manifested as cognitive, as affective, or both. I believe that these may be best understood in the context of a student's mathematics relationality. Understanding these conditions or sets of indicators seemed key to helping a student focus quickly on his core relational challenges.

Mathematics Cognitive Conditions and Relational Counseling

Research and experience have informed me that the cognitive conditions most likely to negatively impact college students' achievement are: (a) a procedural approach to mathematics learning, (b) the lack of a "profound understanding of fundamental arithmetic" (L. Ma, 1999) primarily number and operation sense, (c) weak connections between arithmetic and algebra; (d) underdeveloped understanding of the algebraic variable, (e) poor or counterproductive problem-solving strategies and monitoring and control skills, (f) poor course management skills or (g) any combination of these. As a mathematics counselor I would have to not only assess a student's cognitive standing, considering these categories, but also consider their impact on the development of his mathematics self and his relational patterns. I would then have to prioritize tutoring attention his cognitive conditions in relation to the demands of the course and his limitations.

Cognitive Conditions Related to Personal Cognitive and Environmental Attributes

Students with strong particular learning style^v inclinations may display learning strengths or weaknesses depending on the particular learning environment.^{vi} If there has

been a long-term mismatch between a student's learning style and mathematical learning environments, unless he has been able to be flexible, he may have experienced less success than his potential would indicate, along with an associated loss of confidence in his ability.

Although there is powerful evidence that average children can learn mathematics, many, and especially (but not exclusively) those from disempowered groups, are in classrooms where their ability is judged inaccurately. They are often judged to be lower-ability than they truly are and, perhaps worse, the ability they are considered to have is judged as fixed (Downs, Matthew, & McKinney, 1994; Sadker & Sadker, 1994; Secada, 1992). Most U.S. students have experienced formal or informal tracking into ability groups since the early elementary grades. Likewise students with diagnosed learning disabilities, although cognitively capable, are likely to have been subjected to even lower teacher expectations. Piaget (1973) goes further than Krutetskii (1976) in rejecting the notion that some people have a math mind and many do not, but most U.S. college students have entrenched beliefs about their own math ability that have restricted the development of their ability and led to learning gaps. They may have been put in lower tracks and given less coursework in high school, and they may have taken fewer courses thus jeopardizing their achievement in college (cf. Sells, 1976; Schoenfeld, 1992).

There are complex relationships among students' race, language, ethnicity, SES, and gender, and their mathematics achievement (Secada, 1992). There is no credible evidence that any of these factors or combination of factors affect potential to succeed. There is, however, consistent evidence that schools' differential financial resources, school cultures, and teacher race and ethnicity, attitudes, and expectations negatively

affect persistence in mathematics course-taking, achievement, and especially the academic confidence of students from disempowered groups. Students from a disempowered minority group who have been schooled in a majority setting where teachers who are predominantly of the dominant culture is likely to experience minimal respect for his own cultural norms or for the non-English language he speaks. Should this be true, he has likely experienced minimal mirroring from the teacher and insufficient support for his budding mathematics self.

The development of a student's mathematics self, is affected by myriad personal and environmental factors and their interactions. Students with underdeveloped or damaged mathematics selves tend to blame their difficulties on their own (imagined) intrinsic inability or some other defect because they have been treated as if they are inadequate. It has been relatively rare that a teacher is aware of and takes responsibility for his part in his student's difficulties in learning mathematics. A relational approach to students who have suffered such assaults on their mathematics selves should involve careful attention to what they *can* do mathematically, building on their abilities and understandings using methods compatible with their learning styles, and refuting their "no math ability" theories with evidence of their own work and thinking. In other words, they need teacher-mirroring and support of their vulnerable and undeveloped mathematics selves.

Mathematics Pedagogy and Cognitive Conditions

The mathematics self seems to be the most central dimension in the development of healthy, flexible mathematics relational patterns. The principal means for this healthy development is good mathematics teaching^{vii} in an environment where the student's

mathematics self is accepted, coached, and challenged. When that has not occurred or has occurred intermittently, cognitive symptoms emerge, such as rigid reliance on memorized steps or difficulty in adapting to slightly different wording or appearance that are observable in the adult student's arithmetic, algebra, and problem-solving work in class, on exams, and in the counseling session. How these cognitive symptoms interact with students' affective symptoms and what they tell about the student's overall state of mathematics mental health is investigated in this study. We can expect arithmetical weaknesses and uncertainties to have deeper, more longitudinal and negative implications to the mathematics self (identity) than algebraic weaknesses (if arithmetic is intact). Number and operation sense weaknesses may be especially toxic, depending on their severity and pervasiveness. As an example, Janet's lack of automatic access to her multiplication and addition facts (see chapters 1 and 2) slowed her progress in precalculus and seriously undermined her confidence. However, algebraic weaknesses will invariably also strongly impact present functioning negatively. How cognitive symptoms specifically affect an individual's present ability to learn new mathematical content will be a function of a combination of the course difficulty, the way it is taught relative to the student's needs, the relational and mathematics climate of the classroom, the extra support available, and the way the student's mathematics relational patterns interact with these factors. Vulnerable students may include not only those with cognitive preparation deficits but also some whose cognitive preparation is adequate but who are nevertheless not confident for other reasons.

Researchers such as Skemp (1987) and Buxton (1991) have shown links between affective and cognitive symptoms that have their source in poor mathematics pedagogy.

In particular, predominantly procedural teaching with the teacher as the sole authority on the mathematics leaves the student vulnerable to helplessness and anxiety because he has recourse only to memory or the teacher's logic rather than to the connections he could make himself if he has learned and understood it conceptually.

Cognitive Conditions and Relationality

Attachment to mathematics. Few elementary teachers have what Liping Ma (1999) calls the "profound understanding of fundamental arithmetic" required to understand the problems, and few are able to translate their understanding into practical activities for their students. Thus they have to teach their students procedures rather than concepts. These students tend to develop a narrow procedural knowledge of arithmetic that links poorly with algebra because of the need to generalize beyond procedure to a more abstract statement of relationship. Students' knowledge of and beliefs about mathematics and about themselves doing mathematics may be distorted. If they have not developed a secure attachment to mathematics that can enable them to be flexible and venture into new learning this distortion may be extreme.

EMOTIONAL CONDITIONS AND RELATIONAL COUNSELING

Anxiety

Much of the negative affect that students experience while doing mathematics has been lumped under the label "mathematics anxiety." Educational research supports a relationship between mathematics anxiety and poor performance although that relationship is not unequivocal nor is the effect always significant when it occurs (Hembree, 1990; McLeod, 1992). According to the Yerkes-Dodson (1908) principle (performance related to arousal roughly by an inverted U), students who experience

moderate levels of arousal (whether they interpret that as positive or negative) will do better on a test than those who experience either too little or too much arousal. What exactly mathematics anxiety is and what its causes are have been the matter of much debate and many studies and factor analyses (Ma, 1999; McLeod, 1992). Part of the difficulty is that its etiology, triggers, and expression differ from person to person. A relational counseling approach, I believed, would provide the mandate and opportunity to enable students to reveal and explore these individual differences. But because of semester-long limitations, I wanted an instrument that would differentiate some factors in mathematics anxiety and provide a starting point for discussion with individuals in the counseling situation.

Analysis^{viii} of the literature of attempts to define and measure mathematics and testing anxiety have found a number of dimensions that affect students' performance in sometimes singly, sometimes in combination, and always in relation to other dimensions all in varying degrees. The pertinent dimensions are often agreed to be: (a) the mathematical situations that engender anxiety (e.g., every day life vs. classroom; within the classroom: testing vs. class work versus homework); (b) the type of mathematics involved (e.g., arithmetic vs. algebra); (c) the cognitive precursors to anxiety (e.g., poor exam preparation); (d) whether the mathematics activity is solitary, with peers, or public; (e) to what extent the student suffers from strong chronic anxiety or experiences anxiety easily (trait anxiety); (f) the type and intensity of anxiety engendered by the situation (state anxiety, cognitive worry); and (g) the immediate and long-term physical, affective, and cognitive effects of the anxiety.

I determined that if I understood a student's mathematics affective history and its effects on the different dimensions of his mathematics relationships and, further, if I observed and experienced his resultant relational patterns, I might be able to contextualize his anxiety. I searched among the many formal and informal instruments for one that surveys affective response to mathematics cognitive and situational factors. This seemed particularly urgent because of my perception of the centrality of mathematics cognition in the development of the mathematics self. I chose Ferguson's (1986) *Phoebus* (which I renamed as *My Mathematics Feelings* survey see Appendix B and see endnote ix) to be used in conjunction with the student's and my observation and discussion of his testing behavior. Other pertinent factors would emerge during counseling and their relational etiology also could be explored.

I would first consider normal anxiety that is engendered by a dangerous situation, before looking for a psychological cause originating from a disturbance of mathematics self, internalized presences, or interpersonal attachments (Bowlby, 1973; Fairbairn, 1992; Freud, 1926; Kohut, 1977). In this context such causes as inappropriate placement in the class (indicative of prerequisite knowledge gaps), insufficient strategic preparation for an exam, or poor problem-solving, monitoring and control skills would genuinely endanger the student's chance of doing an exam successfully. These examples constitute appropriate causes of normal anxiety.

Once such normal anxiety has been ruled out, I would consider the relational roots of a student's anxiety.

The Mathematics Self and Anxiety

Anxiety related to assaults on the development of self is what the founder of self psychology, Heinz Kohut (1977) describes as disintegration anxiety, “an ill-defined but intense and pervasive anxiety accompan[ying a sense that the] self is disintegrating (severe fragmentation, serious loss of initiative, profound loss of self-esteem, sense of utter meaninglessness)” (p. 103). I have seen this when a student with a deep sense of his own inability to do mathematics becomes inarticulate and paralyzed when called on in class or experiences panic, mental disorganization, helplessness, even physical pain when taking a test. Could he be experiencing a form of the disintegration anxiety Kohut spoke of? Is this part of himself so malformed or underdeveloped that when his mathematics self is being scrutinized by a public question or a test, especially in mathematics class, he feels his self disintegrating to the extent that it might even threaten the rest of his developing academic self (cf. Lenore in Fiore, 1999; Tobias, 1993)?

I envisaged that counseling help for a student suffering so could take a two-pronged approach. The counselor could help the student to connect with mathematics, to recognize and own his developing understanding, and to expand his tolerance of the anxiety engendered by not knowing or understanding it all immediately; At the same time, the counselor, student, and instructor might explore alternate alternative arrangements in class work or testing designed to alleviate anxiety. For example, the instructor could signal that the student will be the next person to be asked an identified question so he has time to prepare an answer, or exam questions could be given one at a time.

Internalized Presences and Anxiety

When a student has developed and repressed bad internalized presences in response to unsafe and abusive mathematics learning situations, or has established mathematics as a punitive internal saboteur or superego, these internalized presences may cause him to worry that his mathematical products are bad or wrong even when they are not. He may have internalized his frightening third grade teacher who made him stand at the board for long hours humiliated and unable to do the required problem and this teacher's influence may be manifested during the college exam, insisting that he still cannot do it and recreating the mind-numbing anxiety he experienced back then (cf. Terry in Fiore, 1999). During the exam, he may have to contend with the anxiety engendered by the prospect of his exam grade pronouncing judgment on his worth as a person (cf. Buxton, 1991).

Interpersonal attachment and separation anxiety. Involuntary separation from a person's attachment figure often causes distress and creates disturbance in that relationship when the attachment figure returns, no matter how short the separation or how well the separation was managed. If the person subsequently comes to believe there is risk of further separation he is likely to become acutely anxious (Bowlby, 1973). A student may experience such acute anxiety if he has done well in mathematics and enjoyed positive relationships with teachers but has been separated from these good experiences and subsequently had a bad experience. He may have done badly in a course, clashed with or been ignored by a teacher. Separation anxiety is a natural response in children and adults^{ix} whose access to their attachment base is denied or threatened or whose attachment figure is unresponsive. Maladaptive responses to separation, loss, or

change can be an apparent lack of response (i.e., detachment) or an intense response (i.e., extreme anxiety or phobia) (Bowlby, 1973; Freud, 1926).

In a study of instructor-caused onsets of students' mathematics anxiety, Jackson and Leffingwell (1999) found that responses that could be classified as separation anxiety arose from the perceived inaccessibility or lack of responsiveness of the mathematics caregiver, the instructor.^x Experiencing inaccessibility or lack of responsiveness from previous teachers can negatively affect students' responses to their current teacher's offers of help as a secure base. Without understanding and intervention this separation anxiety may persist.

Students who have once experienced success in mathematics but have subsequently suffer a loss of competence because of poor teaching, course placement, or other external events may experience separation anxiety in relation to the mathematics itself. They may be newly uncertain of its accessibility and reliability. Without counseling interventions to reconnect them to their once-secure base in mathematics and their sound ability to negotiate the current course, this separation anxiety may cause them to fail or do poorly in mathematics courses they are capable of mastering.

This exploration of the relational origins of mathematics anxieties led me to see that once the student and I had determined through the *My Mathematics Feelings* survey and conversation that his mathematics anxieties existed and were troublesome, we could go further and distinguish their origin in different relational dimensions and devise targeted interventions that could look quite different depending on the dimension of origin.

Learned Helplessness and Depression

Anxiety is not the only emotional response to mathematics stress. In my experience, students who suffer from mathematics negativity (see above) expressed as learned helplessness or even depression with or without anxiety are just as prevalent. Learned helplessness has been linked with both situational and clinical depression (Seligman, 1975). Dweck and Reppucci (1973) found that a student may come to believe he is helpless under one set of circumstances but not under others. This supports Seligman's (1975) notion of situational learned helplessness or depression. It may not be so much the mathematics itself but the way it has been taught that renders students so vulnerable to learned helplessness in its face (Boaler, 1997; Carter & Yackel, 1989; Dweck & Reppucci, 1973; Piaget, 1973; Skemp, 1987). Mastery-oriented, positive students may exhibit helplessness and depression-like symptoms in certain mathematical contexts. Learned helpless and depressed people believe that the situation they are in is beyond their control; there is little or nothing they can do to change the outcome.

It is not unusual to find one or two students in any class of 30 who view their mathematical past, present, and future with despair. A mathematically depressed student sees himself as mathematically deficient; he considers the present mathematical demands excessive; and he views his future as impossible. He may want to drop the course he is in now and he will seek any alternative to the looming mathematics course to follow.

A depressed person's negative orientation and behavior influence other people whose responses in turn influence the individual (Bandura, 1977). For example, emotional withdrawal may elicit rejection or criticism that in turn aggravates the patient's negative self-cognition and thus his depression. A mathematically depressed student may

avoid classes, homework, or the learning support center. This avoidance behavior may be interpreted as laziness or irresponsibility and result in censure rather than sympathy.

Alternately, a mathematically depressed student may become excessively dependent on the mathematics counselor or the instructor and seem unable to proceed on his own.

Student Beliefs, Helplessness/Depression, and Mathematics Pedagogy

Students develop beliefs about mathematics and their ability to understand it that are closely linked with the beliefs and practices of their teachers and the effects on their mathematics orientation and self concept. In the U.S., the most detrimental belief about mathematics and mathematics learning that has the most far-reaching negative consequences for students is: “Learning mathematics requires special ability, which most students do not have” (Mathematical Sciences Education Board, 1989, p. 10).^{xi} The belief that ability is a trait rather than a malleable quality has been linked to learned helplessness in mathematics learning situations (Dweck & Wortman, 1982). It amounts to a type of mathematics gene theory that is applied in both a positive and negative manner. A student who identifies with a family member who is “good at mathematics” is likely to believe he also has the potential to be “good at mathematics,” but students who identify with a family member or members who “could not do mathematics either” are more common and are likely to find this belief debilitating. It has been found that a student’s beliefs about his achievement lead him to attribute outcomes to one of two central causes: his ability or his effort. Thus a student who believes his ability is low and unchangeable is likely to attribute a poor score on an exam to his (poor) ability. If a student attributes both his failure to lack of effort and also success to his (sound) ability, he is ascribing to beliefs that generally underlie a healthy mastery approach. On the other hand these

attributions may instead be an all-powerful, all-knowing (grandiose) mask for an underlying fear that one might not be able to do it—and that one has no intention of trying because of the risk of being found out (see below).

Student Beliefs, Achievement Motivation, and Helplessness/Depression

Achieving a high grade or some other recognition, also termed performance achievement motivation, often becomes more important and more possible than learning with understanding in the compulsory and competitive U.S. school systems. Piaget (1967) sees learning achievement motivation to be related to two important factors: (a) the “moderate novelty” of the new task, and (b) reasonable proximity and accessibility of learning, given levels of prior understanding. An individual’s curiosity is aroused by the “moderate novelty” of an object in relation to his prior experience; this curiosity motivates him to investigate, learn, and achieve understanding. The students in this study brought many different motivations to their tasks of succeeding in a mathematics course. These stem from their prior experiences, are related to their present ambitions, and affect how they would do in the course.

Both learning and performance achievement motivational patterns have been found to be affected by students’ sense of worth (Dweck, 1986). A sense of contingent self worth and a belief that their intelligence is fixed typically lead students to make performance as their primary goal in learning; they work only to be seen and judged to be successful. They will not approach a task with confidence (mastery orientation) unless they perceive their ability to be high for that task. If they perceive their ability to be low they are likely to become discouraged and even helpless. If they have a choice of learning tasks, some tend to choose tasks that are below their ability in order to ensure good

performance or they will choose tasks well beyond them that no one would expect them to complete successfully.

Students who believe their intelligence is malleable show more adaptive motivational patterns; they typically make learning their primary goal. These students typically approach tasks with a mastery orientation regardless of whether they perceive their ability to be low or high in relation to the task. They choose learning tasks because the tasks are personally challenging rather than first considering whether they are able to do well at them (Dweck, 1986). These students are likely to be discouraged and anxious and become helpless in fast-paced, text-based, procedural classroom where they find learning and understanding the mathematics difficult or impossible (Boaler, 1997). The optimal conditions for learning achievement motivation to lead to understanding and not be frustrated include these principles:

1. Students need to be encouraged to make it their personal goal to solve the problem; the tasks themselves need to be “appropriately problematic” (Hiebert, et al., p. 51);
2. The culture of the classroom must be a secure base that supports and allows time for struggle, reflection, and communication;
3. Students need to see ways to use the tools they already possess to begin the task.
(Hiebert et al., 1997)

Apparently similar classroom behaviors may stem from quite different motivational orientations linked not only to the student’s sense of mathematics self but also to the mathematical tasks and learning environment.

Student Beliefs and Helplessness/Depression: Developing a Survey Instrument

Students' beliefs about their mathematics selves, world, and future have been researched extensively and the links between these beliefs and their mathematics course persistence, behaviors, and achievement have been thoughtfully studied. As noted above I had developed the *JMK Mathematics Affect Scales* (see Appendix A) to quickly gauge students' immediate operating beliefs on a session-by-session basis. I wondered if in addition I could develop or find and adapt an instrument that would survey underlying factors researchers had linked to mathematics negativity or helplessness.

Whereas a mathematics anxiety instrument is intended to assess students' short-term emotional responses, a belief survey looks more at stable long-term underlying beliefs and attitudes. These may help to explain the student's short-term emotional responses as well as established mathematical behaviors. I looked for a self-report survey instrument around beliefs about self and mathematics that included statements about the following:

1. Mathematics as procedural or conceptual;
2. Mathematics self as learned helpless through mastery oriented in mathematics learning situations;
3. Links between mathematics beliefs and mathematics self beliefs;
4. Achievement motivation: performance through learning motivation
5. Personal characteristics and societal myths: Fennema and Sherman (1976), Fennema (1977), Kogelman and Warren (1978), Tobias (1993) and others have shown links between these and mathematics anxiety and debilitation of mathematics achievement.

I did not include locus of control as a factor, although I knew that a student's locus of control (whether he sees himself or some external entity such as luck or the teacher, as the controller of his outcomes) has been found to be an important factor in his mathematics achievement (cf. Nolting, 1990). I preferred to gauge this and also a student's locus of responsibility (whether he sees himself or others as responsible for what happens to him) directly from cues in the counseling setting.

With some modifications, Erna Yackel's (1984) *Mathematics Beliefs Systems Survey* with some modifications fit my criteria. Its chief attraction for my purposes was the careful investigation it provides of procedural versus conceptual beliefs about mathematics, based on Skemp's (1987) analysis (see Appendix B). I reasoned that analyzing clusters of items with the student could help him become conscious of beliefs, attitudes, and conditions whose relational origins we could explore and that he would be free to modify. I then considered what that exploration of relational origins of negative and counterproductive beliefs might reveal. I found the relational dimensions of a) the student's mathematics self and b) his interpersonal attachments to be particularly vulnerable to development of different types of depression. Because of the difference between the origins and hence potential remedies for these depressions I needed to clarify how to distinguish them when their initial presentation was likely to be similar.

The Mathematics Self and Depression

While CT describes the manifestations of mathematics depression, relational theory traces the origins of the depression and points to relational remedies. If a student's mathematics self has been under-stimulated because of chronic teacher neglect, his mathematics self will likely be underdeveloped. When such a student is faced with a

mathematics challenge, he is likely to experience a vague but pervasive sense of depression and excessively low mathematics self-esteem. His depression will feel like emptiness, a sense of not really being alive mathematically. He may believe his mathematical self does not exist apart from the mathematics tutor. He might excessively merge with the tutor. If so it will be the tutors' role to mirror his emerging mathematics competence back to him and to provide timely tolerable frustrations. Then the student can begin to discover his own prowess and learn to do mathematics on his own, with appropriate support. Alternatively, if he believes, even unconsciously, that he is incapable of understanding the mathematics he may try to memorize all procedures and will likely defensively blame external factors when this is unsuccessful. Such a student may avoid seeking help from teachers or tutors citing lack of interest or effort as the reason for his lack of success.

Attachment Theory and Depression

Not all mathematics depressions are rooted in underdevelopment of self. By contrast, a student may be in the depression stage of a grief process. A student who is used to doing adequately but then experiences doing badly may be thus affected. Another student who has experienced a teacher's dislike, rejection, or humiliation after a history of positive teacher experiences may also feel depressed in a subsequent mathematics classroom unless he is helped to work through his depression.

This line of thinking about relational origins of depression had led me to seeing that and how assessment results and counseling interventions for mathematics depressions originating primarily in the self dimension (empty depression) might differ

from those that originated primarily in the interpersonal attachment dimension (grief depression).

Grandiosity

Self Psychology and Grandiosity

Although U.S. students perform much less well than students from other developed countries, a persistent finding in international mathematics studies is that, on average, they think more highly of their mathematics ability than do the students who outperform them (National Center for Educational Statistics, 1995, 1999). U.S. males are more likely than females to think more highly of their prowess than their achievements would suggest to be appropriate (Sax, 1994; Signer & Beasley, 1997). Struggling students with a grandiose (all-powerful, all-knowing) view of their mathematics functioning are rarely seen in the learning support center because they cannot consciously face a need to get the help they need.

Grandiosity may be linked with an underlying poor mathematics self-esteem because of early teachers' failure to provide the student with the tolerable reality that the student is *not* all-powerful or all-knowing, even in his teacher's eyes. This in turn leads to inadequate internal mathematics structure and values needed to curb his grandiosity via idealizing and incorporating his mathematics teacher image. He is likely to deal with a mathematics class or a specific mathematics problem by expressing his belief in his ability to do it while he fails to put in the effort needed to succeed. He seems to be unwilling to risk putting in the effort and risking that he may not be able to do mathematics. That risk is too great for his vulnerable and underdeveloped mathematics self, so he may preserve his unrealistic sense of his ability by doing poorly or failing the

course and attributing this to his lack of effort. The challenges a mathematics counselor might face in trying to help such a student seem considerable. The greatest challenge is persuading him to get help and the counselor has to be very careful initially to accept the student's grandiose view of himself while finding ways to diagnose and remedy his mathematics gaps and deficits.

Mathematics Mental Health Conditions: A Summary and Caveats

When I considered the ways a student might present himself to a mathematics counselor, it was clear that the conditions I discuss above are far from exhaustive. Each describes a dimension or continuum of cognition or affect common to every student. Where a particular student's results are located in one dimension or combination of dimensions will allow the tutor to determine the state of his mathematics mental health from sound through poor. Whatever his state, growth is always possible. I expect that not only students who consider themselves poor at mathematics could benefit from engaging in this process of relational mathematics counseling. It was also clear, though, that a relational counseling approach ensures that even if a student comes to counseling with a condition different from those discussed here, the tutor will be able to understand him well-enough to help him understand himself and improve his mathematics mental health. I became increasingly aware that engaging in this process with a student likely involves not only *his* change and growth but also *mine*. The role of the instruments I developed or adapted must be adjunct rather than definitive; the role of the tutor and the relational counseling approach should be preeminent in the growth and achievement of both the tutor and student.

IMPLEMENTING A BRIEF RELATIONAL COUNSELING APPROACH

Extreme scores on a student's *Feelings* and *Beliefs* in-class survey assessments may alert him, me, and/or his instructor to the possibility of his benefiting by counseling; he may have entered the course expecting to need assistance and comes early to seek the regular help he believes he needs; or he might be prompted to come by a quiz or exam grade below his expectations. Ideally he would begin a course of mathematics counseling early in the semester, enough before the first big exam that at least half of the first session could be devoted to gaining some understanding of his mathematical background and experiences, and his current sense of himself as a mathematics learner. Student expectations about how mathematics counseling might be similar and different from traditional mathematics tutoring might need to be discussed; students are unlikely to be consciously aware of the possible relationships between their mathematics relational patterns (including their sense of mathematics self) and their approach and achievement potential in the current class. Realistically, students are likely to exert considerable pressure to focus on the mathematics content of the course from the beginning so the process of gathering background information and the process of orientating them into a mathematics counseling approach will need to be ongoing through the course of counseling.

In that first session, I would ask the student to create the metaphor whose threads and themes we could explore over the course of counseling. My curiosity about how he came to where he is now would also form a thread running through sessions as we pull apart the mathematics challenges he is facing. The counselor must be alert for his own reactions, and for behaviors in the student that could be elucidated by the student's

Feeling and *Beliefs* responses, his metaphor, his mathematics background and experience, and his present mathematics performance. Class assessment results, the student's responses to them, and the mathematics patterns they reveal are likely to be focal in counseling. The *JMK Affect Scales* filled out at each session would provide regular opportunity to explore links among behaviors, beliefs, and exam results.

I realize that the student in mathematics counseling is part of a complex system of important players. Each, including himself, is faced with multiple roles. In my study I focus on the student and the counselor, but others, especially the instructor, play active roles the student's and the counselor's mathematics relational worlds.

Roles Played in Mathematics Counseling

In this brief relational mathematical counseling approach, it is not only important for the counselor to understand and integrate a great deal of information about the student, but he also has to consider roles of all parties: the tutor/counselor, the student, and other significant players (e.g., the instructor) within the college context. The mathematics counselor or the mathematics counselor and the student together become aware of the student's mathematics dimensional relationships as a vehicle for both to know the student holistically and identify what and how he needs and wishes to change. Approaches, assessments, and therapeutic contributions from each of three dimensions of the relationship have been identified.

At this point I need now to discuss new and necessary orientations and preparation of a mathematics relational counselor. By definition, the counselor undertakes to view the student with unconditional positive regard. He unequivocally believes in the student's existing mathematical intelligence and the potential for that to

grow. The counselor must also understand the counseling and mathematics learning processes to be a collaborative effort. The counselor brings expertise in mathematics, in mathematics pedagogy, and in relational counseling approaches and techniques;^{xii} the student brings his own reality, his mathematics understandings and potential and his willingness to explore, consider, and apply insights that emerge in the counseling process.

*Relational Counseling Role: The Therapeutic Relationship
in Mathematics Counseling*

The following roles emerged for me as ideal yet potentially attainable:

The Mathematics Counselor as Listener and Witness to the Student's History

The counselor listens knowingly (mathematically and developmentally) with curiosity rather than with judgment. He elicits the student's experience of his own mathematics history. To test the efficacy of interviewing for understanding students' mathematics identity (self) development, I developed a semi-structured interview outline and piloted it with basic algebra students at a small liberal arts college in the Northeast (see Appendix A). The interviews I conducted with these students about their mathematics identity development corroborate Buxton's^{xiii} (1991) findings and is the protocol I developed that I use here (Knowles, 1998).

*Transference and Countertransference in the Mathematics
Counseling Situation*

The counselor must be alert to how the student responds to him as a significant mathematics figure from the past (transference); the counselor also watches for ways he unconsciously responds to the student's transference or as a significant figure from his own past (countertransference). This awareness and interpretation of client transference

and counselor countertransference in the counseling relationship are central to relational psychotherapy. Close observation of the counselor-client relationship yields crucial data for identifying relational patterns that are either conscious or unconscious, and that can be either beneficial or counterproductive to the student's sound mental health. In the mathematics counseling setting this requires the mathematics counselor to become conscious of how he experiences the way the student relates to him and seems to expect him to be as a teacher (transference). He must also become aware of his own reactions to and hopes for the student, understanding direct responses to the student and knowing reactions that are based on those from his own teaching or other relational experiences, triggered by the relationship with the student (countertransference).

*Insight, Central Conflict Identification, and Interpretation
in Mathematics Counseling*

The counselor observes and hears patterns and unconscious contradictions among aspects of the student's relationships that may help to explain the student's puzzling mathematics-related behaviors and may yield clues to identifying his central relational conflict (insight). He then discusses and clarifies these with the student (interpretation) so the student may gain insight into his problems;

*Mathematics Counseling Role: The Tutoring Relationship
in Mathematics Counseling*

The counselor models healthy mathematical behaviors and interprets them in relation to his own underlying healthy beliefs about the mathematics, himself, and the mathematics learning situation. He cannot presume that the student will make these connections between behaviors and beliefs without sometimes extensive mutual interpretation.

Mathematics Tutoring as Central to Mathematics Counseling

In standard relational counseling, the focus is both the client and his relational problems; in mathematics relational counseling the focus is the student and his difficulties learning mathematics. The counselor must balance therapy's client-centeredness and counseling's problem-centeredness (Corsini & Wedding, 1995) by adopting the dual focus of relational brief therapy.

Contributions of Conceptual Mathematics Tutoring and Mathematics Course Management Counseling

The counselor is an experienced mathematics learning specialist who is aware of the toxic effects of an exclusively procedural approach to mathematics and the importance of strategic course management in a time limited college course setting. Although these issues may not be focal in the eyes of the student, the counselor must be alert to any need to incorporate them into successful counseling. Understanding the motivations of the instructor (and the department) is also key since ambivalence about what is valued as mathematics outcomes and how the instructor assesses these outcomes may result in confusion for students between getting good grades and really working for comprehensive understanding^{xiv} (Hiebert, 1999; Lee & Wheeler, 1987; Mokros, 2000). The learning counselor typically has little if any influence on the curriculum or the assessment so his role is to help the student adapt to the course in a way that is as healthy as possible for him.

"Understanding is an ongoing activity not an achievement" (Kieran, 1994, p. 589) but its links with mathematical self-esteem places the onus on the mathematics counselor to discern compromises between achievement (of grades) and understanding: In addition

the student needs effective ways to adapt to the present mathematics classroom at the same time that he repairs his mathematics self-esteem and succeeds in the course.

Constructivist Approaches: The Student as Author of His own Growth and Healing in Mathematics Counseling Situations

The relational mathematics counselor believes the student has what he needs relationally, intellectually, and especially mathematically to make the changes he needs in order to achieve good-enough results. The approach to the counseling and to the mathematics is thus a developmental constructivist approach. This, however, does not preclude strategic direct teaching in the time limited setting.

Roles in Relation to Other Key Players

The Student and the Instructor

The student's relationship with his instructor is likely to be revealing not only of his present mathematics learning approaches but also of his historical patterns of relating with mathematics relationships. As the counselor becomes aware of the student's perceptions of the relationship with the current teacher and as they are both able to directly observe the relationship, the counselor may use discussions of the congruence between the two to explore these patterns. How the student perceives himself in relation to his classmates and relates (or not) with them is also likely to be of interest although not as pivotal as his relationship with the instructor.

The Counselor and the Instructor

Effective tutoring involves not only supporting students in learning the content covered in the syllabus but also in helping them understand the instructor's teaching approach, assessment schemes, and priorities. This implies the tutor's knowing or being able to understand the instructor's approach. A relational approach implies in addition

that the tutor/counselor know or be able to gauge how the instructor's pedagogy, classroom management style, and relational patterns might impact the student. The counselor must discuss the instructor's approach with the student (and possibly the instructor), especially if it seems to be detrimental to the student. Ensuring that this happens this is likely to be extremely important to the efficacy of counseling. The counselor has to be conscious of his relationship with the class instructor and may have to use this awareness in mathematics counseling to help the student find ways to negotiate a constructive relationship with the instructor and class.

Supervision of the Counselor by a Person Knowledgeable in Counseling

Because a major source of insight for the counselor is the transference and the countertransference in the counseling situation, he should be under supervision. This means that at least once or twice during the semester he should present himself and his student as cases to a person knowledgeable in counseling in order to confirm or challenge his insights and approaches and to gain insight and inspiration in cases that he continues to find puzzling.

CONCLUSION

I have situated brief relational mathematics counseling in the college learning center context and pointed to the details of what it might look like. I have designed a summary chart that illustrates its components and how I see they relate to each other (see Table 3). What follows in this dissertation is a description of what happened in the pilot study as I applied the theory explored in chapter 2 in ways that I envisioned in this chapter.

In the next chapter I will describe the research methods I used to describe my pilot implementation of brief relational mathematics counseling with students in a statistics for psychology class at a small university college in the Northeast.

ⁱ Because this is an odd numbered chapter I use “he,” “his,” and “him” as the generic third person singular pronouns.

ⁱⁱ Through intelligence testing, “aptitude” tests such as the SATs (Scholastic Aptitude Tests), or standardized achievements tests with percentile rankings interpreted as ability measures.

ⁱⁱⁱ Through school mathematics grades and “ability” grouping, and teacher/school and parental/societal expectation.

^{iv} As noted in chapter 2, people suffering from depression have been found by cognitive therapists to view the world, themselves, and the future through a negative cognitive schema (Beck, 1977; Beck, Rush, & Shaw, 1979). Martin Seligman (1975) has shown that a person suffering from situational depression has almost identical symptoms to those suffering from situational learned helplessness.

^v How he processes and assimilates new learnings, accommodates his cognitive schema to these new learnings, stores them in long-term memory, and retrieves them for application and in appropriating further learning, constitutes the student’s cognitive learning style (Davidson, 1983; Piaget, 1985; Schoenfeld, 1992; Skemp, 1987). Skemp (1987) uses Piaget’s term “assimilation” as the process whereby the learner assimilates the new learning into existing conceptual schema and at the same time “accommodates” the existing conceptual schema to meet the demands of the new situation, resulting in a struggle to arrive at an expanded schema and new greater understanding.

Davidson (1983) found strong links to hemispheric preference and clearly defined analytic (her Mathematics Learning Style I) and global (her Mathematics Learning Style II) learning styles in terms of students’ mathematical behaviors and approaches. Although these learning styles have not been found to be directly related to mathematical achievement, Krutetskii (1976) found that students who had a strong learning style preference and a relative inability with their other mode, found it difficult, if not impossible, to begin a problem using the other mode’s approach. Thus being forced to approach problems using another’s preferred approach greatly disadvantages these students. Learning flexibility, however, can strengthen performance.

^{vi} A strong leaning towards one learning style/processing channel (e.g., visual versus auditory versus kinesthetic) may present as a learning disability in an environment where another is favored but may be celebrated as ability in an environment where the preferred style/channel is favored. Thus there are individuals for whom having a learning disability may be more relative to the learning environment than intrinsic to her.

^{vii} In the high schools, teachers may know the mathematics content, but the pedagogy is often teacher-centered and procedural, as procedural mathematics routines are transmitted to the students. Many members of the mathematics education community have come to believe that exclusive exposure to a transmission model of pedagogy is generally antithetical to the development of students’ mathematical power. Unfortunately it is what most U.S. students coming to college have experienced (Boaler, 1998; International Association for the Evaluation of Educational Achievement, 2001; Skemp, 1987).

^{viii} Analysis of attempts to measure mathematics anxiety reveals problems in the research in understanding what exactly is being measured (McLeod, 1992). Use and development of *Mathematics Anxiety Rating Scale* (MARS) (Richardson & Suinn, 1972) which has been normed and is perhaps the most used in the field, is illustrative of the problem. MARS doesn’t distinguish among different types of anxiety, for example, cognitive worry versus affective emotionality that some theorists differentiate (cf. Ho, et al., 2000) or between state versus trait anxiety (cf. Nolting, 1990). MARS defines anxiety by a single affective response—fright—and asks students to distinguish among five levels of fright in relation to mathematics-related activities and situations. Factor analysis of MARS items yielded two relatively homogeneous factors (15 items each): mathematics testing anxiety and numerical anxiety (Rounds & Hendel, 1980). An additional factor, abstraction anxiety, important for college students but not addressed in MARS has been identified by Ferguson and a resulting three scale (the first two of which use MARS items identified by Rounds and Hendel) test—Phobus—developed (Ferguson, 1986). The last two scales of Phobus (numerical

and abstraction anxieties) differentiate between two types of mathematics, each, however, in different settings: number/arithmetic (outside the classroom in every day settings) versus mathematics involving algebraic variables and other literal symbols (in classroom and college settings). The first scale inquires about mathematics testing-related situations before during and after the test. Items in Phobus can be further classified according to whether the activity is likely to be solitary or public or either. No items inquire about effects of degree of test preparation on anxiety levels nor do these scales ask about effects of anxiety on cognition during testing. A deficits model of testing anxiety proposes that a student who is poorly prepared and has poor test-taking skills will have high anxiety in testing situations (e.g., Tobias, 1985), and an interference model of testing anxiety proposes that in testing situations, anxiety interferes with students' recall and thinking.

^{ix} Freud (1926) believed separation anxiety to be a natural response to separation and loss only in children. In adults he viewed it as pathological.

^x Only 11% of the 157 above-average college students seeking certification in elementary education, surveyed, reported *only* positive experiences in their own mathematics education. Of the others, when the onset of their anxiety was in the 3rd or 4th grade (as for 16% of the sample), among behaviors of instructors cited, instructors were perceived to not respond to students' needs for clarification and tutoring or showed anger or disgust when students asked for help (p. 584). Many of the students whose negative experiences began in high school (26% of the sample) reported the same ignoring, rejecting, or ridiculing of students' needs, as did many of the 27% of students whose problems began freshman year of college (p.584).

^{xi} This is a version of Kogelman and Warren's (1978) Myth 11: Some have math minds and some don't; or Schoenfeld's (1992): Ordinary students cannot expect to understand mathematics; or the National Research Institute's first Myth: Success in mathematics depends more on innate ability than on hard work (National Research Council, 1991, p. 10).

^{xii} In addition to access to a supervisory person knowledgeable in counseling psychologies, for a professional mathematics tutor to engage in the brief relational mathematics counseling described here, some preparation (i.e., coursework or at the very least, directed reading) in counseling psychologies, including CT/CBT and relational conflict therapy would seem to be a minimal requirement.

^{xiii} To understand (and ameliorate) adults' mathematics "panic," Buxton (1991) looked at individual in-depth interviews, group study of mathematics problems, and discussions of affect. His participants' *stories* invariably linked their mathematics panic and failure to achieve to parents, teachers, and their theories about themselves. What they believed mathematics to be and how they experienced mathematics teaching in their lives interacted significantly with their mathematical self-perceptions.

^{xiv} "Understanding can be characterized by the kinds of relationships or connections that have been constructed between ideas, facts, procedures and so on...there are two cognitive processes that are key in students' efforts to understand mathematics—reflection and communication" (Hiebert et al., 1997, p. 15), both of which require the opportunity and time to do so.

CHAPTER IV

METHODOLOGY TO STUDY BRIEF MATHEMATICS RELATIONAL COUNSELING MODEL

LEARNING ASSISTANCE CENTER AND MATHEMATICS SUPPORT: Finding an Appropriate Research Setting to Pilot Brief Mathematics Relational Counseling

My responsibility as the mathematics specialist for the Learning Assistance Center at Brookwoodⁱ State University is to offer support to students taking mathematics and mathematics-related courses. Along with the mathematics peer tutors whom I help to train, I offer mathematics tutoring in individual appointments, in open drop-in tutoring at the Learning Assistance Center,ⁱⁱ and in study groups for specific courses. These offerings are advertised to students via memos to instructors and by initial visits to the classrooms by the peer tutor or me. Some students who need support find their way to the Center in a timely fashion and often enough for the support to help; some come at the last minute (e.g., just before an exam when it is often difficult to resolve their problems); others do not come at all.

We do not see all of the struggling students and generally cannot directly observe how the students we do see are handling their instructors' teaching and testing approaches. For my study of whether and how relational counseling insights could contribute to mathematics support, however, it was necessary to observe the classroom environments and student behaviors and interactions there. I decided that my research should focus on one mathematics course so that I could attend that class and offer individual mathematics counseling to its members. This approach was modeled on a tutoring practice already used in writing-intensive classes at Brookwood, where peer tutors are class-support tutorsⁱⁱⁱ who attend the assigned class and offer learning

assistance both within and outside the classroom. Although this approach had not yet been used in mathematics classes, it was a familiar practice at Brookwood in other subjects, and the advantages for my research seemed obvious.

To pilot my counseling approach I decided to focus on students in a class that was considered to be at risk for high negative mathematics emotionality, withdrawal, and failure. The PSYC/STAT 104^{iv} (Statistics in Psychology) class that I researched was a one-semester introductory statistics course that fulfills the university's quantitative reasoning core requirement. It is also a major requirement for nursing, psychology, and biological science students. This course is taught for fifteen weeks in the fall and spring semesters and for ten weeks in the summer^v at Brookwood State University. Ann Porter^{vi}, the PSYC/STAT 104 instructor, a tenure-track faculty member, agreed to host this research in her classroom.

The specifics of the course, the students, the instructor, and the mathematics counselor were particular to us. However, I was certain that my observations, diagnoses, and the application of brief relational mathematics counseling approaches to Brookwood students' challenges would provide insight into some broadly applicable ways that mathematics students can be supported and shed light on changes needed in traditional college mathematics support. I expected a framework could emerge to help mathematics support professionals to understand and deal with students' mathematics problems in a way that would also promote their mathematics mental health while they are engaged in a semester course. The emergent framework is grounded in relational conflict brief counseling theories and cognitive constructivism.

The Course

The introductory level statistics courses, PSYC/STAT 104 and BUS/STAT 130, are among the most failed and dropped first year college level classes at Brookwood.^{vii} Although there is no stated mathematics prerequisite for PSYC/STAT104, there has been ongoing pressure from academic counseling and academic support personnel on the Enrollment Management Committee to make successful completion of high school algebra at least a strong recommendation.

PSYC/STAT 104 is offered through the psychology department. Some students take it to fulfill the quantitative reasoning requirement for a liberal arts degree. Nursing and psychology majors are required to take it. Nursing faculty see it as a gatekeeper course for the degree: If a registered nurse (RN) is not able to pass it with at least a C, it is thought that she^{viii} might not be a suitable candidate for a bachelor's degree. Enrollment in the summer course is always lighter than in the fall/spring semester courses and the course only takes 10 weeks to cover 15 weeks of material. During the summer, students typically work full-time and take PSYC/STAT 104 and at most one other course. The class offered in the summer of 2000 was typical, with RNs, psychology majors, and others, all hoping to do well enough to be able to proceed towards their larger goals.

THE STUDY SITE AND PERSONNEL

The University

Brookwood State University (not its real name) is a small commuter university college with approximately 1,500 degree and continuing education students. The summer enrollment is approximately 550. It is located in the small New England city of Brookwood. The greater Brookwood area population is almost 200,000 and is

predominantly white with 5.6% non-white or mixed race residents concentrated in the city proper. It is ethnically quite diverse. Thirty-one and one half percent of the population has French or French Canadian ancestry and many maintain their ancestral language and culture. Three percent identify as Latino and 14.4% of the population speaks a language other than English, two-thirds of which are Indo-European languages. There are more than 50 different languages spoken in the local schools. Six point six percent of the population is foreign born, half of these having entered the U.S. since 1990 (U.S. Census, Census 2000). Among these are considerable numbers of refugees from the Balkans, Africa, and the Middle East. The university's college credit and intensive college-preparation summer English Speakers of Other Languages (ESOL) courses attract between 30 and 50 high school students and adults per year, approximately 35% of whom go on to degree programs in the university.

The average age of undergraduate students attending Brookwood is between 26 and 27 years and the student population approximately reflects the racial and ethnic diversity of the greater Brookwood area (Brookwood University records, March 2003). The university offers two-year associate's degree, bachelor's degree, and some master's degree programs. Many students enroll in credit courses as non-matriculated, continuing education students.

Until recently, many of the classes (including PSYC/STAT 104) were held downtown at the Riverside Center, while the Learning Assistance Center and the Computer Lab were located at the Greenville Campus on the edge of the city about five miles from Riverside.

The Researcher and the Course Instructor

At the time of the study, I had been in the field of mathematics education for almost thirty years, the previous 12 at college level. My professional focus had been on the teaching and learning of developmental and first-year college-level mathematics, although I had tutored students and trained tutors across the undergraduate mathematics spectrum. I had developed curriculum and placement testing and had taught mathematics courses at a community college and small four-year liberal arts colleges in New England. I had worked in academic support for these courses with general student populations and special populations that included learning, sensory, and physically challenged students.

I worked at Brookwood State University as the mathematics learning specialist and assistant director of the Learning Assistance Center. Prior to the study, I had only briefly met Dr. Ann Porter (pseudonym), the course instructor, at all-college functions since she worked almost exclusively at the Riverside campus. We communicated by memo and through peer tutors about study group and tutoring offerings for students in her courses. I had not met any of the students in the class except Pierre, whom I knew by name and face through the Learning Assistance Center's work with English speakers of other languages.

I was aware of the negative reputation of the course among students who perceived themselves to be shaky in mathematics because I had tutored one third of the members of the PSYC/STAT 104 course^{ix} at Greenville campus in the spring of 2000. I was aware of another third who were struggling and I perceived the dread of students in the Learning Assistance Center who knew they would have to take it in the future.

Researcher as Mathematics Tutor and Counselor

My role in this study would extend beyond that of researcher doing naturalistic observation to active intervention as a tutor and a counselor, so it was important for me to engage in continual self-reflection before and during the time of contact with the class and during the period of post-analysis of the data. In particular, I needed to reflect on myself as a mathematics tutor and also as an emerging mathematics counselor as I put relational and cognitive counseling theories into use.

Who I am as a Mathematics Tutor

I had always performed well as a student in the predominantly transmission^x teaching, textbook-focused, and procedural^{xi} mathematics classrooms of my elementary and high school education. At university, I became more conscious of the larger concepts underlying mathematics but it was not until I began tutoring students with learning disabilities that I became uncomfortable with the prevailing pedagogy and its implicit assumptions about students' learning processes; I realized how capable my tutees were but also saw how incomprehensible they found much of the mathematics presented to them in class. This began my struggle to understand their ways of thinking, to understand the mathematics more deeply myself, and to find ways to help them understand and achieve in a class that someone else is teaching, over whose curriculum or pedagogy I had no control.

As a tutor I tend to help too much, by teaching and telling, more than to coach the student to find his own way to understand the material. I tend to suffer from “agenda anxiety” on behalf of the students—knowing what they will be expected to cover but

worried that they might not recognize the urgency. I tend to try to push them too fast. I find it hard to let them make the mistakes they need in order to grow.

I feel tension as a tutor of courses that other people teach, and this increases when the curriculum or the instructor's pedagogy seems to increase the students' difficulty in understanding the concepts and connecting related concepts. I feel even more tension when the student experiences the classroom as abusive or unsafe. At times I allow this tension to enter the tutoring session by siding with the student against the curriculum, the system, or the student's past preparation or teachers. I generally do not join students in criticizing the instructor but try to help them find ways to handle these conflicts in a way that is constructive to them. I sometimes find myself defending teachers whom students are attacking.

Who I am as an Emerging Mathematics Counselor

I am a white, university-educated, Australian, female, extroverted mathematics teacher and tutor brought up in the suburb of a large city in a middle class home by parents who were both tertiary educated professionals. In addition to working in mathematics education, I also worked with several groups of Australian Aborigines doing field linguistics and a trial literacy project. It was in the context of that work that I met and married a rural, working-class, New England American who graduated from a technical high school program in the 1960s and works in the building trades.

My faith is grounded in the principle that all humans are made in the image of God and are thus inherently creative and have the potential (indeed the obligation) to learn and grow and understand. It has been an important basis for my interest in and continually emergent acceptance of people whose backgrounds and characteristics are

different from mine. I am aware of my need to continue to work through my class-, ethno-, religion-, gender- and extrovert-centric orientations and grow in appreciation and acceptance of difference. As I explore and understand the challenges and opportunities that my tutees and my characteristics and backgrounds have placed on our development and our ability to understand and accept each other, I continue to find that some aspects of who I am provide potential bridges and others create potential barriers; I know I have blind spots that make understanding difficult.

I am female, non-American and from what is often seen by Americans as an insignificant former British colony.^{xii} I live with a person from a low SES background who experienced low expectations and less education because of this background and still struggles with a sense of powerlessness. My efforts to accept and maximize my potential within these identities give me some empathy with students from disempowered groups—women, racial and ethnic minorities, and people of low SES—whose mathematics selves, internalized presences, and attachments have been negatively affected because of who they are.^{xiii}

My struggles with arithmetic details (I cannot keep my checkbook straight), visual memory, visual-spatial reasoning, and directionality (I cannot tell left from right nor connect the implications of up versus down without verbalizing) enable me to empathize with students with learning disabilities or a strong mathematics learning style preference (and concomitant weakness in the other) who believe their learning challenges prevent their achievement in mathematics. I can also model struggle and success for these students.

My being white, middle-class, university-educated, and successful in mathematics may be initial barriers for students who feel disempowered because of who they are but I have found that self-disclosure of my struggles can help break through. My family background of trying to help a mother struggling with addictions makes me vulnerable to co-dependently take on a student's responsibility to make any changes she needs to, or to excuse his failure to take that responsibility himself. I find the fine distinctions between support and indulgence difficult. On the other hand, I find it difficult to (and would rather not) work with students who appear to be overestimating their abilities or knowledge or who seem to be rigidly adhering to approaches that are counterproductive. Hence, I recognize the particular importance in this study of attention to the student's transference and my countertransference in the mathematics counseling situation.

The Instructor

Dr. Ann Porter is a young, energetic, white woman (in her late twenties at the time of the study). She has a Ph.D. in experimental psychology and was actively engaged in research with a geriatric population at the time of the study. Ann also served as faculty advisor to the Student Government at the university. She had taught this course before. Ann began at the university two years before the study. She stated that she taught with a more "laid back" teaching style than she had experienced as a student (Interview 3, archived). As she described her professors' transmission teaching methods for her undergraduate introductory statistics and her later graduate statistics classes, she told me that she believed students should instead be able to grapple with the mathematical procedures during class with the opportunity to receive guidance rather than merely watch the procedures being done on the board as had been her own experience (Interview

3). She had been comfortable with algebra in high school, had minored in statistics in her doctoral program, and she was finding that teaching it to undergraduates was deepening her enjoyment of the field.

Ann told the class she liked the assigned text, but disliked the required computer program MINITAB—a late 1980s version—because it was “somewhat archaic” (course syllabus; Class 1, May 31, 2000). She told them of a more modern statistics software program she used to analyze her own data and promised to bring it in to show the class. She shared her own struggles with anxiety in a statistics class she had taken. She invited the class to call her by her first name if they preferred. All did.

When I approached Ann before the course began she was hesitant about my doing research in her class because the class time needed to cover the material was reduced by several hours in the summer.^{xiv} She had committed herself to a very busy summer and she was also concerned that my research project would add to her workload. She was worried that my using a counseling approach with students for their “psychological” problems might have unforeseen repercussions on students’ behaviors in the classroom and in relation to her, and make teaching the course more difficult. She did not see students’ affective issues to be within her purview and did not want students to expect that of her. To allay her fears, I designed whole class research explanation and surveying to take minimal time and we agreed to schedule it just before breaks or after exams. Ann discovered her fears that my research would increase her workload were unfounded; indeed, the reverse was true. She found that in most cases students’ negative affect and cognitive struggles actually became more contained because of the support I was offering.

My Roles in the Study

I attended each class primarily as a researcher. In that capacity I took a small amount of class time to explain my project and administer pre- and post-surveys of mathematics affect to the class. Otherwise, I observed and recorded interactions in the classroom—especially instructor-student interactions during lectures and student-student and instructor-student interactions during problem-working sessions.^{xv} Increasingly I took the role of class-support tutor. In that capacity I led a weekly study group for the class and during class I assisted students sitting near me by working the problems in parallel with them, as Ann circled the room helping others. Ann occasionally consulted me on mathematical questions when she was uncertain. I also offered individual mathematics counseling to volunteers from the class and because students were meeting with me, they were generally less demanding of Ann's time outside of class.

THE CLASS AND INDIVIDUAL PARTICIPANTS

The PSYC/STAT 104 class of the summer of 2000 was typically small. There were 13 students (7 women and 6 men) at the first class meeting on May 31, 2000. I have given each a pseudonym to preserve anonymity. All were white and spoke English as their first language except Pierre, a French-speaking black African; at least one (but possibly three^{xvi}) was first in the family to attend college, and most were long-time local residents. They ranged in age from 19 to the mid-forties, and about half traditional-age students. The class average age was around 28 years, somewhat higher than the Brookwood average. All but three were full-time students. Because it was a summer class, nearly half were from other colleges, a greater proportion than is usual in other

Table 4.1

Profile Summary of Students taking PSYC/STAT 104, Summer 2000 (N = 13)

Student	Where Enrolled?	Student-Related Data	High School Math Courses	College Math Courses	Work in Summer
PARTICIPANTS					
Eow ^a -R ^b Autumn 4 sessions	SU ^c	W ^d ; Age ~20; Full-Time; Mj: Psyc Mn ^e : Business to Mj ^e : Business; Mn: Psyc	Algebra I: A ⁺ ; Geom: A; Adv Algebra II: C ⁺ ; Prob&Stat (1/2): A Discrete Math(1/2): A	Finite Math, 2000: A ⁻	Retail~30hrs
Ew-H? Brad 4 sessions	BSU ^c	M ^d ; Age~40s; Pt-time Repeat PSYC/STAT 104	Algebra I, II, Geom: Bs??	PSYC/STAT, 1998: F/AF	Nursing-FT ^f
Ew-H Jamie 5 sessions	SU	W; Age: 20; Full-Time; Mj: Psyc Repeat PSYC/STAT 104	Algebra I: B/B ⁺ , Geom: C/D ⁺ , Algebra II: B/C ⁺ , Precalculus: C?	PSYC/STAT, 1998: D ⁺ Finite Math, 1999: W	Retail~30hrs
Eow-H Karen 5 sessions	BSU	W; Age: 22; Part-time; Mj: Psyc, Mn: Educ Repeat PSYC/STAT 104	Algebra I; C? Geometry (struggle); Algebra II: C?	PSYC/STAT, 1998: F	Assistant Teacher – Elem. Special Ed-FT
Ew-H Kelly 3 sessions	SU	W; Age: 19; Full-Time; Mj: LibArts; MjInt: Soc. Wk	Algebra I; Geometry; Algebra II	Basic Math, 2000: D/C?	Camp Counselor-FT
Eow-H Lee 6 sessions	BSU	W; Age: 19; Full-Time Mj ~Psyc	Geometry: B; Algebra II: A; Precalc/Calc: A	Finite Math, 1999: A	Dental Office Assist~30hrs
Eow-H? Mitch 4 sessions	BSU	M; Age: 23; Full-Time; Mj: EuropeHistory Repeat PSYC/STAT 104	Algebra I: F, A; Geometry: F, C	PSYC/STAT, 1998: F Finite Math, 1999: C	Retail: FT
Eow-R Mulder 5 sessions	OU ^c	M; Age: 21; Full-Time; Mj: Biology	Algebra I; Geom; Algebra II: C	None	Retail~30hrs
Ew-H Pierre 8 sessions	BSU	M; Age~30s; Full-Time; Mj Biology; ESL/French	Algebra through Calculus	Calculus I: D	Residential Support: FT
Eow-R Robin 3 sessions	BSU	W; Age~mid40s; Part-Time; Mj: Nursing	College Algebra A	None	Nursing: FT
CLASS ONLY (not individual participants) Catherine	BSU	W; Age~40s; Full-Time; Mj: Biology, Mn: Educ		Calculus I, 1999: A	
Ellen	BSU?	W; Age~30s;			
Floyd	OC ^c	M; Age~20-30s; Full-Time; Mj: Psyc		Finite Math	

Note. ^a Participant signed up for mathematics counseling: Eow = Every other week; Ew = Every week. ^b Participant's initial motivation for signing up was: -R = to help me with my research; -H = to get tutoring help; -H? = apparently to get help. ^c Institution where student was enrolled: OC = Other College; BSU = Brookwood State University; SU = State U.; OU = Other U. ^d Gender: W = woman; M = man; ^e Mj: Major, Mn: Minor; MjInt =intended Major; ^f FT = full-time work,

semesters. Three of these were from the affiliated State University. All of the participants except Mitch had taken at least Algebra I, Geometry, and Algebra II in high school. Four students were repeating PSYC/STAT 104; three had taken the course within the past 2 years and failed it (Karen, Brad, and Mitch) and one (Jamie) had earned too low a grade to be counted for her Psychology major.

Ann briefly introduced me as a researcher at the first class meeting and in the second I gave each class member information about my research project (see Appendix D) and then administered class mathematics affect pretests (see Appendix C) to all who agreed to be involved in the whole class study. All 12 students present completed the pretests, thus constituting their consent to have me use them and classroom observations of them as data. How they could signify consent was explained in writing in the research explanation and I have archived pretests as consent agreements.^{xvii} Ellen, who was not present, had dropped the course. At this class meeting all students were invited to volunteer to be individual research participants^{xviii} by signing up for one-hour mathematics counseling sessions with me. Nine students volunteered by filling out and signing a volunteer agreement card. Four opted for counseling every week and five for every other week. One other (Lee) initially checked “no” for one-on-one counseling but e-mailed me the day before the first exam to ask to participate. Each of these signed an Informed Consent Form (see Appendix D) during the first counseling session.^{xix} Of the initial group of ten counseling participants, two failed to complete the course—one left before the second exam and another, citing family responsibilities, in the ninth week of the course. A summary of individuals’ mathematics-related characteristics and history is presented in Table 4.1.

The benefit of participating in my study was the individual statistics tutoring and mathematics counseling, so monetary compensation for participants was not necessary.

DATA COLLECTION

The Research Schedule

Table 4.2

PSYC/STAT 104, Summer 2000 Class and Research Schedule

	Course Week										Post
	1	2	3	4	5	6	7	8	9	10	
1 st Class Mondays 6:00 p.m.		Class 2 Pre- tests	Class 4 EXAM 1 June 12 *Pre- SRA	Class 6	Class 8	Class 10 no class meeting	Class 12	Class 14 EXAM 3 July 17	Class 16	Class 18 MINITAB Post- Tests	^b Comp (extra) EXAM
2 nd Class Wednesdays 6:00 p.m.	Class 1	Class 3	Class 5 MINITAB Comput Lab	Class 7	Class 9 EXAM 2 June 28	Class 11	Class 13	Class 15	Class 17 EXAM 4 July 26	Class 19 EXAM 5 Aug. 2	
Study Group Wednesdays 4:30 p.m.		Study Gp 1 4att.	Study Gp 2 3att	Study Gp 3 1att	Study Gp 4 4/5att	Study Gp 5 1att	Study Gp 6 2att	Study Gp 7 1att	Study Gp 8 5att	Study Gp 9 6+att	
Drop-In			Ke, L				B, L	Ka, L, J		Mu, MAT120	
Individual Session		<u>*Ke6/8</u>	<u>Ka6/12</u> <u>A6/12</u> <u>Mi6/14</u> <u>R6/14</u> <u>Ke6/16</u>	<u>J6/20</u> <u>B6/20</u> <u>Mu6/21</u> <u>L6/21</u> <u>Ke6/21</u> <u>P6/22</u>	<u>Ka6/26</u> <u>Mi6/26</u> <u>P6/27</u> <u>A6/28</u> <u>Mu6/29</u>	<u>J7/3</u> <u>L7/5</u> <u>R7/5</u> <u>Mu7/6</u>	<u>Ka7/10</u> <u>B7/10</u> <u>J7/11</u> <u>Mi7/12</u> <u>R7/12</u> <u>P7/13</u> <u>B7/13</u> <u>P7/14</u>	<u>Mu7/17</u> <u>Ka7/17</u> <u>A7/17</u> <u>L7/19</u>	<u>Ka7/24</u> <u>Mu7/25</u> <u>R7/25</u> <u>B7/25</u> <u>P7/26</u> <u>J7/26</u> <u>Mi7/26</u> <u>A7/26</u>	<u>P8/2</u> <u>L8/2</u> <u>P8/3</u>	<u>J8/6</u> <u>P8/7</u> <u>L8/7</u>
My Outside activities in relation to course	Interview Ann Porter May 31						Interview Ann Porter July 10	Present cases to Dr. P. July 20 Study Gp before EXAM w/Ann 4-6 p.m. w/Jill 5-6 p.m.		Interview Ann Porter Aug 3	

Note. ^a After Exam #1, I administered the Statistical Reasoning Assessment (SRA); ^b Students could take and optional comprehensive final after the course ended to replace a lower grade; ^c A = Autumn; B = Brad; J = Jamie; Ka = Karen; Ke = Kelly; L = Lee; Mi = Mitch; Mu = Mulder; P = Pierre; R = Robin.;
^cThe underlining in the table indicates the first individual mathematics counseling meeting for that participant.

Once participants had volunteered for individual mathematics counseling, we negotiated meeting times, and by the end of the fourth week of class I had met with each of the participants at least once for mathematics counseling (see Table 4.2 for complete schedule of the research). The number of counseling sessions ranged from three to eight, with an average of five per participant.

I did not make my choice of individuals for the focal cases until after the course was completed so that during the sessions I would be equally focused on all 10 participants. I audio-recorded counseling sessions and had ones I identified as key transcribed. My roles in sessions varied with the participant and the timing of the session (e.g., the proximity of an exam).

Mathematics, Affect, and Relational Data Collection and Use

Instruments for Assessment and Treatment

Because I was piloting the brief relational counseling approach, I knew I must identify students' relational patterns and both affective and cognitive symptoms to be dealt with in the brief time available. I devised, adopted, and adapted a number of survey, emotional response, and mathematics cognition instruments, some of which I administered to the whole class and others to individuals in counseling sessions. In chapter 3, I discuss my development and choices of individual instruments and indicate my proposed use in counseling (see Appendix B for the individual instruments). Also in chapter 3, I discuss my choices of class survey and mathematics instruments and indicate my proposed use of the instruments in counseling (see Appendix C for the class instruments). The individual case studies in chapter 6 reveal whether and how I actually used them in counseling. Chapter 8 includes an evaluation of the instruments' use in the

counseling process and there I make recommendations regarding their further adaptation and appropriate use.

Mathematics Data Collection and Use

I collected data about each student's mathematics skills using a statistics reasoning test (the *Statistics Reasoning Assessment* or SRA), administered to the class at both at the beginning and the end of the course, an arithmetic diagnostic (the *Arithmetic for Statistics Assessment*), an algebra diagnostic (the *Algebra Test*), all class PSYC/STAT 104 tests,^{xx} and participant-observation notes written during and immediately after classes and individual and group meetings (see Appendix C). With all but Autumn, the greater proportion of each session focused on the course's mathematics content. We used student class and assessment products to identify issues with strategic preparation and course management strategies. For example, exam analysis focused on accuracy of students' perceptions of what would be examined and how, their preparation, type of errors, and troubleshooting behaviors to enhance approaches to the next exam (see Appendix E, Table E4).

Mathematics Affect Data Collection and Use

Each student's conscious affect around mathematics learning was appraised using in-class pre- and post-feelings and beliefs surveys and discussion of his responses. The following is an example of how I used this survey data with a participant in a counseling session: When I pointed out Autumn's low score on the Learned Helpless/Mastery Oriented subscale of the *Beliefs* survey scale relative to the scale and to the class, she seemed a little surprised at first. Perhaps this was because she had offered to meet with me for my research and did not perceive herself to be in need of mathematics tutoring or

counseling. When I explained the concepts, however, she agreed that she had acted in a helpless way and at the same time revealed her disappointment with herself. It seemed that her performance learning motivation and her learned helplessness had conspired together to prompt her to a decision she later regretted.

JK: ... You answered in a way that seemed as if under certain mathematical situations, you would have a tendency to give up—

Autumn: Oh, YEAH.

JK: Or to not go ahead.

Autumn: Yeah. ↑ (laughs)

JK: Okay. All right. That's—that's—

Autumn: Definitely!

Autumn: ... I remember now that when I was in 8th grade ... I was in the higher level math class ... But I was only getting 70s, and I wasn't happy with that so I wanted to go back, so I could get better grades... I went easier.

JK: Easier class?

Autumn: Because I'm a perfectionist, and that wasn't good enough.

JK: ... rather than going and seeing how you could get your grade higher?

Autumn: Yeah ↑ I just gave up and went down.

Autumn: But I didn't challenge myself, so—(Session 2)

Autumn had earlier expressed disappointment with her later mathematics achievement.

Autumn: Um, mathematical achievement. I'm somewhat discouraged because I didn't really challenge myself enough in high school ... I kind of took the easy way out.

JK: Ahh! So you feel that you could have achieved a higher level?

Autumn: Yeah. I definitely could have. (Session 1)

Autumn revealed in further discussion that she really was not “definite” that she could achieve at a higher level; she had “challenged herself” and tried a harder class under difficult circumstances but had not gotten her required A. I surmised this was probably because she did not go for help, but she seemed to have decided it was because of an underlying inability to do harder mathematics—she took no further risks; after that she chose only classes she knew she could get an A in. Autumn had given in. In the conflict between safely preserving her high grades and achieving to what she hoped was

(but feared was not) her potential safety had won but Autumn was not happy. The learned helplessness discussion was fruitful in two ways. First, it showed me that a student's own survey responses, while, in themselves, providing limited information, could form a stepping off point for both the participant and me to explore more deeply. Second, participants, at least in this case, will likely not reveal this type of information about themselves and their motives through direct questioning; use of their survey responses and my explanation of what these responses generally indicated about them seemed to be the prompt for such revelations. To support this conjecture, although I had asked Autumn about her mathematics course-taking experiences in Session 1, she did not reveal her performance achievement-motivated (see chapter 3), course-switching behavior in 8th grade until Session 2 when I introduced her survey responses for discussion.

Relational Data Collection and Use

The principal means I used to collect data that linked affect and motives with relationality of which the participant was less consciously aware were through the metaphor and affect scales and through transference and countertransference.

Metaphor and Affect Scales Data Collection and Use

I gathered through individual metaphor surveys administered at the first counseling session and as part of the *One-on-One Evaluation* at the end of the course, an individual mathematics affect scales instrument administered at each counseling session, an individual mathematics learning history interview protocol, and classroom and individual meeting participant observation. Jamie was so unobtrusive in class that the instructor, looking back at the end of the course, wondered if she had started three or four classes later than the rest of the class (Interview 3). And she quietly slipped out of the

room whenever other participants were making their appointments. It was in discussing Jamie's metaphor that the role of her mathematics history and her personality in her present puzzling behaviors became clearer.

JK: Yeah. And then what happens during the storm? How do you, like, handle the storm?

Jamie: Um—stay inside. [both laugh]

JK: So how does that relate to the math?

Jamie: Um. Well, you have to prepare for tests. I don't know how staying inside does. (Session 1)

Jamie went on in discussion to talk about an elementary teacher who had yelled and explained her reaction:

Jamie: Yeah. You want to sit down and shut up so you don't bother her.

JK: So maybe ...you know, your presence in a classroom is very connected to that?

Jamie: Yeah. ↑ (Session 3)

Bringing this history, her metaphor, and my observations of her current classroom behavior together enabled Jamie and me to realize that she was "staying inside" in this class almost as if she were still in her 5th grade class not able to do anything but survive the storm, but this behavior was jeopardizing her chances of success in the class. That conflict became our counseling focus.

Transference and Countertransference Data Collection and Use

I noticed transference and countertransference in individual participants' relationships with me as counselor and tutor. In some cases, we discussed it, providing data about participants' and my own subconscious mathematics-related relationship orientation. Here is an example of me slipping into a countertransference:

JK: Maybe then your resistance is: you say, "This is conceptual. I don't have to do that." Maybe if you could say, "Ah this is *not* conceptual."
Rename it: "This is just mathematical."

Mulder: Pain in the butt!

JK: Am I a pain in the butt? [startled]

Mulder: No, that section of the test

JK: Well, you are doing a nice job of resisting, which is good ... (Session 5)

I was almost certainly included in Mulder's "pain in the butt" classification. Here, as in previous sessions, I was giving advice, trying to fix his problem for him like a mother of a child rather than trusting him or allowing him to find his own way, and Mulder was actually resisting my countertransference with his rebellious teenaged-son-to-mother transference "Pain in the butt!" as much as he was resisting the cognitive challenges posed by the multiple-choice questions. In previous sessions I had scolded him and pushed him to overcome his resistance to mastering the conceptual multiple-choice part of the exams. In this session I continued:

JK: Come on! Keep going! You've got a bunch of these to do. You are really resisting very well! ... And what it does to me is like I'm thinking this guy is so smart he could do so well and the mother in me comes out and it's like "If I could only persuade him."

Mulder: Yeah, I don't think you can do this one. [ignoring me]

Mulder: Hey, I'm done, I'm done.

JK: Oh, but look—there are these.

Mulder: Oh, YEAAAHH! Right on!! [very sarcastically]

JK: There are not too many!

Mulder: You make me really not want to come back here. (Session 5)

When I recognized Mulder's "teenaged son" transference and admitted to my countertransferential indulgent but thwarted mothering approach I was able to recognize the inappropriateness and ineffectiveness of this cycle we were in and soon after, I removed myself from the cycle so Mulder could focus on his mathematical challenges instead of on the power struggle with me (see chapter 6 for further elaboration). This excerpt is an example of the transference and countertransference data collected and shows how I used my understanding of the transference and countertransference in the counseling process.

After each counseling session I examined and filed dated products from the session and completed a *Mathematics Counseling Session* reflection (see Appendix B). I noted transference, countertransference, and relational dimension incidents. At the end of each day I audio-taped further reflections on the class, individual counseling sessions, study group, or other interactions that occurred that day.

Efforts to Obtain Triangulation of Data

Because much of the data I was gathering was subjectively experienced, and because understanding the interrelationships among data was essential for effectively helping participants' progress, I determined to work with a supervising counselor. There were several participants with whom I was struggling, and my own blind spots were almost certainly preventing me from seeing difficulties with others. After I had met several times with each participant, I met with a psychological counselor, Dr. P., presenting each participant and my experience of her for clinical supervision, for an expert perspective on subjectively experienced data and my responses to it, and for support and suggestions for ongoing counseling interventions.

This meeting served the purpose of supporting, challenging, and focusing my emerging counseling efforts with participants. It also served as a key triangulation tool for the case study data, that is, it ensured that each participant's and my relational data was experienced by another knowledgeable person who actively participated in the relationship. The relational dyads between each participant became triangular—among each participant, me and Dr. P. Dr. P.'s later responses to my analyses of courses of counseling when the pilot study was completed further supported this triangulation purpose.

Triangulation was also provided by the instructor's perspective on the progress of the class and individual students. By the design of the study, in order to ensure that students' course outcomes not be compromised, the instructor was blind to survey and counseling data students gave me. I interviewed her before, during, and at the end of the course to learn her perspective on her teaching, on the students in her class, and on the effects of my presence in the class. Correlating her and my experiences of the classroom provided valuable insights into students' processes and changes in the classroom and assisted the progress of counseling. All individual meetings, the supervision session, and interviews were recorded on audiocassette. One class and the lecture portion of another were video-recorded. All material is archived.

Data Collection Summary

By the end of the summer course I had collected approximately 75 hours of audiotaped data from 48 counseling sessions and nine study group meetings, and an additional 25 hours from interviews, the supervision session, and my after-class reflections. I had 56 class exams (all the exams taken by each class member) approximately 20 completed pre- and post- feeling and belief surveys, 36 mathematics assessments and approximately 50 in-counseling affect/relational assessments from the ten participants. In addition, I had almost 100 pages of divided page course and observations notes, and approximately 40 class seating, lecture interaction, and problem-working session interaction charts. I also had copies of Ann's worksheets and the worksheets I devised for use in counseling.

ANALYSIS OF DATA

Mathematics Educational Analysis

Increasingly, mathematics education research recognizes the value of carefully conducted qualitative studies of teaching and learning processes and outcomes (McLeod, 1997). In this study, a case study analysis based on the considerable amounts and levels of qualitative and quantitative^{xxi} data that I gathered and analyzed, best served to illustrate the model of brief relational mathematics counseling, developing a full picture that allows both researcher and reader to generate hypotheses that may be tested by further cases and more experimental approaches.

Despite the quantity of quantitative data I compiled, the conclusions I draw are as much those of a therapist as of a social scientist. Fundamentally, my research is an exploration of students' subjective experiences of mathematics, and of my subjective experience as a tutor and counselor helping her. In the words of Pierre Dominice, "The scientific model we have tried to respect in the educational sciences does not allow us to explore the vividness of subjectivity" (Dominice, 1990, p.199). The scientific experimental method is not usually possible with human participants because all variables except the being investigated cannot be held constant. Therefore, in education, a quasi-experimental method is frequently used. Studies using this method attempt to hold constant as many variables as possible while causing the one or two variables in focus to change. In such studies, a multitude of variables, complex interrelationships among variables, the uniqueness of each participant, are all seen to be difficulties or variables to be reduced or at least evened out as much as possible to produce the uniformity necessary to show the effects of one variable on another.

In this study, however, not only do I choose not to ignore the complexities of interactions among variables and the uniqueness of each participant, but I embrace them. Complex human beings struggle with the influences of their conscious and subconscious existence on current mathematics practices and outcomes; simplicity would be a reduction of their educational reality. Case study analysis using both qualitative and quantitative analytic techniques is therefore the optimum choice. Nevertheless, findings from some elements of this pilot study may lead to the need for future quasi-experimental studies to establish their effectiveness.

Dynamic Psychological Counseling Analysis

Psychotherapy research offers this justification for the case study method:

[T]he primary means of clinical inquiry, teaching, and learning has been and still remains the case study method grounded in the tradition of naturalistic observation. Statements about psychotherapy that are derived from group data typically have little direct relevance for clinical problems that are presented to the psychotherapist.^{xxii} (Jones, 1995, p.99)

Advances in quantitative methodology in single-case research are leading to greater rigor and greater generalizability of the findings from such single cases (Jones, 1995).

Additionally, psychoanalytic research tradition development in standardizing interpretation and treatment of clients' core relational challenges while taking care not to minimize the uniqueness and complexity of each person seem to me to be directly applicable to the mathematics counseling setting (Kernberg, 1995; Luborsky, 1976; Luborsky & Luborsky, 1995).

Counselor-participant match is an important factor not only in counseling efficacy but also in psychoanalytic research analysis of counselor and participant insight, interaction, and change (Kantrowitz, 2002; Kernberg, 1995). Counselor-participant match

can be assessed in terms of particular conflicts that arise and more importantly, in terms of characterological similarities and differences that may hinder or support participant progress. In this study, the *same* mathematics counselor (I) met with each of ten participants involved in the *same* focal endeavor—the PSYC/STAT 104 course. These constants thus reduce to manageable proportions charting individuals' progress and comparing their issues and changes through counseling. In psychotherapy, supervision is considered crucial for helping counselors to identify blind-spots in their countertransference (Kantrowitz, 2002). Because of the number and variety of participants in this study, as discussed above, I turned to supervision by a person knowledgeable in counseling psychology to help me become aware of patterns of relationship with participants that were helpful for some but counterproductive for others.

The patterns that emerged helped me identify characteristics and mathematical relational patterns of students who elicited similar or different countertransference reactions in me. For example, the motherly reaction that Mulder elicited in me was different from the one that Jamie elicited. I responded to Jamie with a nurturing, controlling mothering reaction after I had overcome her “mathematics teachers are dangerous; stay away from me” transference. On the other hand I responded to Mulder with an indulgent but thwarted mother countertransference. A key part of my method in relation to these focal participants was to analyze the countertransference they evoked in me. It became clear that a focus of study was the student-counselor dyad rather than the student or the counselor separately.

Integrated Cognitive and Relational Analysis

Used in this Study

Analysis of each participant's data and my relationship with him was ongoing and evolved through the summer. I mapped the mathematical and emotional paths the class, the individual students, the instructor and I walked, using data gathered principally to be analyzed and used with students during the study to inform the direction of their mathematics counseling. Data were also used in post-analysis of the study and in post-analysis of the effects on participants during mathematics counseling.

During the course, I studied the audiotapes, observation notes, and student products continually so as to develop strategic cognitive, affective, and relational interventions.

Relational Episode Analysis

A focal unit of study was the mathematics *relational episode* (cf. Luborsky & Luborsky, 1995, and see Appendix E). Each episode was analyzed and triangulated with other data to determine what it revealed about the participant's central mathematics relational conflict. In acknowledging that it is her unresolved mathematics relational conflict that is preventing her desired achievement, it is important to see that this means that the student is struggling with a conflict of which he is only partly conscious. She is likely then to say and do things that appear contradictory, but it may be these very contradictions that reveal most about his central conflict (see Appendix E). To identify this central mathematics relational conflict, relational episodes were juxtaposed that revealed insights into each of the participant's three personal dimensions identified by Mitchell (1988): the mathematics self, internalized presences, and interpersonal

attachments. In Appendix E, I provide a discussion of analysis categories and the procedures and theory used to develop them.

Conversation Analysis

In order to communicate what transpired in a mathematics counseling sessions I needed to find ways of coding session transcripts to not only indicate transcription technicalities such as impossible or uncertain transcription, but also to indicate a sense of timing, emphasis, and degree of agreement, and to allow for explanation of concurrent activity. I found some of the conversation conventions developed by Deborah Tannen (1984, p. xix) and those used by Anne Dyson (1989, Figure 1.1., p. 4) to be useful. I developed some of my own for functions they did not address, modified some where their Table 4.3.

Conventions used in Presentation of Transcripts

↑	marks enthusiastic agreement with other speaker
↓	marks hesitant or minimal agreement with other speaker
=	marks somewhat agreement with other speaker
(+)	marks positive affect in tone of speaker
(-)	marks negative affect in tone of speaker
ɿ	marks a glottal stop or abrupt cutting off of sound.
NO	that is, capitalized word or phrase, indicates increased volume.
{ }	includes parallel or immediately contiguous speech of the other person of the counselor-student dyad. If it is a person other than the counselor-student dyad speaking, that person will be named.
* *	indicates intentional waiting or pause time.
...	indicates omitted material.
//	with no text included indicates that transcription was not possible
//	with text included indicates uncertain transcription.
()	includes notes referring to contextual and nonverbal information, for example (laughs), (surprised), or (unconvinced).
[]	includes explanatory information inserted into the quotation later by me.

[I use conventional punctuation marks (periods, question marks, exclamation points) to indicate ends of utterances or sentences, usually [marked by conventionally agreed intonation changes and] slight pauses on the audiotape. Commas [indicate] pauses within sentence units. Dashes (--) indicate interrupted utterances (Dyson, 1984, Figure 1.1., p.4).

distinctions were too fine for my purposes, and changed some for ease of word processing (see Table 4.3.).

Mathematics Behavior and Product Analysis

I developed different coding categories for student verbalizations and behaviors during the major different in-class experiences. From analysis of the class lecture session data I developed the following coding categories for student questions, answers, and comments: (a) timing,^{xxiii} (b) accuracy/relevance, (c) topic,^{xxiv} (d) level of certainty (affective and cognitive), (e) frequency, and (f) development. From analysis of student behaviors in problem-working sessions, I developed the following coding categories: (a) topic/task, (b) seating, (c) tools,^{xxv} (d) interaction with instructor, and (e) interaction with researcher. From class exam data I developed coding categories for individuals and for the class: (a) pre-exam input (class treatment, student reaction and counseling preparation), (b) student's out of class preparation, (c) errors,^{xxvi} (d) trouble-shooting efforts, (e) instructor grading, and (f) post-exam counseling (see Appendix E for chart organizers of these coding categorizer schemes). What the analyses of class lectures, class problem-working, and class exams revealed about the student's personal mathematics relational patterns and central conflict I considered as the course

Counseling Use of Analysis.

As the study continued I devised ways to integrate data of different types so that I could use them to counsel participants and clarify their challenges. They were also used as interventions (e.g., *Survey Profile Summary Sheet*, see Appendix B and chapter 6). With each participant I used insights and suggestions from the supervision discussion of

their data into following counseling sessions. The integration of data in supervision discussion increased my efficacy as a counselor.

Post-analysis of all data, including participants' final evaluations and exams focused on relational episodes and their cognitive and affective links to relational patterns. The timing and fit of the ongoing analysis and the researcher's understanding of each participant's central relational conflict and related counseling interventions were determined. It was then that the three focal cases for deeper post-analysis and presentation were chosen in order to illustrate the brief relational counseling approach.

BRINGING IT ALL TOGETHER INTO A CASE STUDY ANALYSIS.

The PSYCH/STAT 104 Class as the Individual Case Context

In chapter 5 I narrate the story of the class as a whole. That narration provides the basis for analysis of individuals' interpersonal relational patterns in the classroom context. Since the focus of this study is on the individual counseling and the student-counselor dyad, the particular value of examining the classroom context lies in the context it gives for the focal student case studies I present in chapter 6. In addition, when I conducted a comparative analysis of all participants' mathematics cognitive preparation and relationality, I expected a student classification to emerge not unlike Tobias' tier scheme. Tobias (1990; personal communication, March 16, 2001, May 20, 2003) formulated a tier analysis of science and mathematics undergraduates as they appear to academic support personnel. Given that Tobias's tier classification is accepted in the field of developmental mathematics education, it is, in a sense, the null hypothesizes classification scheme. As such I decided to use it for comparison purposes in describing the classification scheme that emerged from this study. In addition, and perhaps more

importantly, I considered that Tobias's tiers describe students she sees to be increasingly more vulnerable and in increasing need of academic support in order to succeed. It seemed advisable for me to take this into consideration in choosing my focal cases: When I choose from the ten participants, I chose students from vulnerable tiers. I had also to consider however that my study might identify other criteria that should influence my choice of focal cases. Tobias describes students in her tiers as follows:

The First Tier

Students of the first tier are those who enter college well-prepared and confident, that is, with mathematical power (NCTM, 1989, 2000). They have developed conceptual understanding, are procedurally competent and are ready for new mathematical learning. Academic resource centers or mathematics centers frequently recruit mathematics peer tutors from this group.

The Second Tier

It is mostly the students in the tiers below who come to the attention of academic support personnel. Tobias identifies students in the second tier as capable students who have become convinced they "can't do mathematics." She observes that many of these students have learning styles different from the learning styles favored in the traditional mathematics classroom. They may be more verbal; they more often favor right-brain and visual thinking; and they are usually divergent thinkers and global (in contrast with analytical, cf. Witkin, Goodenough, & Karp, 1967; Davidson, 1983). It is not so much the mathematics subject matter but the pedagogy that has been the stumbling block for them. Depending on when and how these students experienced, "I can't do mathematics," they are more or less mathematically prepared. Almost invariably they believe they do not

have mathematical minds. Because most of these students are college bound, however, they may have struggled through three or even four years of traditional high school mathematics, often through precalculus.

The Utilitarians' Tier

Students in the next tier, whom Tobias has designated utilitarians, have in her words “learned to play a mathematics game.” According to her, they are procedural learners who are competent but not interested in understanding the mathematical concepts. They may have succeeded in traditional mathematics classes that emphasized procedural competence but may be unprepared for and resistant to the greatly accelerated pace and greater conceptual demands of some college mathematics courses. They may become angry if they fail or do poorly and they may be resistant to suggestions involving changing their ways of approaching mathematics.

The Underprepared Tier

In high school, many of these students were either not expected to attend, or did not intend to attend college, or if they did they did not expect to have to do mathematics in college, so they did little or no algebra. Others attempted some algebra in high school but were never engaged or did it a number of years ago. Still others “succeeded” in poorly taught or lower track classes. Whatever the reason, the underprepared have serious gaps in their knowledge base and often a poor mathematical self-concept.

The Unlikelies' Tier

These are students Tobias designates as those “we can never reach.” They include students who are hostile and “won’t give us trust” (Tobias, S., personal communication, March 16, 2001). But with the “unlikelies” Tobias hesitates to cite lack

of mathematical ability as a cause of their difficulties and poor prognosis. Academic support personnel typically err on the side of faith in the ability of each student to transcend her difficulties, given the right combination of circumstances, change of heart, and support. However, most academic support personnel can point to students who would not or could not budge. In my experience, students least likely to succeed were those who are unable to confront their own difficulties honestly.

Choosing the Focal Participants

I chose three students, Karen, Jamie, and Mulder, for deeper case study analysis, using a number of criteria. My most important consideration was how their mathematics counseling illustrated different dynamics between the student and me involved in finding a central relational conflict and how we used this insight to improve the student's mathematics mental health and success in the course. I also considered Tobias's tier analysis, however. With respect to Tobias's tier analysis, Jamie would probably be classified as second tier and Karen and Mulder had characteristics of the underprepared and unlikelies, and, even in some senses, utilitarian tiers. Their stories are presented and analyzed in chapter 6.

The focal participants were in many ways typical of students in need of support in their college mathematics course. Jamie and Mulder were traditional college aged, full-time students and had at least one parent who had a bachelor's degree; Karen was a little older, a part-time student, and the first in her family to pursue a bachelor's degree. Karen had previously failed the class and said she had always been poor at mathematics; Jamie had previously earned a D^+ in the class and reported an uneven mathematics history, doing well or badly at different times. Mulder had not previously taken a college

mathematics course, and reported a history of not trying in high school mathematics classes and just getting by with Cs.

These students reported family theories about their mathematics ability—Karen reported that hers was a reading and writing type family, Jamie said her mother’s theory was that the women in her family were not good at mathematics, and Mulder speculated that he was probably capable of doing mathematics because his uncle and father were “smart.” Not only were the focal participants similar and different in their histories, families and attributions, they also appeared immediately typically needy but for different reasons and in different ways from the perspective of the learning support center. In chapter 6 when I present the counselor-student dyad cases with Karen, Mulder, and Jamie I will discuss further these similarities and differences and their significance to my case selection.

I present these students in the context of the class in the next chapter and zoom in on their courses of counseling in chapter 6 in order to illustrate the development and application of brief relational counseling to identifying and treating central mathematics relational conflicts.

ⁱ I have given all institutions and locations mentioned in this study fictional names to preserve confidentiality.

ⁱⁱ The Learning Assistance Center has copies of all the mathematics course texts, student study guides, and student and instructor solution manuals. Instructors are requested to file their syllabi and class handouts with the Learning Assistance Center so that the peer tutors and I can keep pace with the courses as they progress through the semester.

ⁱⁱⁱ This class-support tutor has been variously labeled class-link tutor and class tutor. Typically this person would be a peer tutor (usually an undergraduate who has successfully completed the course), but it is not unheard of for a professional tutor to fulfill this role (M. Pobywajlo, personal communication, January 24, 2000; Petress, 1999).

^{iv} All course numbers have been changed to ensure confidentiality of the institution in which the research was conducted. The first digit used here is designed to indicate the level. For example, the number 104, with 1 as the first digit is a first year college level course. The course is described in the course catalog as follows:

PSYC/STAT 104 (freshman level)

Design, statistical analysis, and decision making in psychological research. Substantive problems as illustrations of typical applications and underlying logic. No credit for students who have completed BUS/STAT 130 or BIO/STAT 105 (fulfills quantitative reasoning general education {core} requirement). Special fee. 4 cr. (From the on-line Brookwood State University Course Catalog)

^v The class was scheduled for Monday and Wednesday evenings 6:00 p.m. to 8:20 p.m. on the second floor of the Riverside campus building and ran from Wednesday, May 31 through Wednesday, August 2, 2000.

^{vi} The names of all persons in this study have been changed to preserve their anonymity.

^{vii} The average attrition rate (drop, withdraw, fail) from 1995 through summer 2000 for PSYC/STAT 104 was 26.6% over all. This breaks down to an average 31.4% attrition rate for Fall/Spring semester courses and a much lower 14.75% attrition rate for the summer courses (archived grade reports, Brookwood State University).

^{viii} In keeping with my former practice, as this is an even numbered chapter I use “she,” “her,” and “hers” as generic third person singular pronouns.

^{ix} This course was taught by an adjunct psychology professor.

^x The teacher tells about and the students are expected to passively absorb the new material. See also chapter 2.

^{xi} See chapter 2, endnotes xvi and xvii.

^{xii} Although Australians are considered racially and ethnically similar and are generally well-liked by Americans, there is an assumption that America and things American are bigger and certainly better than things Australian, and that Australia and therefore Australians are cute but inconsequential in anything that matters and are expected to agree and admire. I thus struggle with belonging in the U.S., with maintaining an Australian identity, and with feeling “less than” because of who I am. On the other hand, because I am not a white American, I am not implicated in the oppression of disempowered groups here (though I am in my country of origin). Now after 23 years here, I am more an Australian American than an Australian but continue to have a connection with people who for whatever reason do not feel that they belong comfortably because of who they are.

^{xiii} I realize that merely belonging to a disempowered group or being married to someone from such a group does not necessarily mean that I understand the challenges others from the same group face, nor how to encourage them to achieve their potential nevertheless. Indeed, for example, if one is at a low level of identity development, one is likely to buy in to the majority's negative assessment or low expectations of one's group and/or be trying to distance oneself from one's group and be trying to be like the majority (Ivey, Ivey, & Simek-Morgan, 1993; McNamara & Rickard, 1989).

Americans have particular trouble with SES—few admit to having a low SES, that is, to belonging to the working class—and appreciation of values and cultures of the working class are rarely espoused (Frankenstein, 1990). The deficits are well-known: students from low SES backgrounds with parents who have not gone to college are less likely to go to college themselves or to succeed in college if they do. The Federal TRIO grant program provides extra support for such students in post-secondary education. My husband's experience of discrimination because of his SES background continues and we struggle with appreciating each other's different class strengths and weaknesses. Again identity developmental level (in this case class identity) is an issue, as is also an understanding of what might be involved (for my daughters and for my students) in learning about and negotiating the culture of power—the predominant culture in society and in academia (Delpit, 1988).

^{xiv} The summer 10-week session allowed for 4 hours and 40 minutes per week for 9 weeks and one class of 2 hours and 20 minutes in the first week. Ann did not hold class on the Monday of the week of July 4. The total class time available was then 42 hours. In contrast, during a regular semester she would have between 45 and 48 hours of class time to cover the same material.

^{xv} From Class 2 on, I used a music-scale like form and class layout form to record professor-student interaction for some portions of the lecture or lecture-guided problem portions of the class. During the class I noted the time at regular intervals during the class. I used these forms in subsequent classes and I developed an informal 2×2 charting procedure for diagramming interactions among students with each other and with Ann during the problem-working portions of class. After each class I tape-recorded my reflections on the class, professor, students, and on myself and my plans for the next class (I have archived these notes and recordings). See Appendix C for copies of the forms.

^{xvi} Parents' college experience came up incidentally in counseling with some participants, but because I had incomplete data, I surveyed participants in November 2000 by e-mail. Of the six who responded, Karen, replied that her parents had not attended college. Lee's mother had an associate's degree from a technical school but neither parent had attended a four-year college. I believe that of the others who did not respond, Robin's parents (and possibly Kelly's) had not attended college.

^{xvii} I sought approval for conducting the research from both the Lesley University Committee on the Use of Human Subjects in Research, and from the Office of Sponsored Research's Institutional Review Board for the Protection of Human Research Subjects for the state university system to which Brookwood State University belongs, and was granted that approval. I have archived the official approval documents I received.

^{xviii} In this and following chapters I use the term "participant" to refer to students in the PSTC/STAT 104 class who participated in individual mathematics counseling with me.

^{xix} I have archived all original completed forms.

^{xx} Ann followed department policy in not returning exams to students. Instead she briefly went over exams in class with students and had them returned to her. However she agreed to allow me custody of exams to use with participants in counseling sessions and gave me all students' exams at the completion of the course. I have archived these materials.

^{xxi} I determined that the primary use of new and adapted instruments would be descriptive; early use for individual affective, cognitive, and relational pattern recognition could be invaluable in helping the student

and me become aware and prioritize interventions. Already normed instruments might be useful to develop realistic goals in the context of a course. Post testing using the instruments should give students indications of change in the factors surveyed, but the most concrete indicators of effectiveness of the mathematics counseling, for the students at least, would be improvements in exams or quizzes. Causal factors for change may be difficult to determine in such a study so hypothesized relationships among factors will need further study.

^{xxii} In the past, research in psychotherapy into *outcomes* that involved pretreatment and posttreatment experimental designs resulted in findings that do not account for the real complexity and non-linear experience personal processes. Research into *process* that involved time-sampling strategies and averaging of readily quantified units such as grammatical categories of speech produced findings that seemed disconnected from the actual clinical experience and the theory behind the treatment. In any attempt at quantitative research, the problem of quantifying the “relationship between therapists and patients” arises but the fact is that this relationship “regularly appears in reviews as an important moderator of treatment effects” (Russell & Orlinsky, 1996) (p. 713). More recently, “researchers have turned to systematically conducted naturalistic studies to assess treatment effectiveness and clinical significance” (p.710). There is an important trend for researchers to “sift through the complexities of interactional and relational meaning” (p.711) and outcomes are being seen more as parts of a process rather than different phenomena.

^{xxiii} Timing is judged in terms of the extent to which the student’s verbalization is linked in a timely manner with the instructor’s utterance. For example, on a number of occasions Robin answered Ann’s question with the correct answer to a *previous* question; her timing was off.

^{xxiv} Subcategories of topic developed were: (a) current content (mathematics; application; personal), (b) course strategy, and (c) grading.

^{xxv} Subcategories of tools developed were: (a) text, (b) items provided by instructor, and (c) student provided aids such as calculator, notes, ...

^{xxvi} Subcategories of errors developed were: (a) defining the problem: concepts, (b) planning the solution: procedures, (c) carrying out the solution: algebra, (d) carrying out the solution: arithmetic, (e) conclusion: Checking and reporting

CHAPTER V

AN ACCOUNT OF SUMMER 2000 PSYC/STAT 104 CLASS

In this chapter I will briefly reintroduce the students, introduce the physical setting of the classroom, and then discuss features of the class and teacher that were salient to the mathematics mental health of the students. Those include the curriculum and the text, the instructor's pedagogy, her view of statistics and mathematics, the emotional and mathematical climate established in the class, and how the students interacted with the instructor and with each other. I will show how these features played out in the first few classes of the term and several typical or importantly different classes. From that picture, I will discuss each participant's experience of the class in relation to mathematics counseling interventions, highlighting the interactions among students' relational patterns and the classroom dynamics.

Students

The class consisted of 8 traditional aged students (18 through 25 years of age), all but one full-time. The remaining 4 (5 if I include Ellen) non-traditional students who ranged in age from early thirties through mid-forties were part-time bachelor's degree students. Seven (or possibly eight) of the students were enrolled at Brookwood University; three were enrolled at State University; and the other two were enrolled at private colleges. All for whom I had data (I do not have that data on Catherine, Ellen, Floyd or Mitch) were working during the summer, five at vocational positions they maintained all year round, and four at temporary summer positions (see also chapter 4, particularly Table 4.1).

Students¹ had differing degrees of familiarity with the college mathematics courses. All had completed at least a year in college. Mulder and Robin were the only students in the class who had not taken a mathematics course in college. Of those who had taken college mathematics courses, only Autumn, Catherine, and Lee had been successful; the rest had either failed or earned Ds.

Three students were repeating PSYC/STAT 104 because they had previously failed it in the summer of 1998. I found out after the study was completed that Jamie was repeating it because of a D⁺ on her first attempt in freshman year, not an acceptable grade for a course in her psychology major. Eight of the students were required to take PSYC/STAT 104 for their degree programs: Robin and Brad for nursing; Pierre and Catherine for biology; and Floyd, Ellen, Jamie, and Karen for psychology. For the other six, the motivation for taking the course was less clear. Two began the class with a psychology major in mind, thus requiring PSYC/STAT 104, but one changed her mind during the summer. The other began to waver on a psychology major, making it unclear whether PSYC/STAT 104 would be necessary for her. Another was taking it for elective credit to transfer. Mitch was taking the course to redress the messy situation of having to repair his GPA because he had failed it before, even though he said he believed that he should not have taken it in the first place. Kelly only needed to pass any college level mathematics course, something she had thus far failed to do.

Only two students knew each other before the class began: Lee and Mitch. Mitch was the only student in the class who knew Ann the instructor, outside of the classroom setting; he was a member of Student Government for which Ann was faculty advisor.

Physical Settings

The class usually met in a room on the second floor of the renovated former mill building that was the Riverside campus. The only classes not conducted in this room were Class 5, the MINITAB computer orientation class run by Ann and Pat, the computer lab assistant, which was held in the computer lab at the Greenville campus. Classes 4 and 9 when Exams #1 and #2 were given were held in a classroom across the hall which had individual seats and attached desk-tops. Class 10 was not held as a class so that MINITAB project partners could meet during that week.

Otherwise all classes were held in the same room. The space was almost entirely filled with six 2.5 by 5 foot tables arranged to make one 5 foot by 15 foot table, with 14 or 15 chairs arranged around it (see Figure 5.1).

I found that, although there was considerable variation in students' choice of seating, there were patterns that seemed to be connected to relational alliances, to technical constraints (e.g., Pierre's audio-taping), to the timing of a student's arrival, and

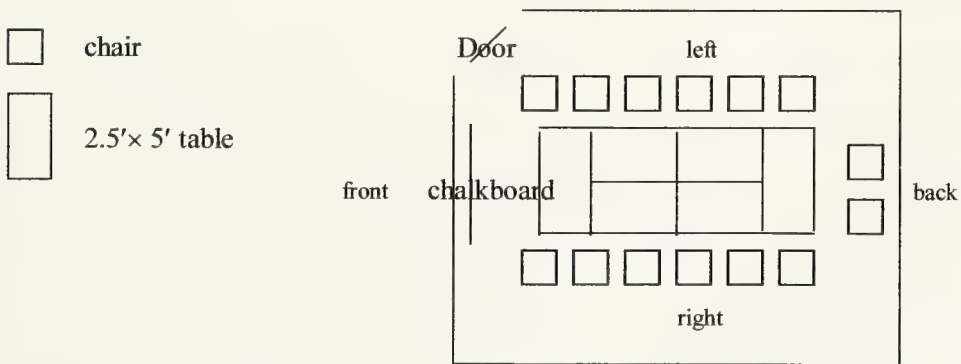


Figure 5.1. Room and furniture configuration for PSYC/STAT104 class, second floor, Riverside Center, Brookwood State University, summer 2000.

to other less obvious relational factors. Ann had previously taught only in classrooms with individual seats and attached desks facing the front and the chalkboard. She reported that her students consistently sat in the same seats. In this setting she was surprised by what she perceived to be almost random seating choices by students.

My own choice of seating was largely driven by my desire to observe the class and individual members most strategically. I know that my seating choices affected students and their experience of the class and also undoubtedly affected what I saw of the class (particularly during problem-working sessions). In Appendix F I detail and discuss seating choices—both the students' and mine.

PSYC/STAT 104: COURSE ELEMENTS AND EVENTS

Class Presentation Organization

Classes began at 6:00 p.m. The first part of a typical class consisted of Ann's presenting theory or as she said, "the concepts," with the overhead projector and the chalkboard. Ann always stood at the front of the room during the lecture portions of the class, moving from her notes on the table to the board or the overhead projector and back. The lecture took as long as the whole class period (i.e., from 2 to 2 hours and 20 minutes) if there was lecture guided problem-working interspersed but more typically went until break at around 7:00 or 7:15 p.m. (i.e., $\frac{3}{4}$ hour to 1 $\frac{1}{4}$ hours). Following the lecture, Ann usually handed out worksheet/s requiring the application of the theory just presented. She moved around the classroom checking over students' shoulders to see if they were on track. If a student seemed to be struggling, Ann would sit with him. She usually carried the worked solution so she could tell or show the students where and how their solutions

differed. When she had these worked solutions she would give me a copy so that I could help the students in the same way.ⁱⁱ

Ann used arithmetical accuracy as a quick indicator of whether a student was proceeding correctly. When I had my graphing calculator with me I would use it to enter and analyze the data. Ann was impressed with this as a quick way to find the arithmetically correct answers when she hadn't previously worked them out.

When more than one topic was being covered during a class period, Ann typically lectured on one of the topics and had students do a worksheet that was sometimes lecture guided and sometimes done with her roving help. She then proceeded to lecture on the next topic, go to another worksheet and so on (Class 2, for example). During the lecture portion of the class Ann did not usually work problems on the board. Instead, she had the students use her worksheets, their texts and her over-the-shoulder help to work them out, sight unseen, during the problem-working portion of the class. In an interview, she told me that she this was a preferred method because it forced the students to find out how to do each problem themselves (Interview 3).

The Curriculum and Textbook

The text Ann used was *Understanding Statistics in the Behavioral Sciences* (5th edition), written by Robert R. Pagano (1998). It is an introductory non-calculus based statistics text using a typical sequence. The book treats descriptive statistics in the first 6 chapters, followed by inferential statistics in the subsequent 12 chapters. Probability, random sampling and hypothesis testing concepts introduce the inferential section, followed by a "cookbook" of parametric and non-parametric tests. Ann's curriculum covered all but chapter 17 of the text, although some chapters were only partially

covered. Ann lectured from notes that kept quite closely to the text; at times she dictated directly from it.

Students were expected to read ahead in preparation for the lecture and to practice procedures and solutions after the class.ⁱⁱⁱ First a narrative introduction explains the theory, next step-by-step procedures are provided, and worked examples are given, and finally problem sets are assigned in each chapter, in that order. Material for the worked problems and problem sets is situated in realistic behavioral science settings.

The first stated goal of the course is to familiarize students with the tasks and tools of descriptive and inferential statistics so that when they take a subsequent research methods course, they can assess others' use of statistics and begin to learn to design their own studies. It is not expected that they do these things in PSYC/STAT 104; the problems posed in the text and in problem-working sessions have all been worked through to isolation of variables. There are no open-ended questions or non-routine problems. The text contains no projects to give students experience with the process of conceptualizing a hypothesis through data-gathering; the assumption is that these will come later in the research methods courses. Nevertheless, the department had designed MINITAB computer projects^{iv} where students analyze given data sets and learn to interpret results. Ann also teaches the Research Methods in Psychology (PSYC 220) course and she told me that her expectations of how much was retained from PSYC/STAT 104 were fairly low. If students have developed a basic idea of the rudiments of descriptive and inferential statistics and their differences, she is prepared to re-teach other pertinent PSYC/STAT 104 material during PSYC 220 (personal communication, September 12, 2000).

In order to “increase understanding and critical thinking about the statistics that the media presents” (PSYC/STAT 104 Syllabus, see Appendix G) Ann raised some common misconceptions around statistical ideas and discussed these briefly with the class. She took time in Class 1 to introduce such a problem using a misleading advertisement. I took this to indicate that she considered discerning misleading statistical information as an important theme for the class. In Class 6, Ann distributed an article^v that claimed, that an increase in excise tax on beer would “lead to” a reduction in the gonorrhea rate amongst teenagers, based on a correlational finding. She pointed out to the class the misattribution of a causal relationship, where a possible link was all that could be claimed. Lee was the participant who showed the most curiosity about these issues and was very eager to spend more class time than was given to explore them

Pedagogy and Student Responses

Ann’s approach to mathematics teaching cannot be easily categorized. She did not demonstrate how to do procedures; instead she employed a student-centered exploratory, problem-working approach to mastering them, expecting that students had the capacity to do it, with herself as coach. This approach would be considered pedagogically sound from a cognitive constructivist point of view. Because of class time limitations and the applied statistical focus of the curriculum, a compromise had to be made between presenting conceptual links among and within procedures to the whole class and giving students the opportunity to struggle with procedures so that they could master them. Ann chose the latter alternative but helped students with conceptual questions and difficulties on an individual basis during problem-working sessions.

Lee's experience illustrated the difficulty an under-confident but conceptually oriented student may have in a course like PSYC/STAT 104 even when the instructor and the mathematics counselor are affirming of a conceptually curious orientation. Ann admired Lee's inquisitive approach and her penetrating questions about the statistical concepts but at times Lee was not able to articulate her question clearly or there was not enough class time to pursue it. Lee's initially sound understanding that correlation cannot be assumed to imply causation as well as her sense of Ann's ability to provide a secure mathematics base were each undermined by her perception of Ann's and the text's position.^{vi}

Ann's non-directive worksheets provided students with in-class experience of working through problems on their own^{vii} with her guidance (see Appendix G). This process often challenged and even frustrated students. At the same time, each student *did* experience successful completion of at least one problem of each type. I made note to discuss in counseling both the appropriateness of their heightened emotions under such circumstances and also the pedagogical benefits of this approach. Jamie claimed to be a visual learner and said she found the worksheets very helpful, especially the ones with the columns, because she felt they complemented her learning style. Mulder also preferred to use visual learning approaches and found the worksheets helpful but he used them unconventionally and studied by visualizing his successfully worked examples on them.

Students with sound mathematical foundations (e.g., Lee) responded well to the challenge of this approach and at times went beyond mere procedure on their own.^{viii} Students whose mathematical foundations were poor (e.g., Karen and Kelly) found the

exploratory, problem-working approach difficult and became anxious. Used to having procedures demonstrated, Karen^{ix} felt abandoned and helpless when she was expected to negotiate such procedures on her own. Both Karen and Kelly complained that Ann had not been “thorough” in covering the material before the first test. It may have been the absence of familiar solution demonstrations they complained about. I was able to support some students as they worked through frustrations with Ann’s exploratory approach (e.g., Karen). With this help, they found that they eventually benefited from having to struggle to master the procedures on their own.

Students reacted differently to what appeared to them to be a *laissez-faire* approach to linking the statistical concepts with their underlying mathematical basis and to understanding the formulae to number to concept links. All students in the class except Lee, Robin, Catherine, and perhaps Pierre were used to following a procedural approach to mathematics. Because Ann allowed students to use formula sheets in exams with some verbal identifiers and charts, there was a reduced load on memorization of formulae but an increased call for understanding differences and similarities among formulae. Because the concepts were not uniformly connected to procedures during class, some students found learning new formulae and procedures to be onerous and memory-dependent, because they seemed new and different rather than being rooted in previously mastered material.

Even though the more procedural learners were used to this experience, the fact that they depended on their memory of dimly understood, individually mastered procedures kept them vulnerable. Generally they were without the mathematical tools for monitoring and checking and this kept them anxious and dependent on factors they often

felt were beyond their control. Students like Karen and Brian tended to approach each inferential test as if it required an entirely new procedure—another observation I used to inform my mathematics counseling.

Conceptually oriented students found Ann's indirect approach to the conceptual linking difficult in some ways, especially if their confidence in their own ability to discover these conceptual links was shaky. Lee was the most vocal of the participants about her difficulty with this approach but she struggled to make connections herself—she attended study groups and met with me to explore and seek answers to her questions. Lee spent little time doing homework on her own (20 minutes a week, see Appendix H, Table H1) and expressed high anxiety. This may have been related to her difficulty in acquiring a secure conceptual base more or less by herself.

Pierre used an opposite tactic to try to gain a conceptual understanding of the material. He spent many hours (17 per week at least, see Appendix H, Table H1) studying the text and other materials he got from Ann and meeting frequently with Ann and me. This broad-based, over-inclusive approach was done at the expense of mastering the procedures to be tested and, therefore, at the expense of earning a good grade (at least through Exam #3).

A challenge for me in counseling was to support students' strategic pursuit of the conceptual links that were not provided in class and to help them embrace rather than resist the real benefits Ann's approach afforded them in mastering the material.

Mathematical and Statistical Challenges

Ann was confident in her grasp of the statistical concepts, but she was less confident of her grasp of the links between the statistical concepts and the mathematics

used to explore them. The mathematical challenge of this course lies principally in being able to understand, decode, and link data, and information about data, with appropriate symbols or formulae, and in being able to adapt and apply mathematical understandings to an unfamiliar problem situation. For example, the *order of operations* agreement requires that to compute ΣX^2 one must square all the Xs first before one adds them (i.e., work exponents before multiplication or division, which is, in turn, worked before addition or subtraction), whereas for $(\Sigma X)^2$ one must add the Xs first and then square the result because of the parentheses that require attention to operations inside before doing anything else (essentially allowing one to cut in line). In algebra an equivalent situation might look like $X^2 + Y^2 + Z^2$ where $X = 2$, $Y = -3$ and $Z = 1$ compared with $(X + Y + Z)^2$ when $X = 2$, $Y = -3$ and $Z = 1$. If order of operations is not made explicit, students often make errors that they would not if they were simply doing algebra. Because the text does not make explicit the equivalencies despite the unfamiliar look, I realized that I should include that discussion in counseling sessions.

Statistics and the Use of Already Derived Formulae

This course required very little^x manipulation of algebraic variables as is typical in a non-calculus based introductory statistics courses; there was a heavy emphasis on the use of already derived formulae. A conceptual approach to instruction might involve exploring the forms of these formulae in relation to their derivations and uses. Formulae such as the one used to find the percentile rank of a score (see Class 2) comprise all the steps of a multi-step process in one formula; this could be too complex for algebraically challenged students because of the intricate interactions among letter symbols and operations. I believed students might understand the finalized complex formulae if they

explored and mastered the process using estimation, proportional reasoning and dimensional analysis. In Study Group 1, that is how we approached it (see below). We extended beyond using a formula for a percentile rank to find its corresponding score using the text's step by step approach, and it seemed that the work in Study Group 1 did complement the text and class work and forged conceptual connections for some of the students.

Early in the course Ann showed a preference for using an empirical (rather than computational) process and formula for finding the standard deviation. She said she wanted to help students develop a sense of how and why the formula was derived and is used. She had students work the procedure in Class 3, but time constraints and most students' procedural orientation led to a predominantly procedural focus for most students. In Classes 6 and 7 the concept of deviations and squared deviations from the mean reappeared (now in the context of two rather than one variable in correlation and regression analysis). Now Ann had students use the computational formula rather than the empirical one, and did not link the idea to students' prior work on deviations. This was perhaps because it was now being applied to two variables and between the variables, in two dimensions rather than one. Although the standard deviation concept was the same, the uses and interactions may have been more complex than Ann felt the students needed or had the time to explore. There were other mathematical themes that Ann did not point out to students such as the fact that the function of all the z and t statistic formulae is the same.^{xi} I resolved to address these strategically in counseling. For example to demonstrate the equivalencies of the t and z formulae, I decided to use

comparative diagrams (for all) and algebra (only with students who had a level 4 understanding of the variable on the *Algebra Test*).

Multiple Uses of Letter Symbols

The multiple uses of letter symbols seemed to be the cause of much confusion even for relatively algebraically confident students (as noted below in my discussion of Class 2, Exam #1, and Class 13). These different uses are not usually discussed in application classes like this one, yet they are particularly salient in introductory statistics courses because of the heavy emphasis on the use of already derived complex formulae. Philipp (1992) notes that current teaching practice in algebra does not address these different uses of letter symbols explicitly. In introductory statistics courses instructors do typically discuss the symbol classifications of random variable (a true variable), parameter (constant for a particular population) and statistic (constant for a particular sample). What Ann did was identify names and meanings of important letter symbols as the text did. She required accurate memorization of these on the tests, giving up to ten percent of test grades to symbol identification and meaning. However, she did not discuss classes of symbol, nor draw attention to the multiple uses in one formula, or how the symbols differed in their uses, and how they were related to the mathematical content and each other.^{xii} Because Ann did not provide secure base support in this for students to explore and develop these connections, I took it to be part of my complementary teacher-parent role to do so in mathematics counseling sessions.

Group Learning

From the first class, Ann provided opportunities for students to work together both informally and formally. The ways students did or did not take advantage of these

opportunities or form alliances independent of Ann provided important data about their mathematics relational patterns that informed counseling. Although there was no effort to organize students to work in groups in class, a paired getting-to-know-you interview in Class 1 and pairing up to work on and present the MINITAB computer modules at the second to last class presented opportunities for students to form study alliances. Ann encouraged students to use the class contact sheet with e-mail addresses and phone numbers to contact one another. The only pair of computer project partners to work together on other aspects of the course was the Lee-Mitch pair who had known each other before the class began.

Whether students worked together during the problem-working portion of each class seemed to depend on where and beside whom they were sitting and on their established interpersonal relational patterns. Lee (a social learner^{xiii}) initiated and maintained contact with Mitch; Robin with Brad (both older and nurses) worked together. They formed pairs that fairly consistently sat together and worked together on the problems. Lee and Mitch were also MINITAB computer project partners. Mulder (who was also a social learner) would work with whoever sat beside him unless it was a loner who would not engage. Autumn, Karen, Catherine, (and Mitch if he weren't with Lee) Jamie, and Pierre were all loners, rarely working with others, especially other students. Autumn, Karen, and Catherine (and Mitch) seemed to be loners by choice (voluntary loners), but Jamie and Pierre worked alone more because of constraints they seemed to feel precluded choice (involuntary loners). Jamie and Pierre appeared to want to be more involved with others.

During problem-working sessions, in particular, how these distinctions played out in class was affected by student seating choices and apparently affected the relative value students received. For example, when Mulder, a social learner who found the lectures difficult to process and relied on the problem-working session, sat between loners Autumn and Pierre, he worked on his own (Class 3). That he did poorly when that material was examined in the first exam may have been related. These distinctions also seemed to affect the amount of support students received from Ann during problem-working sessions. For example, because Jamie rarely used body language that would invite Ann's intervention, such as moving to allow Ann to see her work as she went by, Ann checked her work and offered her assistance less than she did the other students in the class (cf. video-recording of Class 16, archived). Because I observed how students related (or not) with Ann (and me) in the classroom and I discussed with the student in counseling, what that revealed about their teacher attachment patterns, some participants were able to recognize and modify such behaviors they now recognized as counterproductive.

My analysis of student seating choices indicates that, contrary to Ann's perception of randomness, most students were quite consistent in their seating choices and that my choices did not appear to influence theirs. My seating choice did affect the level of interactivity of my immediate neighbors during problem-working sessions, however, especially voluntary loners like Karen who would not work with her peers but would work with me. Seating choices of those who were not loners did seem to be related to and affected the level of collaboration during the problem-working sessions (see Appendix F).

Classroom Emotional Climate

The PSYCH/STAT 104 class had a generally positive emotional climate. The course was taught in a manner that had the potential to develop, maintain, and repair attachment relationships between teachers and students and between students and mathematics. Ann provided the elements of such an environment, but that did not mean that each student was aware of it nor received it as a benefit.

In Ann's course, Jamie and Karen, for example, each of whom came to the class with a history of mathematics classroom experiences that had negatively affected them, did not initially perceive Ann's classroom as safe for them and could not benefit from her positive offerings. In counseling sessions, I saw an aspect of my role as helping them investigate whether this classroom climate might be different and even positive for them.

Dimensions of a positive emotional climate emerged as (1) the creation and maintenance of a positive *interpersonal relational* climate and (2) the creation and maintenance of a positive classroom *mathematics* climate.

Creation and Maintenance of a Positive Interpersonal Relational Climate

There were three crucial elements to the positive interpersonal relational climate that Ann created in the class: herself as a secure teacher base, the classroom as a secure base, and fairness in testing.

The teacher as secure base. Ann provided *herself* as a good-enough, emotionally secure base for her students so that they can find acceptance and reassurance when they are uncertain, as well as the courage to move out to explore without fear of censure for going away or for making errors. Ann set the scene in the first class by self-disclosing; she described her own struggles with statistics learning and also how she managed to

overcome her uncertainties.^{xiv} Ann did not hesitate to consult with me in class if she were uncertain on the mathematical material, modeling an open exploratory approach that did not require students or even teachers to have perfect understanding. I, too, openly expressed my puzzlements.^{xv}

Another feature of Ann's approach was that she did not call on individuals for responses to questions during the lecture discussion. I drew Jamie's attention to this during counseling and she was then able to acknowledge to herself that, in this class at least, she was safe. She came to realize that she did not have to worry that the instructor might call on her.^{xvi} This recognition freed her to relax and even to ask a question of Ann in class.

Although Ann made herself available to meet with students and to help them with the course material (because she believed mastery itself would allay anxiety) she did not believe it appropriate for her to get involved directly with students' emotional problems with mathematics or the class. She neither invited nor required student disclosures.

The classroom as a secure base. Ann modeled and monitored interpersonal classroom behaviors to ensure that all students were safe. The way Ann dealt with incorrect or half-correct responses during the lecture sessions set the tone. She considered the response, found what was reasonable in it, responded, and moved on respectfully. Whether a student perceived this positively depended on his^{xvii} already established interactional patterns. Karen gave an incorrect response to a question during the first class but despite Ann's respectful response, subsequently responded only to questions requiring a non-mathematical response.

There were no incidences of student to student disrespect during the course. Lee did object to the fact that (in line with department policy and for statistics education purposes) Ann distributed a histogram of exam scores after each exam. Because it was relatively easy to identify each person's grade given the small class size, Lee felt that this was not respectful to students who did not wish to reveal their grades.

Fairness in testing. Ann seemed to make it a priority to be explicit and fair but Karen and Kelly, for example, did not see that. This became a focus in counseling because while they were externalizing their difficulties and scape-goating the instructor they were not taking the control they needed to negotiate the course.

Before each test Ann was careful to give a study handout with a list of the symbols that would be tested and specific homework problems from the text. She also handed out solutions to even-numbered problems from the teacher's edition of the text (for an example, see Appendix G). More importantly she made sure to teach everything that she tested; in particular she made certain that each student completed each type of problem correctly in class. Ann allowed unlimited time as well as the use of a formula sheet on tests. She provided helpful organizers, including the list of six steps of hypothesis testing, so that students could incorporate this into their formula sheet (see Appendix G).

If a scheduled test time was inconvenient, students could take exams early, though not after the scheduled time. Ann's optional comprehensive final could also replace one missed exam and could be used to replace a poor exam grade during the course.^{xviii} The relative proportions of the grade allotted (Ninety percent of the grade was earned from exams and 10% from computer analysis projects.) seemed to accurately parallel the effort

and emphasis required in the course. The heavy weighting of exams may have contributed to the class' collective mathematics testing anxiety remaining considerable: It changed from 2.9 (on a scale of 1: not at all frightened, to 5: very frightened), to 3.0 on that scale. Only two individuals' testing anxiety levels fell substantially during the course while three individuals' anxiety rose substantially and the others' remained substantially the same (see Appendix C for the surveys and Appendix H, Table H3 for student changes). One whose anxiety abated somewhat still expressed elevated anxiety (3.6); in fact of the 9 students remaining in the class, 7 expressed anxiety levels of 3 or above.

The Creation and Maintenance of a Positive Mathematics Climate

Ann provided herself and the classroom as a secure relational base, but even that was not enough to create a good-enough mathematics classroom climate. Her attitudes towards her students' ability and potential to learn mathematics and the way she taught mathematics and supported students were also essential. In particular, her belief in every student's potential to master the statistics (given adequate support) and her promoting the authority of the mathematics over her own authority were key. This was evident in Ann's willingness to acknowledge her own uncertainties about the mathematics and refer to others (me in this case) who could not only help her understanding but also was there to support her students mathematically.

In Ann's assessments of students' likelihood to do well in this class, her central consideration was whether their mathematics background was adequate and whether they would apply themselves sufficiently to succeed. She made no trait judgments that might have locked students into doing poorly because she expected it. She did not believe that

some people could do mathematics and others not. Ann's expectations seemed to be influenced by students' classroom behaviors^{xix} and by a constellation of age, gender, and particular major. For example, she (and I, initially) expected Robin, an older (in her 40s) nursing student who often appeared flustered and confused in class, to have trouble and perhaps do poorly. On the other hand Brad whose classroom behavior was confident and apparently relaxed, Ann expected to do well despite his being an older nursing major (Interview 2). In each case Ann's expectations were challenged by the student's achievements—Robin did well while Brad struggled to get C's. However, I never observed that Ann's expectations affected how she related to or graded a student.

This applied course was taught by psychology rather than mathematics faculty, and because of that an important complementary role emerged for the mathematics counselor. When uncertain about the mathematical bases for the statistics, as noted above, Ann was very open about drawing from my mathematical expertise in class. Her pedagogical approach, especially her use of problem-working sessions, reinforced the statistics/mathematics as authority rather than the instructor. Mathematics counseling was pivotal in complementing and supporting Ann's mathematics teaching because of varied student comfort with and responses to it at least initially.

An important part of students' developing a sense of mathematical safety was the support offered *outside* the classroom, especially for those whose low confidence made it difficult for them to study and practice on their own. Ann repeatedly offered extra time and help to students. Because I was so available and she was at State University in another capacity several days a week, most students saw me more than her outside class but they were well aware of her openness to helping them. She stayed after class to help

anyone who came and helped out in several study groups before tests. All six students who filled in the *Class-Link Evaluation* at the end of the class responded positively to my contributions as a class-link tutor but only Lee, felt that Ann had relied too much on me to give support to students (see Appendix C for the form; student responses are archived).

CHRONOLOGY OF PSYC/STAT 104, SUMMER 2000

The class chronology underscores the significance of understanding students' mathematics relationality within the whole class system. To describe class process, interactions, and student outcomes, I will describe in detail the first three weeks of the course through the first test and I will discuss how this was the first of several cycles of class, study group meetings, individual counseling sessions, that culminated in an exam. I will then sketch key events that occurred during the remainder of the course (see Appendix I for a complete calendar of events for the class).

The First Cycle through Exam #1

Class 1

The first class consisted of introductions along with an overview of the syllabus, course schedule, and assessment procedures and an interactive lecture on the first chapter of the text. All of the 13 students Ann expected were there except for Mitch who would be at the next class.

Ann began to establish the relational climate that accepted struggle and acknowledged the importance of collaboration and mutual support by self-disclosing her own statistics anxiety (see endnote xiv), by asking the class to pair off, interview each other, and introduce his interviewee to the class,^{xx} and by organizing an exchange of e-mail addresses and phone numbers.

After break, Ann used the overhead projector to show an advertisement for paper towels that used misleading graphics and numbers to compare with its rival. Lee was quick to respond accurately to Ann's questions about it. During the lecture, Ann directed her questions to the class as a whole, not to individuals; if there was no response within two or three seconds, she answered them herself. Ann asked and then explained what statistics was, using the definition given in the text: "A way of organizing, summarizing, and understanding data." Data is "information collected and generally understood at a numerical level." All of the students wrote the definitions in their notes. Next the class discussed the scientific method and Mulder responded by referring to his research project on caterpillar aggression.

The classroom interactions proceeded in the following pattern: Ann presented a concept, she asked a question about it of the whole class, a student or group of students responded (or Ann when there was no quick student response). Ann responded to student responses, and then cycle began again. Ellen, Robin, Mulder, and Brad responded during this discussion. Robin seemed to have some concepts confused but Mulder, Brad, and Ellen appeared to have a good grasp of the big ideas. Karen responded to a question incorrectly. Ann dealt with this by respectfully considering Karen's answer, correcting it, and moving on. Neither Catherine nor Autumn offered any responses but they appeared to be actively and knowledgeably engaged in observing the interactions and they were taking notes. Pierre also did not offer any reactions but he was working at his notes and attending to the interchange. Jamie alone did not seem to be involved. She kept her eyes lowered, not making any eye contact. She did take notes but at times I wondered if she

were asleep. I did not observe any interactions between students during this class other than the paired interviews.

Class 2

The plan for the class was to cover chapters 2 and 3 focused on basic measurement concepts and frequency distributions including finding percentile ranks and percentile points.

Before beginning the lecture, Ann introduced me as researcher and academic supporter (class-link) for the class. I invited students to a weekly study group before each Wednesdays' class in room 207, where the class met. Mitch had joined the class but Ellen was absent.

In the first half, the class worked on a teacher-directed, lecture-guided data sorting exercises, classifying and sorting different types of data according to measurement scale^{xxi} and finding the median, mode, and mean of a set of ratio data (time in seconds for 20 rats to run a maze). Next came sorting data into a grouped frequency distribution. Ann provided worksheets for these exercises and she used the overhead projector to gather class responses. The problem-working interactions were almost exclusively between individual students and Ann, rarely among students.

Just before break, according to prior agreement with Ann, I administered the surveys I had prepared—the *Mathematics Beliefs Survey*, the *Mathematics Feelings Survey*, and a short mathematics background survey (all class surveys are in Appendix C) and invited volunteers for the individual mathematics counseling sessions. Nine students of the twelve present volunteered to be participants.

The focus of the second part of the class was to learn to use the grouped frequency distribution to find a score given a percentile rank. For example, we had to find the number of seconds it took a rat to complete the maze given that it was at the 40th percentile rank in relation to the other rats' scores. Ann commented, "Students say this is the hardest math in the course," but assured us that "the math gets easier; the concepts get harder." Ann lecture-guided us through the steps delineated in the text as each individual worked on the problem and reported his findings.

No one in the class, with the possible exception of repeating students, had seen this procedure for finding a score in grouped data, given the percentile rank. During the procedure I felt lost; I did not have a sense of the end from the beginning, the rationale for each step, nor any visual connection with the data—a very uncomfortable experience for a conceptual learner like me. I made a note to explore the logical and visual connections with students in study group and in counseling.

Perhaps more importantly for students was the fact that in the formula there were six unfamiliar letter symbols, five of them with subscripts. I suspected that students might find this procedure difficult on the test the following week.

By the end of class time the direct process had not yet been tried—finding the percentile rank for a given score. We were assigned this as homework—to find the percentile rank of the rat that took 82 seconds to do the maze. A formula was given in the text but there was no step by step procedure.

Study Group 1

The group formed before class at Riverside Center; Brad, Jamie, and Lee were there, and Pierre came in a little after we began. The group gathered at the end of the

table near the chalkboard. The first exam was scheduled for the Monday following, on chapters 1 through 5 of the text. Students' pressing concern was solving the problem that had been assigned at the end of Class 2.

I wanted the study group to be a setting where the students did the work while I coached. I expected that the students might want help with strategic planning, knowing what and what not to concentrate on for the test. Although I had tutored students taking PSYC/STAT 104 in the semester before this, the instructor was different so like the students, I was uncertain how we would be tested. Our natural questions were: Would Ann examine students on what had been covered in class or on any concepts included in the first five chapters? Would the problems be straightforward or tricky?

Even before the exam, we had some reassuring evidence that Ann's test would be fair. She had handed out a study guide for the exam that included instructions on what could and what could not be included on a student's formula sheet and a list of 13 symbols, including Σ , σ , μ , P_{50} , and z , whose definitions were to be examined. She had included an example of an acceptable definition. I had observed Ann carefully checking her lecture notes, apparently to ensure that she had covered everything. She assigned only certain homework problems from the text and handed out solutions to any even numbered ones whose solutions were not in the back of the text. With this evidence I speculated with the students at study group that Ann would test only on what had been covered.

There had been no opportunity for students to explore the derivation of the formulae for percentile point and rank using proportional geometric reasoning, so I thought the study group could try that. I put the grouped data on the board. Brad took the chalk; I coached pointing to and drawing in the geometric proportions on the board; he

had taken this class before and said that all he wanted was to know how. He did not want a deep explanation.

As students worked, Lee found an anomaly in the formula to find percentile rank of a score, namely $f_{\frac{i}{I}}$ in which the i that is the subscript of f in the numerator locates the interval in which the X score in focus is found (different for different values of X), whereas the i in the denominator stands for the size of each class interval which is constant for the distribution. The other students in the study group were struggling to sort out the other symbols so were not engaged in Lee's and my discussion of this point. She decided to use different symbols to keep them straight: an upper case I for the constant size of the class interval and the lower case i for the subscript that indicated whatever interval we were interested in. I was impressed with Lee's interest in and good analysis of this use of letter symbols and showed my enthusiasm for her approach.

Jamie was actively following the discussion of the process but was not saying anything, so I asked for her answer at one point. Although her face turned red, she answered correctly. I hesitated to direct many more questions to her because of what seemed to me to be her obvious discomfort, though I did some further questioning.

After completing the homework problem we tried one going in the opposite direction and finally stopped as other students came into the room for class.

Class 3

Ann handed out written instructions on how to construct a formula sheet for the test next class and a reminder of the symbols we needed to be able to define by giving both a description and a statistical meaning. On the board, she went over the homework

question on finding the percentile rank of the time of the rat that ran the maze. Karen was the only student apart from Catherine and those in the study group who had done it.

Ann went on to chapter 4 on measures of central tendency and variability. Following a lecture discussion of mean, mode, and median, Ann handed out a worksheet to teach the process of finding standard deviation from the mean using the rat maze time scores in a column labeled X , and two blank columns labeled to facilitate correct interpretation and use of the standard deviation formula.^{xxii} There was a brief discussion of the different formulae for standard deviations of samples versus populations and Ann called the Greek symbol for population standard deviation (σ), “omega.” Mulder corrected her telling her it was called “sigma.” Ann accepted the correction but erroneously applied it to s , the symbol for *sample* standard deviation, and continued to call σ , “omega.” I felt awkward as she continued and, perhaps somewhat inappropriately, discussed the problem with Lee who was sitting beside me. She was becoming confused especially when Ann began calling s sigma.

This was a very real dilemma. I knew that letter symbol classification caused students difficulty and now the instructor was confusing their names and was also mixing up the Greek versus Roman letter symbol categorization. A relatively consistent convention in statistical symbolization is to assign Greek letters as symbols for *population* parameters, corresponding to the first sound of the item labeled, for example μ (Greek lowercase “mu”) for population **m**ean, and σ (Greek lower case “sigma”) for population standard deviation. Roman letters are used to symbolize sample statistics, for example, \bar{X} is used for sample mean and s for sample standard deviation. In my experience, explaining this convention to students struggling with many unfamiliar

symbols had helped them considerably. I felt compelled to clear up Ann's confusion with her so the students would not be confused but on the other hand, I was very aware of the delicacy of trying to balance my multiple roles of participant as a student, class-link tutor, researcher, and colleague in relation to Ann's instructor role, and I was not sure how Ann would react to my introducing this concern.

When I did address it with Ann privately after class her reaction reaffirmed my prior assessment of her healthy self-reliance (cf. Bowlby, 1982) and of an appropriate way for a class-link tutor to approach such an issue with a self-reliant instructor. Rather than my telling her of the error, I pointed it out by using the text as the authority. Although a little embarrassed, Ann reacted positively to my addressing it in private and to now knowing the correct designation for σ and s . The way I handled the incident seemed to contribute to Ann's confidence in my expertise because she encouraged me to contribute it in the future. At the next opportunity Ann addressed the confusion directly with the class. I was then able to emphasize the Greek-Roman letter distinction for population versus sample symbols with individuals without involving them in an apparent conflict between believing what the instructor taught or what I told them. Kelly was the only student in the class who remained confused labeling σ "omega" in Exam #1.

Ann had filled in the bottom total row of the $X - \text{mean}$ column on the worksheet with the mathematical statement, $\Sigma(X - \text{mean}) = 0$, that is, the sum of the deviations of scores from the mean is always zero no matter what scores are analyzed. This indicated to me that Ann wanted the students to attend to that concept particularly but she didn't stress it in class. Its mathematical inevitability, its universal application throughout statistics, and its usefulness for checking ones' mathematical processes seemed so

important, however, that I took mental note to point some of this out to students in individual sessions.

At this point in my classroom observations I was using the *dialogue* and the *class plan* as recording devices (see Appendix C) for interactions between instructor and students during lecture discussions. I was not yet systematically recording interactions among students during problem-working sessions. Ann circled the room helping individuals and there seemed to be little interaction among students. Later I realized that students, who in later classes worked together, were not sitting together in this class, except for Robin and Brad. Mulder who later would work with anyone who was willing was between Autumn, who never worked with a fellow student in problem-working sessions, and Pierre who rarely did. This setup definitely did not lend itself to conversation.

After break Ann covered chapter 5 (*The Normal Curve and Standard Scores*) in lecture discussion using the rat maze mean times and standard deviation computed before break to compute a transformed z score on the board. She also demonstrated use of the standard normal tables to find the probability of obtaining a particular time score or less.

At the end of the class students who had formed pairs for the MINITAB computer module project were given their projects. Lee had already arranged to partner with Mitch. Otherwise, women sitting beside or near each other in the previous class seemed to have already paired off when Ann made the announcement. (Kelly worked with Autumn and Catherine with Karen.) Men who were sitting beside each other in class 3 paired off then: Mulder partnered with Pierre and Brian with Floyd. Robin was left out—she seemed flustered by the situation and initially said to me that she would just do the project on her

own. When I told her that I understood that it was a requirement to partner with someone, she agreed to partner with Jamie who did not yet have one.

Individual Sessions

The first test was to be held at the next class. I had individual mathematics counseling meetings with Kelly on the Thursday and Karen on the Monday just before the test. Both were very anxious, each seemed to have some fundamental arithmetic confusion with decimals and percents, and both found the large amount of material to be covered in the test overwhelming.

I had also been involved with Pierre who had to take his first test early because he would be away when it was scheduled. He was going to take it in the library (located across the hall from the Learning Assistance Center at the Greenville campus) on the Thursday (June 8). He took the day off work so that he could study. I was expecting that he would come to the Learning Assistance Center. At one point in the mid-afternoon when there was a lull in Learning Assistance Center activities, I went looking for him and found him in an otherwise empty conference room. He said that he had looked into the Learning Assistance Center earlier adding, "You were busy." I encouraged him to come in next time and begin working so I could help when I was free. There were several similar situations in the following two weeks until we finally managed to meet for his first mathematics counseling session towards the end of the fourth week of class.

On Monday, the day of the first test, Karen and Kelly came to drop-in mathematics tutoring at the Learning Assistance Center. Karen came only briefly before her mathematics counseling appointment at 4:00 p.m. Kelly seemed so overwhelmed that she was seriously considering dropping the class. I felt as if she was trying to get *me* to

tell her if she should. I told her that she had to make that decision but that perhaps taking the test might help her decide. She decided to meet with Ann to discuss her options and perhaps get *her* to make the decision for her. Kelly's mother had already called Ann and me to discuss Kelly and we had independently encouraged her to let Kelly work it out.

Exam #1

The first test was held in a room across the hall from where the class usually met. The exam room had individual seats with attached small right-handed desks all facing the front of the room. I expected a rather high level of anxiety before this first test. Karen had expressed some of this, perhaps seeing Ann as the cause, but students always face unknowns that cause anxiety in all courses even when the instructor makes an effort to prepare them, as it seemed Ann had.

On the surveys I had administered in Class 2, the class average level of *Mathematics Testing Anxiety* of 2.9 on a scale of 1 (not at all scared) through 5 (very much) with a range of 1.5 through 4.1 indicated a higher level of anxiety than for either *Number* (2.2) or *Abstraction* (2.8) and high^{xxiii} if compared with Suinn's (1972) norms on the Mathematics Anxiety Rating Scale (MARS) from which the items were drawn. Jamie's reported testing anxiety level was the highest in the class at 4.1 and Karen and Kelly's were almost as high at 3.6 and 3.5 respectively (see chapter 6 and Appendices H, K, and L). In counseling, we explored relationships among participants' testing anxiety scores and factors such as their past experiences in mathematics exams, their preparation for the exam, and their perceived ability to achieve on the current exams.

Everyone was in the classroom by 6:00 p.m. except Kelly. Kelly was considering dropping the course but Ann had just persuaded her to try the first exam so she came in a

few minutes late. Each student had to hand his formula sheet^{xxiv} to Ann and pick up part I, the part that Ann called conceptual consisting of multiple-choice questions and symbol identification questions. Once a student completed part I, he took it to Ann who was sitting at the table in the front of the room, and picked up the computational part of the exam with his formula sheet that Ann had checked to make sure it met her criteria.

Students took between 20 and 30 minutes to complete part I of the test. Part II consisted of 14 questions requiring various descriptive statistical analyses of small sets of data, all from chapters 2 through 5 of the text. All required procedures had been covered in class. Students had had the opportunity to struggle through all the procedures using worksheets in class except for the z -score questions. Those had been covered in Class 3 on the board by lecture discussion only.

Ann had agreed that I could ask students to complete Joan Garfield's (1998) *Statistical Reasoning Assessment* (SRA)^{xxv} after they completed their tests. I intended to use this as a pre and posttest to gauge changes in students' statistical conceptions. As each student finished her test she took the SRA. Autumn had arranged her first individual session for immediately following the test so when she finished the SRA (around 7:20 p.m.) we went across the hall to our usual classroom and began. Autumn told me that she thought she had done well on the test, that she had done quite a bit of statistics before, and that she didn't think she needed any help on the current course. As the other students completed their SRAs they came in and gave them to me. Catherine pronounced it "very hard" and seemed anxious about it.

Results of Exam #1

The students' test results are shown in Figure 5.2. My first impressions gathered in class, in the study group, and from individual contacts led to my being surprised by Jamie's high score because of her expressed anxiety, by Robin's high score because of her apparent confusion in class, and by Lee's relatively low score because of her insightful conceptual approach at study group. (On the other hand she had e-mailed me the night before the test expressing considerable testing anxiety and asking for my help.) Mulder's low score also surprised me, particularly his score of 22 out of 40 on the conceptual part (55%), the lowest in the class, because he had been outgoing and articulate in class and seemed to have background in the use of statistics in research. I found Brad's relatively low score was both surprising and not. On the one hand his confident demeanor and class participation bode well for a high score but on the other hand, his professed procedural approach in study group with the "only wanting to know how" and his admission that he was repeating the class did not bode well.

Error Analysis of Class Performance on Part II (computational)

I expected students to have the most difficulty on the question asking, "What score is at the 50th percentile point?" not only because of the complex nature of the formula, the multiple uses of different types of letter symbols and the largely procedural way this was approached in the text and in class, but perhaps even more because it was an inverse mathematical procedure requiring students to begin in the right hand cumulative percent column that they had to create themselves and proceed left to identify the score corresponding to the given percentile. My expectation proved to be well founded.

Only two students (Jamie and Catherine) did this problem correctly. The other nine on whom I have data (I do not have Mulder's Part II of Test #1) lost at least half of the points given for this question, and more students made errors on this than on any other question on the test. Three of these made the inverse error—that is, they began with

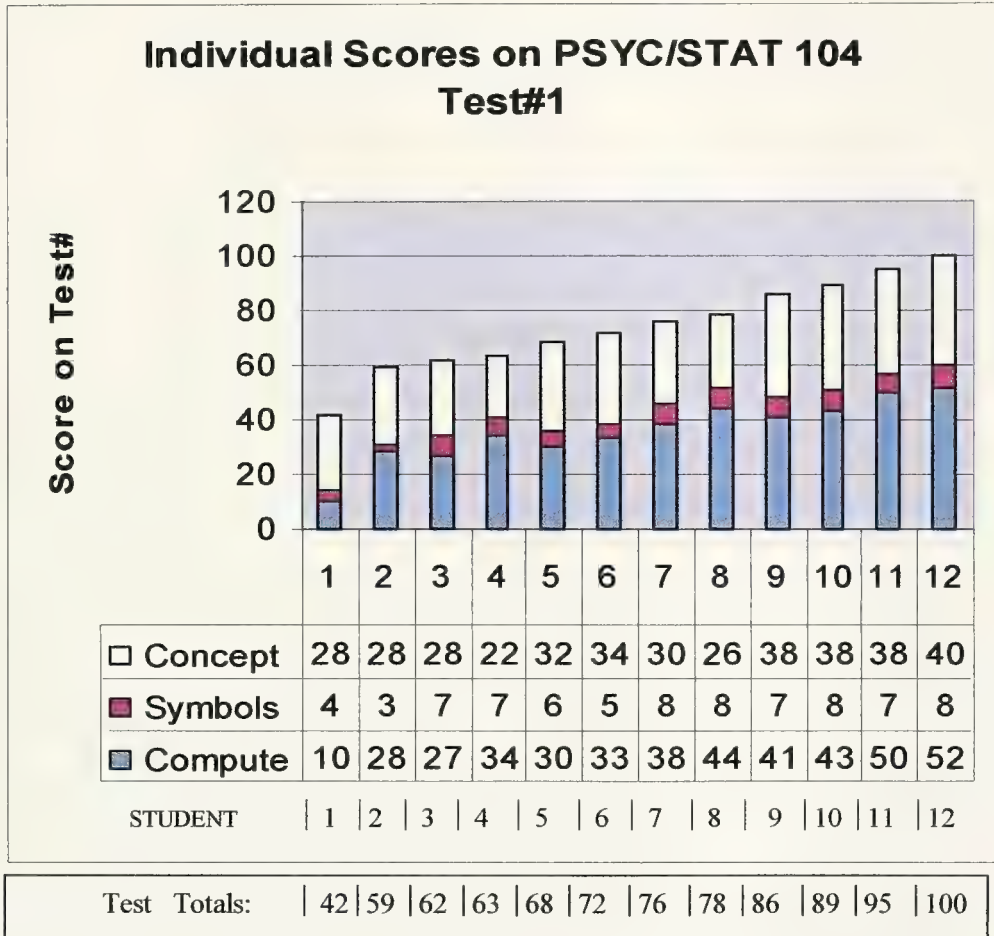


Figure 5.2. Individual's scores on Test #1, with each student's score broken down into his Conceptual (multiple-choice Part I) score, out of 40; his Symbol score (on Part I) out of 8; and his Computational score (Part II) out of 52, total possible 100.

Note: the X-axis numbers refer to individual students as follows: 1: Floyd; 2: Kelly; 3: Karen; 4: Mulder; 5: Pierre; 6: Brad; 7: Lee; 8: Mitch; 9: Autumn; 10: Robin; 11: Jamie; and 12: Catherine.

a score of 50 instead of a percentile rank of 50, but all three had not made a cumulative percent column from which they should have begun.

Multiple uses of letter symbols seemed to be the cause of much confusion in this question even for algebraically confident students. Students made the most errors identifying the correct number indicated by the letter symbol, especially when it was a multi-level symbol such as $\text{cum } f_L$ that not only requires careful interpretation, but also requires a multi-step approach to computing.^{xxvi} Six of the students made errors in correctly identifying or computing the $\text{cum } f_L$. Of the five who identified $\text{cum } f_L$ correctly, three had attended the study group.

Jamie and Lee were the only students in the class to find and correct their own initially wrong substitutions as they were taking the test.

Grading and Instructor Response to Test #1

Ann returned the tests to students at the next class with a summary of class results in the form of a grouped histogram (see Appendix G) with the mean (74.2%) and standard deviation (16.7%) of the scores. Students looked over their tests and then had to return them to Ann. Since I was to be working with individuals, I arranged to get their tests from Ann if they wished so we could do error analysis during individual sessions.

Ann seemed to use arithmetical accuracy as the main indicator of correct procedure. No more credit was given for correct process with an inaccurate result than for incorrect process with an inaccurate result. For example, in question 10, Brad lost 3 of the 4 points because he had replaced his initially correct cumulative frequencies and percents with incorrect ones and used those with the correct process to find the score given the percentile rank. Floyd, Mitch, and Pierre also lost 3 points on that question when they had

made the much more serious inverse error, treating the given percentile rank as if it were a score. With individual counseling participants who focused on grade as *the* gauge of their mathematics ability, I felt an important strategy would be to affirm logical, conceptually sound, albeit arithmetically inaccurate work as a more accurate way to gauge their understanding and ability than the points they earned (or lost).

Relationality and Implications for Counseling in the Next Cycle

Before Exam #1 I had met only with Kelly and Karen for individual mathematics counseling. After Exam #1, I met with the others who had signed up at various times and by Exam #2 on June 26, I had met with them all at least once (see chapter 4, Table 4.2). Kelly and Floyd had stopped attending class by Exam #2. All of the individual counseling participants who took it except Jamie either maintained or improved their scores on Exam #2 (see Table 5.1). Lee and Mulder's improvements were dramatic (up two letter grades); Autumn, Karen, and Mitch improved by one letter grade; the others made more modest gains. Jamie's decline was as dramatic as was Lee and Mulder's improvement—by two letter grades.

Student MINITAB partners had been working with each other since the computer lab class which was held in the computer center at the Greenville campus, but there was no evidence in class of these partnerships leading to study alliances.

The first test marked an important point in the trajectory of the class not only for the students but also for me. The test was a key piece in confirming my initial judgment that Ann was providing a positive classroom climate where vulnerable students could progress with will and strategic support. It also signaled who was vulnerable but in a crude way, the grade signifying quite different things for different people. For example,

Jamie found her 95% quite unexpected, whereas Catherine's 100% was not a surprise at all; Kelly reacted to her 59% flustered, casting about for someone to tell her whether to continue in the class or not whereas Karen and Mulder with similar grades refused to take them as a verdict on their course outcomes. Whether students perceived this positive classroom climate or not depended on their past mathematics experiences, the status of their mathematics preparation, and the relational patterns in which they were imbedded. That was a challenge for counseling: to help students experience the current positive relational reality rather than a negative reality from their past.

In this first cycle through the first test I found that Ann's provision of a positive relational climate gave me the space to negotiate a comfortable position for myself as part of the class community, and her healthy self-reliance made it possible for her to access my support comfortably. As a result, in class and in counseling there had developed an easy sense of our working together for the benefit of students. In counseling it seemed that I would not have to do damage control for current relational assaults but rather develop my role as a mathematics complementor of inevitably underemphasized or missing material from class while the student and I explored his particular relational challenges.

The Post Exam #2 through Exam #3 Cycle

Exam #3 was based in chapters 10 through 14, moving into inferential statistics with hypothesis testing using sampling distributions. Ann confessed to not being really clear on sampling distributions and how to explain them to the class. She invited me in class to offer my further explanation. I was somewhat nonplussed, not being sure what she found confusing or what she felt students did not understand. Her explanation seemed

clear to me and I said so but I knew that unless students actually created their own sampling distribution from a finite population the connections would likely not be clear to them—telling is no substitute for experiencing the mathematics. There was no opportunity for that now, however, except perhaps with Lee in a study group that only she attended.

Class 13

The material to be covered in this class on Wednesday, July 12 was chapter 13 on the Student's t test for single samples and chapter 14, on the Student's t test for correlated and independent groups. The next class would be Exam #3 on all inferential statistics and hypothesis testing through this class.

The first part of the class was a short lecture discussion. Ann explained the use of the single sample t test for sampling distributions, comparing and contrasting it with the single sample z test covered in the previous class. She briefly mentioned using the sample mean to find a confidence interval to estimate the mean of sample means of a sampling distribution of the population. She discussed power and Type II errors, and then wrote on the board a t test confidence interval problem and handed each student a problem sheet requiring hypothesis testing using a single sample t test.

Because of time pressure, Ann did not go beyond helping students work through the procedure for finding a confidence interval estimate for a population mean. Students then alternated between working alone and with the person beside them (except Jamie, Autumn, and Pierre who only worked alone) on the single sample t test problem without too much apparent difficulty.

After break beginning at 7:35 p.m., Ann gave a ten minute overview of two-sample t test hypothesis testing comparing the means of two samples. Students were directed to work on a worksheet problem, using the text for formulae and as a procedural guide. As the students began working on the independent samples t -test, individually (Autumn, Jamie, and Pierre) or in pairs (Brad and Robin, Catherine alone and Mulder checking with her, Lee and Mitch, and Karen and me), there was an audible reaction to the formulae on page 331 (Pagano, 1998). Karen growled, Mulder sputtered in disgust and they both proclaimed, “Yuck!” Robin frowned harder than usual and sighed. Mulder demanded, “So where’s the short version of this?”

There were a number of potential trouble-spots in the independent samples t statistic formula, especially the complex subscript for the estimated standard error. Although Ann had not explicitly taught the idea of subscript-as-label, students had generally succeeded to this point, apparently by ignoring the subscripts that were *monomials* (e.g., 3 in X_3 or $\alpha/2$ in $t_{\alpha/2}$). But they found this new *binomial* subscript (i.e., two terms as in $\bar{X}_1 - \bar{X}_2$) with terms that themselves had subscripts, very confusing or rather distracting. Now that they could no longer ignore the subscript, instead of understanding that the subscript’s function is labeling only, some students tried using it as part of a formula, in this case to compute estimated standard error of the difference between means ($S_{\bar{X}_1 - \bar{X}_2}$). Autumn, who always worked alone and rarely asked even Ann a question during problem-working, had done this with the subscript. In an unprecedented move, she got up from her seat and came around the table to me because she knew that what she had done with this formula was wrong but she was not sure why. She had given the subscript a numerical value of 0 (since she knew that the null

hypothesized population mean difference was zero, that is, $\mu_1 - \mu_2 = 0$), had written the population σ instead of the sample S and had finished with $\sigma - 0$ as her interpretation of $\frac{S}{\bar{X}_1 - \bar{X}_2}$. Instead she should have seen this as the symbol only and computed its value using a series of formulae given in the text. This type of error and confusion by even confident and high-achieving students led me to predict that symbol and formulae issues would cause more difficulty for students on the next exam than anything else (with the possible exception of correctly deciding which statistical test was applicable).

As each student completed the independent samples t test problem Ann checked it, and moved him on to the correlated samples t test. This did not cause as much consternation as the independent samples t test probably because the formulae are less complex and include only monomial subscripts. With each of these unfamiliar statistical tests, students were able to use the familiar six-step hypothesis testing procedure protocol that Ann had provided them in Class 11—this provided welcome consistency.

The first students to finish left at about 8:30 p.m., ten minutes after the class officially ended, and others were still there after 9:00 p.m. This was the only class that ran over time; as the Exam #3 was to be given at the next class students did not seem to react negatively.

Individual Sessions

Between Class 13 and Exam #3 I had individual meetings with Pierre and Karen twice and with Brad and Mulder once. Pierre and I had had two long sessions and I realized that he was trying to master *all* the material in the text, not just what Ann was requiring. While this seemed admirable it was leading to his not mastering in sufficient detail any of the material, especially the material Ann was requiring, so his grades were

poor (D^+ and C^-) and I expected this exam to be the most challenging thus far. I suggested he focus more on what was being covered in the course so he began to work on the course specific materials I prepared. Mulder met with me the early on the morning of the exam. He said he hadn't done much, if any, study and said he was feeling stressed because he had to work for the rest of the day and would have no further chance to study.

Karen had met with me the week before the exam and we had worked on the Mann-Whitney hypothesis test. On the day of the Exam #3 we met again after she had already been at Drop-In mathematics at the Learning Assistance Center at the Greenville campus for three hours. Jamie said she would come to Drop-In too but didn't. However Ann was offering a special pre-exam study group/drop-in session in our classroom at Riverside from 4:00 p.m. to 6:00 p.m. and Jamie was there when Karen and I arrived at about 5:00 p.m. from Greenville. Lee had been there since 4:30 p.m. and was working with Ann. Lee had called me earlier in the day panicked because she had to work all day after being ill all weekend. She had hoped to meet with me at 4:00 p.m. but I was already scheduled to meet with Karen then. Autumn and Catherine were also there but each was working on her own. Mitch arrived about 5:20 p.m. and seemed unprepared. He had been hosting a visitor and said he had not been focused on his work.

Exam #3

My anxiety level on behalf of the students was higher for this exam than for any of the others. I was especially anxious about how to counsel and tutor those who sought help. I was also anxious about those I believed should have sought help and did not. My anxiety stemmed not only from the fact that the inferential statistical and sampling distribution material being examined was new to most students and was considerably

more complex than the previous descriptive statistics but also from my undoubtedly inappropriate feeling that I could not quite trust the students to take responsibility for themselves. I was also concerned with the number of statistical tests and procedures the students had to master and wondered with them if they would be expected to identify the applicable one from the problem statements on the exam. I e-mailed Ann to this effect and she replied that she was not sure; maybe she would identify some and have students decide on others.

To help students prepare for this exam I used my analysis of content of the previous exams to develop strategic practice materials. Because the previous two exams focused on problems like the ones worked in class, I erased the test labels on each of the problems we had worked in class and copied them for individual counseling. I added a normal z test problem from the text as we had not done one of those in class. I also modified the decision flowchart the text used for choosing the appropriate statistical test (Pagano, 1998, Figure 19.1, p. 473) to create a simplified flowchart including only tests being examined on this exam, leaving the test name boxes blank for the students to fill in (see Appendix J). Ann agreed that this modified flow chart, filled in by the student, would be acceptable as a formula sheet to use in the exam.

The student who took most advantage of these materials was Karen, during her 4 hours at the Learning Assistance Center on the day of the test. On her second test she had made significant errors because of missing work columns on her formula sheet and because she had not sufficiently practiced all the problems to be tested. She remedied both of these problems for this exam and was rewarded with an almost perfect score on

the computational section. She was one of only two students whose grades improved from Exam #2 to Exam #3, *before* the extra credit was factored in (see Table 5.1).

Ann's strategy of having the students work through problems in class without first showing them what to do forced each to negotiate the procedures required. The lack of systematic group discussion of links among various aspects of the process and of known trouble spots, except with individuals, left some vulnerable to memory lapses or confusion in the exam unless we addressed these in a counseling session. For example, explicit discussion of the direct relationship between the null hypothesis statement and the relevant symbols and parts of the t statistic formulae may have prevented the error some students made.^{xxvii} In the exam Mitch, Mulder, and Robin (almost one third of the class) used their non-zero mean of sample differences \bar{D} for μ_D instead of zero even though they each correctly stated in their null hypothesis statement that there was *no* difference or change in the population scores before and after. They each then had to cast about for improbable \bar{D} s because they had used theirs for μ_D .

Four of the ten students who took Exam #3 made errors in choice of degrees of freedom in the independent samples question on the exam and one in the correlated samples question. Others made errors negotiating the t table. This procedure was introduced to students in Class 13, and its application is complex; it is somewhat different for each of the three t tests taught in that class and different in significant ways from the familiar procedures for using the normal z table. Guided questioning in the form of an assignment sheet might have helped students become more conscious of these differences. In subsequent counseling sessions I noted the importance of walking participants through the use of unfamiliar tables. Karen had resolved her initial

difficulties with the tables during her afternoon of preparation and she had simultaneously designed her formula sheet to prompt correct usage.

Pierre was the only student who misinterpreted the subscript of the standard error as part of a formula instead of as a label. Instead of calculating $S \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s^2}{n_1} + \frac{s^2}{n_2}}} (= 1.77)$ using formulae on his sheet, he used $\bar{X}_1 - \bar{X}_2$ not as a label, but as a factor, multiplying it by S to get $S (\bar{X}_1 - \bar{X}_2)$, that is, $9.73(43 - 39)$ or 38.92 . The large size of his standard error should have given him pause. Because there was no opportunity in class for discussion of the expected relative sizes of the statistics, in relation to the mathematical processes involved, in individual sessions I realized that it was important for me to model and encourage students in this type of questioning and checking.

Grading and Instructor Response to Test #3

A new feature of Ann's grading emerged with the focus on hypothesis testing. In her scheme, a certain number of points, typically 3 or 4, were allocated for the correct decision at the end of the process (i.e., whether to reject or fail to reject the null hypothesis) and for its meaning in terms of the problem at hand. On one question in Exam #3, four students made errors in their calculation of the statistic and found its magnitude to be *less* than the magnitude of the critical value. They therefore logically decided to *fail* to reject the null hypothesis.^{xxviii} Ann penalized them the full amount because they made the incorrect decision, even though it was the one demanded by their results. I was concerned in individual counseling to affirm students' sound mathematical decision-making in a situation like this, and try to allay the negative impact of the lost points on their self concept. At the same time as we discussed the validity of the

instructor's emphasis on the need for the correct decision^{xxix} and, therefore, the importance in subsequent exams of checking the accuracy of one's computations.

Table 5.1

Grades Throughout the Course of all Individuals in PSYC/STAT 104, Summer 2000

	Exam #1	Exam #2	Exam #3 ^b	Exam #4	Minitab Module 1	Minitab Presentation	Final Exam (#5)	Optional Comprehensive Final	TOTAL
Percent of TOTAL GRADE	20	20	20	20	2	8	10	Replace lower exam grade	100
Autumn 4^a	86%	96%	(90+6)%	95%	100%	100%	100%		94.6%
Brad 4	72%	69%	(56+4)%	72%	100%		62%		
Catherine	100%	92%	(91+6)%	100%	100%	100%	100%		97.8%
Ellen									
Floyd	42%								
Jamie 5	95%	74%	(84+6)%	76%	100%	100%	100%	71%	87%
Karen 5	62%	74%	(85+6)%	88%	100%	100%	96%	57%	83%
Kelly 3	59%								
Lee 6	76%	97%	(83+6)%	81%	100%	100%	100%	77%	88.8%
Mitch 4	78%	87%	(62+6)%	82%	100%	100%	92%		82.2%
Mulder 5	63%	81%	(76+5)	91%	100%	92%	94%		81.96%
Pierre 8	68%	72%	(60+6)%	91%	100%	92%	96%	72%	79.56%
Robin 3	89%	87%	(77+6)%	88%	100%	100%	96%		89%

Notes: ^a Names of counseling participants are bolded and the number beside their names is the number of their counseling sessions. ^b Because more than two thirds of the class experienced grade decline, some severe, on Exam #3, and more showed a fundamental lack of understanding of the concept of statistical power, Ann gave an in-class, open-book assignment worth up to 6 points to be added to the Exam #3 grade.

Karen and Jamie's scores on Exam #3 showed an improvement of one letter grade over their scores on Exam #2. Everyone else except Catherine (whose score remained about the same) dropped from one half to two letter grades. Ann was concerned not only with the drop in scores but also with the evident lack of understanding of the concept of statistical power. In Class 15 she assigned an open book extra credit assignment for 6 points on the topic of statistical power and the factors that influence it (see Table5.1).

From Exam #3 through the End of the Course

The ten students remaining in the class were all passing with grades ranging from a D⁻ (Brad) through A (Autumn and Catherine) after completing Exam #3. Karen was showing steady improvement in grades and Jamie was recovering from her big dip in Exam #2. Mulder's score on the multiple-choice conceptual section remained a significant problem but he had done quite well on his computation despite his lack of preparedness for the exam. Brad seemed quite crushed by his low score and I felt the urge to "rescue" him from himself, convinced that he was sabotaging his own chances of succeeding. Pierre had not followed my advice to focus on course material only and did poorly again (a D⁻ before the extra credit).

The nine remaining counseling participants continued to meet with me individually. Some also attended study group, Lee every week and others if the study group was just before an exam. Their course grade progress is shown on Table 5.1. Their progress as mathematics learners and course strategists and other changes in their mathematics mental health are discussed in chapters 6 and 7.

END OF COURSE SUMMARIES AND DISCUSSION

Student-Instructor Interactions during Lecture Discussions

Analysis of the interactions between the instructor and individual students during the lecture discussion portion of class revealed patterns relevant to the emotional climate of the classroom and the individual's perception of it. In general Ann asked questions of the whole class; she directed questions to individuals only in relation to a prior issue they were discussing. At times several students responded together to Ann's whole class questions. When students asked questions, some raised a hand to draw Ann's attention

(e.g., Lee in Class 16); others spoke into a silence or out of puzzlement with what Ann had just said (e.g. Karen, Mulder, and Robin each asked Ann to repeat or clarify what she had said in several instances).

There was almost no correlation between the number of students' responses or questions and their grade in the class (see Table 5.2). There was, in fact, a small negative correlation ($r = -.244$) between a student's average number of responses or questions and final grade (for those who completed the course).

In addition, apparent accuracy and pertinence of student response was often incongruent with grade. These phenomena make it very likely that any judgment of student competency based only on class interactions could be quite misleading.

Table 5.2

Number of Individual Utterances During Lecture Portion of Classes and Final Grade

	Class Number																		Av. & Final Grade
	1	2	3	(4)	(5)	6	7	8	(9)	(10)	11	12	13	(14)	(15)	16	(17)	(18)	
Autumn	0	4	3			3	0	1			0	0	0		2	0			0.82; A
Brad	4	2	2			12	2	2			0	0	3		1	4			2.9; D
Catherine	0	0	1			1	1	0			0	0	0		0	0			0.27; A
Ellen	3																		
Floyd	3	0	1				0												1
Jamie	0	0	0			0	0	0			0	0	0		0	0			0.0; B ⁺
Karen	1	4	3			2	1	3			2	3	2		1	3			2.27; B
Kelly	1	3	2			1	0	2											1.33
Lee	2	4	4			5	2	8			5		3		1	3			3.36; A ⁻
Mitch		3	1					2			1	1	0		2	1			1.38; B ⁻
Mulder	6	3	3			3	0	3			7	5	1		1	2			3.09; B ⁻
Pierre	0	4	0			1	1	2			1		1		1	1			1; B ⁻
Robin	3	4	5			7	1	2			4	8	3		1	3			3.73; A ⁻

Participation alone is clearly not enough. Factors related more closely to classroom interaction were student learning style and preferred modality, personality, and previous experiences in a mathematics learning environment.

Brad and Mulder interacted in a way that gave the impression of familiarity with and grasp of the material, while Robin gave the opposite impression. Ann's initial judgment of Brad's competence was dramatically modified by his poor grades and his struggles in the class problem-working sessions. Her initial judgment of Robin's incompetence persisted however, even despite her consistently good grades. Both Robin and Mulder seemed to have difficulties with auditory processing of lecture material but Robin's struggles were clearly discernable in her often puzzled demeanor, her checking with Ann to see that she had understood correctly, and in the tentativeness of many of her correct responses.^{xxx} In contrast, Mulder responded to questions only when he was certain of the material; he dealt with his struggles to understand the lecture presented concepts by focusing on parts of the lecture and ignoring others or by giving up and working instead on the computation with fellow students during problem-working sessions if they were willing. After Karen gave an initial response that showed confusion from then on she restricted her responses to supplying data (e.g., her beer preference, Class 16; or sports data).

Jamie was the only student who *never* asked or answered a question during the lecture discussions (see Table 5.2). Her shyness was obvious from the beginning and she typically kept her head down and eyes lowered. By the end of the course she was raising her head and making eye contact but she still did not speak. The strongest students in the class grade-wise, Catherine and Autumn, were among the quietest. Catherine (average

responses 0.27 per class, see Table 5.2 above) had an air of quiet confidence that accurately reflected her easy mastery of the material. Autumn (average responses 0.82 per class) was more responsive during the first half of the course than later, perhaps in part because she was more familiar with the material at the beginning and she was careful only to respond when she was quite certain.

All students except Robin seemed to try to restrict their responses to answers they felt they knew. For example, Mitch usually held his head stiffly on his hand and would respond barely audibly, when he was sure of the answer (see Appendix E, Table E2 for the criteria I used to analyze student's utterances during lecture discussions).

IMPLICATIONS FOR MATHEMATICS COUNSELING

I found that being in the class, doing the statistics, observing the students, and taking the exams provided me with good data to use in order to plan and provide strategic tutoring in the statistics/mathematics and the counseling of participants' relational issues.

Study Groups and Cooperative Learning

Because the study group was open,^{xxxi} attendance fluctuated from one to seven students, apparently according to whether there was an exam immediately following (see chapter 4, Table 4.2). Because attendance was not consistent, a group working approach was difficult to establish. No one in attendance, except perhaps Lee, was oriented towards working with peers to investigate a mathematics problem. They each related directly with me and seemed to show little interest in others' responses unless I directed them to evaluate those responses. It was easier to have the students work together on a problem when there was not an exam immediately following but even then the pressure to master the procedures precluded open group explorations. The focus was on working assigned

problems with my guidance and coaching and with students taking turns in presenting solutions to the group.

The study groups that met before exams were more like drop-in with me (and Ann before Exam #3 and Exam #4) moving from individual to individual helping each with his particular questions. An exception was the one before Exam #5. That was a round-table discussion of problems Ann provided and I supplemented, during which I asked individual students in turn (including Jamie) to respond with their solutions.

Mathematics Counseling and the Classroom—Relational Foci

I found that how a student interacted in class with the instructor, the mathematics, and his peers provided me with important data. When considered with material that emerged in individual counseling sessions, it helped me narrow and define that student's relational focus—in other words, his core relational conflictual pattern. By itself, classroom observation was certainly not adequate to identify students' entrenched mathematics relational patterns as they affected their mathematics mental health and prospects for success in PSYC/STAT 104. Observation seemed to even add to the confusion at times. However when I used the relational dimensions to organize my classroom observations, the data that were initially confusing often became important clues to students' core relational conflict (see Appendix E, Table E3).

The Classroom and Issues of Mathematics Self

During lecture discussions, students' ways of interacting gave some clues to their sense of mathematics self. Were they willing to reveal ignorance or only knowledge? Were they interested in growing, in performing, or in merely surviving? Lee and Robin were the most public, in different ways, in their attempts to grow and their willingness to

reveal ignorance for that purpose, indicating to me that they could have healthily developing mathematics selves and learning motivation for achievement. Others, like Autumn, Mitch, Karen, and Mulder, who only revealed their knowledge and were silent when they were uncertain, signaled a more fragile mathematics self. Autumn and Mitch acknowledged their performance motivation for achievement in their *Mathematics Beliefs* survey (as I expected from their other behaviors), while Karen, and Mulder surprisingly revealed more learning motivation. It seemed that their silence in class (except when certain) may have been self-protective, with survival taking precedence over their underlying desire to learn. Mulder, Floyd, and Brad all spoke and acted with confidence in their own knowledge that their exam grades belied.

The student behavior I observed during problem-working sessions (particularly with respect to arithmetical and algebraic comfort and level of confidence tackling new material) gave clues to students' mathematical self development. In class discussions, Karen just seemed to want to survive. There were further clues in problem-working sessions that her difficulties could be related to an underdeveloped mathematics self (e.g., her poor sense of decimals, poor operation sense, and low level understanding of the algebraic variable).

The Classroom and Issues with Mathematics Internalized Presences

A discrepancy between how the student was experiencing the classroom and the reality of the classroom was sometimes a clue to the effect of the student's internalized mathematics presences from the past skewing the present experience. Jamie serves as an effective example. Given the small class size, and the community-style seating arrangements, and the positive classroom emotional climate created by Ann, Jamie's

almost complete lack of participation (in fact, her quite successful hiding) was a clue that internalized past negative experiences might be skewing her perception of the present class and making her feel unsafe in a safe environment. Likewise Karen's observable defensive detachment and difficulty with the class in the beginning may have been related to her difficulty with separating herself from past experiences, especially that of previously taking the class. My awareness of the possible implications of these students' behaviors prompted me to explore further in counseling (see chapter 6).

The Classroom and Mathematics Interpersonal Attachment Issues

Students' attachment patterns to teacher and mathematics also became apparent in class. Kelly's behaviors indicated that she was anxious, disorganized, and dependently clinging to Ann and me; she seemed to have an insecure attachment to mathematics and to mathematics teachers. Although Lee seemed to have experienced a secure attachment to mathematics at times and had a history of generally secure attachments to mathematics teachers, she exhibited a lack of confidence in her ability to develop understanding on her own. This and her difficulty with Ann's approach showed in her spending up to two and a half hours a week with me in study groups and counseling while spending only about 20 minutes doing homework on her own. Karen was detached and defensive and kept Ann (and me initially) at a distance—indicating the possibility of either a lack of secure attachments to mathematics teachers in her history or a traumatic severance of such an attachment with no subsequent reconciliation. Her confusion with decimals and her errors with simple arithmetical procedures in problem-working sessions indicated a lack of secure attachment to mathematics, almost certainly contributing to her expressed anxiety during these sessions. In contrast, Jamie's anxious and disorganized attachment pattern

seemed to imply prior positive mathematics experiences with intervening negative ones so that she was now uncertain and now had little sense of a secure base in the mathematics or in mathematics teachers.

The ability to negotiate change was a particular issue for some students who were repeating the class, particularly Karen and Mitch. I used my knowledge of the statistics and my observations of Ann's teaching to build bridges from their past to their present experience and facilitate their adaptation to this new course. However, the main challenge was to help them acknowledge their conservative impulse reactions (cf. Marris, 1974), recognize the differences between the current and their previous class, and take responsibility for adjusting, rather than externalizing their discomfort by attributing responsibility to Ann.

Whereas student's ways of relating with Ann and me seemed to fall into the category of attachment relationships, their ways of interacting with peers seemed to fall into the category of relationships of community (Weiss, 1991). These relationships of community were evidenced in how students related to each other—as social, independent, voluntary loner, or involuntary loner—and were apparent in problem-working sessions and in study groups. In mathematics counseling, I explored further how a student's pattern of relating with peers affected or was affected by his mathematics mental health.

In the next chapter we will move from the classroom to the counseling setting where I describe and analyze in detail the course of mathematics counseling with three focal students.

ⁱ Excluding Ellen about whom I have no data.

ⁱⁱ I remained seated during these problem-working sessions in order to observe, but I also assisted students around me if they asked me.

ⁱⁱⁱ Pagano (1998) does provide his own links between concepts and procedures in the text, verbally and with diagrams, graphs, and illustrations (e.g., explaining the normal curve, pp. 81-86). He does not invite exploration nor pose open questions for his readers to find links themselves.

^{iv} In the text, there is reference to statistical analysis computer software packages and optional companion manuals for MINITAB or Statistical Packages for the Social Sciences (SPSS) that may be used with the text, but the only reference to them in the text is in chapter 1 (Pagano, 1998, p. 11) in a brief discussion on the use of computers in statistics. Pagano, in his preface to the 5th edition, notes that he had removed the cross references to computer software programs from the text at the request of teachers and students (Pagano, 1998, p. xix).

The psychology department had developed a program of computer analysis projects, independent of a text, using an old version of MINITAB to be completed by PSYSC/STAT 104 students. Because of the accelerated timetable in the summer, instead of every student having to do each of 7 required MINITAB computer assignment modules, Ann required Module 1 for everyone, worth 2% of the final grade. The class then paired off to do one module per pair from modules 2 through 7 and these were presented at the second to last class. This was worth 8% of the final grade.

^v The text also addresses the issue of common misuses of and misconceptions about statistics in the seven "What is the truth?" inserts scattered through the text where the author links statistical concepts to an analysis of real-life mathematical or logical claims of advertisements, research reports, or news items in an attempt to link the text with and perhaps challenge the student's reality. However, the answer is given and there is no invitation for the student to examine his own beliefs and reactions to the material. Ann did not use these in class but instead distributed a copy of a newspaper article that she invited the class to critique—if there had been time. As it was she pointed out the errors in use of the statistics.

^{vi} Lee challenged two of the four possible conclusions from a high correlation coefficient that Anne dictated from the text (i.e., *X caused Y* and *Y caused X*) but eventually resolved this issue for herself by putting caused in quotes so as not to be associated with what she knew to be an erroneous step of attributing causation where "possible relationship" was the only valid conclusion. What Lee did not understand was that Ann and the text were correct in giving possible real connections between variables that would lead to a high correlation, whereas Lee was rightly objecting to concluding that *X caused Y* or that *Y caused X* because of a high correlation between them. Ann did not have time to resolve this to Lee's satisfaction. Lee's subsequent response to a question about this showed that she had become confused whereas on the pre test she had shown a correct understanding (question 16 on the post SRA, see Appendix C). In counseling I did not attend to the real vulnerabilities of Lee's mathematics self-revealed by this situation, so we did not address her conflict in counseling. This situation may also point to her vulnerability to authority over reason.

^{vii} The worksheets, with the exception of the one factor and two factor χ^2 worksheets, consisted of a question and, in some cases, an empty table with column headings. However, the one factor and two factor χ^2 worksheets were different. Ann had posed the question at the top of the page and then provided a step by step, fill in the blank, procedural format. Not surprisingly Lee objected strongly to it. Lee relied on working out the procedure for herself, in class, to guide her in the tests; she found the fill-in-the-blanks format confusing and distracting. She subsequently made an uncharacteristic and serious error on the two-factor χ^2 on the test which she attributed to the worksheet. Instead of apportioning the expected frequencies proportionally among the four cells using the formula, $f_e = (\text{Row Total})(\text{Column Total})/(\text{Total Observed Frequency})$, Lee apportioned them equally. Jamie and Robin both made an even more serious error adding the column and row totals to get a total expected frequency double the total observed frequency—clearly not reasonable if they had thought it through. These errors seemed to be not only related to the more directive worksheet that did not require the student to work his own way through the

procedure, but perhaps also to the absence of class or individual discussion of the mathematics or logic inherent in the formula. Ann stated that in other classes she had taught, students preferred the directive worksheets.

^{viii} Lee's relatively sound arithmetical and algebraic background seemed to help her in this, but there was not time in class for deeper conceptual explanations and connections. The lack of these made her anxious so that when she came to study group and individual sessions our focus was on connections and mathematical meanings (Level 4 on the *Algebra Test*, see Appendix H, Table H1 and Appendix C).

^{ix} Karen had arithmetical difficulties (for example, uncertainties about values of decimal fractions and placement of the decimal point) and her algebra background was shaky (Level 2 on the *Algebra Test*, see chapter 6, Tables 6.1 and 6.2 and Appendix C).

^x The manipulation required is largely linear and usually direct, except, for example, when one has to find a particular score given its percentile rank in a normal deviate distribution, which is an inverse procedure requiring manipulation of linear terms. Ann didn't expect students to do this. In the least squares linear regression analysis section, which she did require students to do, they are required to derive a linear equation in two variables and use it to find particular points. This latter process requires only the substitution of numbers for variables in the derived equation.

^{xi} To transform the independent variable by translation and compressing or stretching in order to convert the probability density function of the data in question into a standardized form whose area (i.e., probability) values are accessible on statistical tables in the text (or statistical software package).

^{xii} In statistical formulae the extensive use of *subscripts* as labels is complicated by the use of numbers, single literal symbols, and even variable expressions as subscripts (see discussion of Class 13). In addition in descriptive statistics the \bar{X} (the mean of scores, \bar{X} , in a sample) in the first part of a course is a statistic, that is, a constant for that sample; the score X is the independent random variable in this distribution of scores. However, in inferential statistics, the \bar{X} (sample mean) becomes the random variable because sampling distributions are distributions of the *sample means* of all possible samples from the population of a particular size (refer to the discussion on Class 13). Ann's sense of students' difficulty in understanding sampling distributions and her own may have stemmed, to some extent, from the lack of discussion in the text or elsewhere of this transition of the \bar{X} from being a constant to being the variable (see *From Exam #1 through Class 12* discussion).

^{xiii} I use *social learner* here to refer to a student's evidenced preference for collaborating with other students in contrast to preferring to work alone (a *solitary learner* or *loner*). This categorization should not be confused with Belenkey et al's *connected* versus *separate* knower which refers more to a student's preference for personal connection with the material being learned. It also should not be confused with Skemp's (1987) categorization of *relational* versus *instrumental* mathematics learner which refers to a preference for conceptual understanding in contrast with a preference for procedural (only) competence. In addition to being a social learner Lee was also a connected and a conceptual (relational) learner, whereas Mulder, who was also a social learner, was a separate and procedural learner by preference or at least by socialization.

^{xiv} Ann then addressed what she expected to be some anxiety about the class by telling of her own experience in learning statistics as a graduate student. In particular she referred to her metaphor of statistics as a beautifully painted mural with all the elements separate, distinguishable, and in their correct places in relation to each other. In an exam, under stress, however, it was as if the separate elements began to run together to form a horrible brown indistinguishable mess; she couldn't tell one procedure from another. She said she had recovered from this disaster, going on to master the subject at doctoral level. She went on to explain to the class how her experience of teaching this course a number of times had increased her confidence and her enjoyment of statistics and that she hoped the students would find taking the course an

“okay experience.” She urged as a remedy to anxiety that students do their homework, study their notes, and ask for help until they had “over-learned” the material.

^{xv} The only caution against this openness is the possibility of disturbing the trust of students in the received knowledge phase (the first phase of epistemological development) who believe that the teacher or text—the external authority—is the repository of all knowledge (Belenky, Clinchy, Goldberger, & Tarule, 1986; Perry, 1968). Such a belief tends to develop too in students enculturated in transmission, teacher-as-authority mathematics classrooms (see chapter 3). To help these students find their own ability to understand the teacher as she expresses uncertainty can simultaneously model exploring the mathematics and discovering it as a secure base.

^{xvi} I did call on Jamie in study groups and though that made her uncomfortable, in the small group she was able to respond. In a classroom setting when I am the instructor, my practice is to call on students for their responses in order to ensure that students who do not voluntarily participate are involved. With shy students who exhibit discomfort (and sometimes cognitive confusion) when called upon, I make prior arrangements, letting them know ahead of time of the question that I will be asking them to respond to.

^{xvii} Because this is an odd numbered chapter, I use “he,” “his,” and “him” as the generic third person singular pronouns.

^{xviii} The fact that this extra exam was comprehensive, covering all the procedures and all the statistical concepts from the course ensured that it was *not* equivalent to a course exam. Instead it was more difficult to do well on, especially without in-class review and specific preparation. Ann reported that no one in her prior classes had taken advantage of this offer so she was surprised that more than a third of this class (4 students) chose to take it. My being available to provide preparation help was perhaps a factor. All scored below their course average (at least one letter grade below). Two of the students scored just well enough on the exam for it to replace a lower course grade and ensure that they moved up into a higher final grade category (Lee from a B⁺ to an A⁻ and Pierre from a C⁺ to an B⁻).

^{xix} My initial impressions of students were similarly affected. Meeting participants individually and gathering multimodal data about them modified my first impressions however. See also final discussion of student-instructor interactions.

^{xx} Starting at the left front and going anti-clockwise, Ellen and Pierre, Lee and Robin, Jamie and Catherine, Floyd and Brad, Autumn and Karen, and Mulder and I interviewed each other. As we were beginning, Kelly rushed into class late, so she joined Mulder and me. Kelly and I interviewed Mulder and Kelly introduced Mulder to the class, Mulder interviewed me and introduced me to the class. I interviewed Kelly and introduced her to the class.

^{xxi} The four types of measurement scale are: nominal, ordinal, interval, or rational scales.

^{xxii} The blank columns were labeled $X - \text{mean}$, and $(X - \text{mean})^2$ respectively This sheet was designed for students to compute the deviations of scores from the mean and then the squared deviations in order to compute the sum of squares (of differences of scores from the mean) and from that the variance and finally the standard deviation of scores from the mean. This procedure and

thus the formula from it, namely $\sqrt{\frac{\sum(X - \text{mean})^2}{n - 1}}$ (for a sample; for a population the denominator is N)

are labeled “empirical” because they reflect the actual process for finding how the scores vary from the mean. Ann alluded to the alternative “computational” formula that does not, and told students “I like this [empirical] way.”

^{xxiii} Overall, an average score of 3.2 (on the 98 items of MARS) is at the 95th percentile (Suinn, 1972), indicating extreme mathematics anxiety (for further discussion of these scores, see chapter 6).

^{xxiv} Each student prepared his own formula sheet to be used when doing part II of the exam, the “computational” part. We could include formulae and descriptions of symbols but not their definitions, as well as visual layouts for a procedure such as the labeled columns for finding the standard deviation, but no worked examples.

^{xxv} I had obtained permission to use it from Dr. Garfield and from the Office of Sponsored Research’s Institutional Review Board for the Protection of Human Subjects. See Appendix C.

^{xxvi} The cumulative frequency of the class group just below the class group from which you are trying to finding the score for the corresponding given percentile rank. To identify this correctly required students to first create a cumulative frequency column, interpret the subscript L to mean *below* (although L refers to *lower limit* of the current interval when it is the subscript in X_L), and then find the cumulative frequency immediately below the one for the interval in focus.

^{xxvii} To compare two independent sample means, the null hypothesis is that there is *zero* or *no* difference between the two population means (μ_1 and μ_2 for independent samples), so that in the t test formula, $(\mu_1 - \mu_2) = 0$. For the population mean of the differences for correlated samples the null hypothesis states that there is zero mean of differences (μ_D), or that there is no change, translates in the t test formula that $\mu_D = 0$.

^{xxviii} For Question 3 in Exam #3 the correct decision was to reject the null hypothesis and conclude that there *is* a relationship between amount of relaxation and hot or cold baths. This decision is based on the magnitude of the t statistic being greater than the magnitude of the critical value of the t with which it is compared.

^{xxix} Ann’s thinking might have been, in this case, that the importance of coming to the correct conclusion in research justifies a severe penalty for the wrong one, in addition to the penalty already incurred for making mechanical errors.

^{xxx} Robin asked “on task” questions and answered Ann’s questions *correctly* approximately *twice as often* as she questioned or answered incorrectly, tentatively, or off task. This was not substantially different from Brad, for example, who made almost the opposite impression on Ann and me in class. Robin’s almost constant frown of puzzlement and flustered air seemed to be related to her relative difficulty with auditory processing of verbal material and her compensatory propensity to ask questions or check her understanding whenever she was uncertain that she “got” it. Robin also seemed to be exhibiting the well-documented tendency of women to be considerably more tentative about what they know than a man typically is (and Brad certainly was).

^{xxxi} In contrast are closed study groups to which students commit at the beginning and other students may not join following the commitment period. These groups then have a consistent membership. Lack of attendance may lead to a person being excluded.

CHAPTER VI

UNCOVERING MATHEMATICAL RELATIONAL PATTERNS: THREE
PSYCH/STAT 104 CASE STUDIES

I have described the class in its context in the previous chapter; now is the time to zoom in on the courses of brief relational mathematics counseling with the participants from the PSYC/STAT 104, the focus of this study. What actually happened? As I looked at the participant and at me and at us in a way that was different, that is, relationally, and we explored the participant's relationality about mathematics as I supported her doing her statistics coursework, what did that look like? *Was* it different in process or outcomes from a traditional series of tutoring appointments? If so, how? In this chapter I present three counseling cases in order to address these questions. Initially I wrote each case as a profile of a student in the process of mathematics counseling within the context of the class. But then I realized that although the student is the focus of attention in traditional mathematics academic support, with this new relational approach I, as the counselor, also came into focus. It struck me that it was, in reality, we—the student and I, and our developing relationship—who were the object of this study. Before I present the cases though, I will briefly review the counseling activity in the study and explain further my rationale for choosing Karen, Jamie, and Mulder from the ten.

Each mathematics counseling participant and I undertook the task of understanding mathematics relational patterns (in particular central mathematics relational conflicts) and pinpointing issues salient to a good-enough resolution of that conflict while she¹ was taking the statistics for psychology course. The approach we used was different from the typical treatments in its focus on joint understanding: That is, students' class assessment results and survey responses became the object of discussion,

modification, and deeper mutual understanding rather than pronouncements that locked them in—in their minds and in mine. The relational counseling explored both conscious and unconscious forces the student and I were experiencing, and the cognitive counseling stressed continual conscious interventions using the insights we gained.

The ten mathematics counseling participants had between three and eight individual sessions each, averaging close to five per person. I expected that only students who were anxious or saw themselves as “bad at math” would volunteer to meet with me for individual mathematics counseling. Instead almost the whole class signed up. The group included students who were extremely anxious, some who were not particularly anxious, and those who were somewhat ambivalent. Some wanted help with the mathematics while others who did not think they needed mathematics help signed up to help me with my research. Some might have accessed mathematics academic support if I had not been in the class; others definitely would not.

I found that the distinctions among the participants that were most indicative of the soundness of their mathematics mental health were the level of mathematics preparation (in terms of arithmetic [number and operation sense in particular], and in terms of understanding of the algebraic variable), which seemed to directly affect their mathematics self-esteem and interact with that to produce their particular condition of mathematics self. It was in talking about their mathematics learning histories and seeing connections between those histories and their present patterns of mathematics relationship, that participants’ central relational conflicts around mathematics became apparent. These conversations raised to the surface participants’ and my awareness of

these conflicts and supported some resolution. They provided key factors both for the course and for their mathematics selves that could profit from brief therapy.

THREE CASES STUDIES

At the end of chapter 4 I alluded to my approach to choosing Karen, Jamie, and Mulder as focal participants. Here I will explain more fully. Karen and Mulder were mathematically underprepared students who acted quite differently but whose relational patterns seemed to stem from a similar source. They, Karen more than Mulder, were among the students most cognitively and relationally vulnerable to withdrawal, failure, or inadequate grades—the students whom mathematics learning specialists most struggle to understand and help in order to avoid disaster, often to no avail. Jamie, whose mathematics background was more substantial than Karen and Mulder's, was, however also surprisingly vulnerable to failure, even with relatively sound cognitive preparation. She had serious relational challenges that jeopardized her chances of success. Karen might, Jamie might not, and Mulder probably would not have accessed the traditional mathematics academic support offered by the college. Each had mathematics learning issues that emerged from different dimensions of their mathematics relationality. All three had learning styles that had affected their mathematics relational patterns differently and impacted how they were negotiating the present course. Though each is unique, taken together, they represented a typical range of student issues that the Learning Assistance Center sees.

I faced quite different challenges dealing with Karen, Jamie and Mulder and understanding myself in relation to them. I experienced Karen's holding me at arms length as a challenge but I also found it frustrating and worrisome—I had to be content

with her setting boundaries that I had to respect even when I believed they might be counterproductive to her progress. Jamie's shyness and obvious discomfort when in focus evoked my sympathy and protective impulses at the same time that I felt I needed to tiptoe around her, anxious that I might harm her. Mulder was opinionated and stubborn and he and I sparred—I found myself on the side of the opposition—which felt as if it included Ann, the instructor, and perhaps his Mom. Each taught me about myself as a tutor, a counselor, and a person; each learned about him or herself as mathematics learners; and we all overcame mathematical and personal challenges to achieve success in PSYC/STAT 104. Before I tell our stories I will quickly review the theoretical bases that formed the framework for the relational counseling I employed.

Theoretical Bases and Case Presentations

The theoretical bases for brief mathematics relational counseling were discussed in chapters 2 and 3. Essentially my approach involved embedding cognitive constructivist, problem-solving, strategic tutoring in a brief relational conflict counseling framework. This was a dynamic process that differed considerably from participant to participant. What emerged from each participant's course of counseling, however, was a common phenomenon that, while providing me with a pivotal key to understanding his or her central relational conflict, also gave me a central organizer for presentation of these three focal cases. That key was each participant's metaphor for mathematics or themselves doing mathematics.

In presenting the cases then, after introducing the participant and me and our relationship, I begin with the participant's metaphor and discuss the mathematics relational implications of the metaphor that we discovered. This discussion leads into

consideration of the participant's mathematics relationality and how we understood and worked with it through the course of counseling. The participant's present ways of relating with me, the instructor, and mathematics—his or her relational patterns—illuminated each of the dimensions of relationality that Mitchell (1988, 2000) identified and that I adapted to the college mathematics learning support context, namely, the mathematics self, mathematics internalized presences—teacher/s (or parent) and mathematics, and teacher and mathematics attachments. Disturbance in the development of one or more of these dimensions led, for each, to present mathematics-related emotional conditions, understanding which, in turn further clarified for us the participant's relationality and central relational conflict. Understanding a participant's central relational conflict, in the context of his or her mathematics relationality, helped me develop a counseling focus. Finally, I follow discussion of this counseling focus with a summary of the course of counseling, session by session, to illustrate the processes, demonstrate the changes we made, and present outcomes.

KAREN'S COURSE OF COUNSELING

Karen "*had* to pass [PSYC/STAT 104] this time." I found this out by the vending machines during break of the third class meeting. As we were choosing our snacks, I commented on her being one of only two in the class apart from the study group to have done an extra assigned homework problem. She told me then that she needed the class for her psychology major but had failed it two summers previously. She sounded somewhat desperate. Even then, before I had met with her one-on-one, after observing her only over two and a half class meetings and despite her doing the homework problem, I had an ominous feeling about how she would do. I had already observed her keeping all

classroom personnel at arm's length, including the instructor and me. She seemed to be positioning herself defensively. Karen's working alone during in-class problem-working sessions seemed intentional and she had not attended the study group; instead she came early to the classroom, sat at the back, and worked on her own while the study group worked with me on the board (see chapter 5, Figure 5.1 and Appendix F).ⁱⁱ

I was not sure how to interpret her signing up for counseling. It seemed incongruous with her distancing stance but consistent with her expressed need to pass this time although she did limit herself to signing up for once every other week not once a week, which was the option I expected from someone who had already failed the class. I wondered how it would be. I wondered if mathematics counseling would be any use. I was worried that the task, that Karen's needs and her defensiveness, would overwhelm both of us. I was anxious that Karen would especially resist my relational counseling approaches but knew that these approaches had the potential to help her succeed this time.

What I did in counseling was to go ahead anyway, tackling the statistics and working side by side with her as we looked at the mathematics, I heard her voice and together we challenged her negative sense of herself doing mathematics. At the same time we evaluated the grounds for her defensive relational patterns. And I realized that my initially overwhelming negative sense of her doing mathematics was also challenged. Karen made better and better choices as she discovered a competency she had not previously recognized and teacher support she had initially rebuffed. Her expertise and confidence increased and her grades improved from a D⁻ on the first exam to B⁺'s and 'A's at the end with an overall 'B' for the course.

As I worked with Karen I learned to attend to and manage my countertransference reactions to Karen's initial defensive negativity. I experienced her transference as her teacher who would "know" as she did the severe limits on the mathematics she could do. She seemed very negative about her prospects for learning mathematics. "That's how I am. I can't/won't be able to...I can plug in the numbers but I don't know why ..."

(Sessions 1, 2); and I felt firmly rebuffed as I imagined her former teachers did if they tried to make a difference. In my countertransference I surmised that Karen's teachers before me may have accepted as I had begun to do that she was unlikely to succeed; this made me feel desperate and overwhelmed. But I (and she) challenged my countertransference reaction and I chose to believe and act differently. By looking at Karen from a relational perspective, I was able to help her find a real but underdeveloped mathematics self and develop it further. By the end of the course, neither of us thought of her any longer as someone who could not do mathematics. I was also able to challenge her defensive detachment from Ann and me; Karen began to experience us as secure bases on whom she could rely and from whom she could eventually venture out on her own. Indeed, Karen still had mathematical challenges, true, but she could face them knowing that she had found herself able to do well enough to succeed in this course.

Karen was a tall, blond, 22 year-old white, elementary school assistant teacher who had dropped out of State University after a year and a half and was pursuing her degree part-time at Brookwood State. She was the first in her family to pursue a bachelors' degree although she reported that her parents had taken some post-secondary technical courses. Karen wanted to become an elementary teacher and was majoring in psychology but only because the university required prospective elementary education

students to major in a non-education field (Class 1). In beginning this second attempt to pass PSYC/STAT 104, Karen stated that she hoped for a B but expected a C⁺/B (*Pre-Test Mathematics History Survey*).

Karen's Metaphor: Mathematics as Cloudy

As the most representative of Karen's metaphors for mathematics: "black," "stormy," "cloudy," "bear," she chose "cloudy" "because there are some aspects of math that are more clear to me, but mostly math is my worst subject and has always been hard for me to understand" (*College Learning Metaphor Survey*). That she chose what seemed to me to be the mildest image from her list surprised me. Karen's rather diffuse, somewhat depressed, image of a cloud seemed to contrast with her almost aggressive defensiveness, which made me expect her to select the image of defending herself against a bear rather than seeing her way through a cloud. Still "cloudy" did seem congruent with what I sensed as a resigned desperation, which to me felt as though she was experiencing groping around in a cloud as fruitless.

As we proceeded with mathematics counseling I understood better what Karen meant by her distinction between "more clear" and "cloudy" mathematics—it was partly about the type of mathematics: "I'm better algebraically than I am geometrically...I can't do geometry at all"(Session 1). But perhaps it was even more about Karen's sense of her own limits: "[Mathematics is] my worst subject ...always hard for me to understand," She clarified this further by responding "nothing" when I asked what she understood about a new concept, explaining "see that's how I am"(Session 2). Her use of the word "always" seemed to indicate a long-term and global negativity about herself as a

mathematics learner. When I asked her about it, Karen confirmed that ‘always’ meant “Back—all through school” even in first grade.

Karen’s Mathematics Relationality

Student-Mathematics Relationships: Karen’s Mathematics Self and Cloudy Mathematics

Since first grade Karen said she had found mathematics cloudy, “hard to understand.” In mathematics counseling when I asked her as an adult about her mathematics metaphor her first statement to me was, “I hate math.” I wondered what her experience of mathematics had been through school for this to be the outcome.

JK: How have you been historically with math, you know, through the grades?

...

Karen: It depends on what kind of math it was. If it was like geometry or something like that, I did horribly {okay} but Algebra and Algebra II, I didn’t do too bad on. I just don’t like math {yeah} at all. I never ever, ever have.

JK: Even in elementary school?

Karen: Nope I’ve always, I like reading and writing not math or science

At our first meeting Karen expressed an antipathy to mathematics requiring interpretation of visual material (e.g., graphs and diagrams): “I’m better algebraically than I am geometrically. I can’t do geometry at all” and later “I hate those bell curve things.” Karen told me she had turned away when Ann had drawn a bell curve in the last class (Class 3) because she disliked them so much. She believed, however, that conceptual learning of algebra was beyond her. “If it’s algebra, and it’s just a matter of plugging numbers into certain formulas, I can do pretty well with that... I can plug all those things into that and I have no idea why, or what that means” (Session 1). I mentioned to her that the study group had been working at understanding how and why the percentile point and rank formulas worked and suggested that she might feel more in control if she understood. Karen demurred, “Not necessarily; sometimes it’s easier if I

don't know why—I can just do it” (Session 1). I interpreted Karen to be saying that an attempt to *understand* the procedure might undermine her tenuous grasp of how to *do* it. I wanted to help her discover that she could understand, at least how this formula made sense, but she did not want to risk it.

It was clear to me from these data that Karen's mathematics self-esteem was quite low. She communicated that by describing her low confidence in her mathematics capabilities (“That's how I am.”), her low expectations (“I'll bomb the conceptual portion.”), and almost global negativity—possibly to protect her mathematics self from further disappointment. She had little of what self psychologist, Kohut (1977) calls “a storehouse of self confidence and basic [mathematics] self-esteem that sustains a person throughout life [in the mathematics classroom]” (p. 188, footnote 8).

How realistic or accurate were her negative self judgments? Did she have enough arithmetical and algebraic competence to build new learning on? Was she actually more firmly attached to mathematics than she believed or felt? I gathered a more systematic picture of Karen's arithmetic and algebra competence during posttesting and this confirmed what I had found through observation of Karen's work in counseling and the classroom during the course. Particularly with fractions and decimals, Karen's number sense and operation sense were very weak (see Table 6.1). This made it difficult for her to evaluate the appropriateness of the numerical results of her data analysis or to troubleshoot her work in order to self-correct an error. In addition, Karen was operating at a level 2 understanding of the algebraic variable, and here she was the lowest in the class (see Table 6.2). That meant that she was able to coordinate operations with letter symbols as objects but that she did not understand letter symbols as specific unknowns or

generalized numbers (and in some cases as variables) and could not coordinate two operations on them.ⁱⁱⁱ Given this Karen was likely to find understanding and using letter symbols in complex statistics formulae difficult. How she prepared the formula sheets to use for exams could be crucial.

*Student-Teacher Relationships and Cloudy Mathematics:
Karen's Mathematics Struggles*

I wondered how Karen's mathematics self development had proceeded for her for her to have such crucial mathematics deficits and to feel so negative. What part had her family and teachers played? Perhaps there was a family connection to her "always" finding mathematics "hard...to understand", I thought. It seemed that she had never reflected on it before, but now she began to see it.

JK: What about your parents? Are they more like that [reading and writing, not math people] too?

Karen: Yeeeah? [considering] Yeah=, yep↑ (Session 2, see chapter 4, Figure 4.3 for coding conventions used)

Because of what I experienced as Karen's reticence in talking about anything personal, I took the enthusiastic agreement I heard in "yep↑" to indicate that yes, she had experienced her family culture as one where her not having an interest in nor doing well in mathematics were accepted, perhaps even expected. I brought it up later and Karen said, "I'd say we're more of the reading, writing type, the whole family" (Session 5), thus confirming her sense that doing well in mathematics was not part of her family scene. When she *did* do well on an exam (Exam #3) they were all surprised and delighted at her success.

What about her teachers, then? What was their part in the development of Karen's mathematics-as-cloudy self? I asked her:

JK: Any teachers in math who, you know, who made you feel bad or better about yourself?

Karen: No, not really. I mean I was never like worst in the class, you know. I was always in the middle, middle to lower scale but I suppose I concentrated less because I didn't like it as much so, you know? (Session 2)

Karen seemed to have managed to get by in class by being unremarkable. She was not the worst so she did not attract negative teacher or peer attention, and she was certainly not the best. But it seemed that she had not received positive attention either. She had managed well enough to avoid attention, despite her perhaps defensive "concentrate[ing] less." If how she was relating to Ann and me was any indication, she had defensively kept her distance from them and teachers had let her be, accepting her limitations as real and essentially neglecting her mathematics self development. This in turn likely led to Karen's blaming herself, seeing herself as intrinsically bad (at mathematics) and not seeing the teacher as responsible (cf. Fairbairn, 1972).

Confirming this, when I inquired whether there had ever been a negative incident with a teacher she shifted the answer to herself by implying again that her present mathematics situation was of her own making: "I was not interested in math at all. I don't like it. That's why I don't do as well" (Session 5). Karen seemed to be using lack of interest to avoid acknowledging what she really believed to be the reason: her underlying lack of ability. Karen never spoke of a relationship with a mathematics teacher in either a positive or negative sense. The only teacher Karen spoke of at all was her instructor from the first time she took PSYC/STAT 104, and then it was to compare her teaching approach with Ann's.

I considered the absence of direct information from Karen about her experience with teachers, despite my probing, and realized that how she related to Ann and me in the

present course might give me the clearest sense of the relationships she had with mathematics teachers through the years. Karen was upset that Ann's teaching was unlike that of her previous teacher for this course whom she described as "more thorough." She believed that Ann would, nevertheless, expect her to know and use all the material in the text even if it had not been covered in class. Later I realized that the discrepancy Karen found most disturbing between the teachers was that her previous teacher had demonstrated on the board how to do each type of problem (perhaps her idea of "thorough") while Ann had each student tackle the problems herself in problem-working sessions that were sometimes lecture-guided but more often accomplished with her roving coaching help. Because of Karen's lack of confidence in her own ability—based on her low mathematics self-esteem—Ann's approach made her feel anxious and insecure despite what I perceived to be Ann's adequate coaching support. Most prominent for Karen seemed to be a sense of Ann's not being there for her in a way she felt she needed. She seemed to feel abandoned. Past experience with mathematics teachers appeared to have promoted her adoption of defensive detached patterns that seemed to have been activated in this class.

At our July 10 interview (Interview 2), Ann had expressed disappointment with her relationship with Karen: "I thought we would be closer." Karen sat as far at the back of the classroom as possible and she did not connect with Ann outside of class time. On several occasions (at least once in Ann's hearing) she expressed hostility about her perception of what had been said about what to expect on the next test in contrast with what Karen believed *should* have been said. This had to contribute to Ann's sense of Karen's hostility and deliberate distancing. I considered this aggressive detachment to be

largely unconscious rather than deliberate on Karen's part. Keeping her distance seemed to be Karen's established defensive way of negotiating a situation that exposed her vulnerable, underdeveloped mathematics self.

I began to believe that Karen had not ever developed a secure attachment to a mathematics teacher. No mathematics teacher had offered herself as a secure base in a way that she felt safe to connect with. She had learned to care defensively for herself and expected little from the teacher. Such low expectations seemed to have made her angry and anxious, even hopeless, because she knew she did not have what was necessary to do it on her own and she needed support from the teacher. Ann's and my experience of her defensively holding us at arm's length suggested that her demeanor may then have become a factor inhibiting even good-intentioned teachers from reaching out to her. Karen's defensive distancing may have been exacerbated in the college setting by the fact the she was the first in her family to go to college. It was unfamiliar territory and she did not have family experience and advice to help her negotiate it.

Emotional Conditions: Anxiety, Learned Helplessness, or Depression?

How did Karen respond emotionally to what seemed to be the underdevelopment of her mathematics self? Was her reaction consistent with a diagnosis of underdevelopment of mathematics self, expressed in underconfidence and defensive detached relationality? Were her emotional responses interfering with her approach to PSYC/STAT 104 to an extent that warranted emergency attention? The way I experienced Karen at the first session felt confusing—I experienced her anxiety, negativity (even hopelessness) and anger.

Anxiety

Karen admitted to being very anxious before the first test. Her scores on the *Mathematics Feelings* pretest survey confirmed that she consciously experienced excessive anxiety in mathematics performance and testing situations. On her *Survey Profile Summary* I had circled all three anxiety scales because they were all at almost the top of the class range (see Appendix K, Figure K1). The combination of her *Abstraction* and *Number* anxieties, however, especially in conjunction with what I had observed of her issues and approach did seem to be directly related to her underdeveloped mathematics self, particularly her inadequate number and operation sense and low level understanding of the algebraic variable (see Figures 6.1 and 6.2). Her testing anxiety (second highest in the class) seemed also to be related to the inadequacy she felt when she tried to recall how to do procedures she dimly understood. At least for the first exam Karen's inadequate practice and unstrategic preparation contributed considerably to her heightened anxiety. It seemed that the anxieties Karen experienced in mathematics situations were normal reactions to threatening situations for which she felt inadequate. It also seemed that her anxieties could be considerably alleviated by more strategic and thorough preparation.

Depression and Helplessness

Karen expressed negativity about her mathematics self, mathematics, and this class. I analyzed her responses to the *Beliefs Survey* that she completed during the second class for underlying beliefs or constellations of beliefs that could better pinpoint her negativity as well as others that could show healthy positive orientations. Karen's average pre-score on the learned helplessness versus mastery orientation scale was

worrisome: below the middle of the scale it was the third lowest in the class. On all three belief scales, her responses fell below the class average. Nine of Karen's 14 *Learned Helplessness vs. Mastery Orientation* responses were 2 or below, reflecting her belief that learning mathematics involved having to be taught and then memorizing different procedures for each new type of problem. This belief would make her helpless if she did not memorize the right things. On the mastery oriented side, although she agreed that some people could do mathematics while others could not, Karen believed that her mathematics ability could improve, so it seemed that she was not locked into a fixed trait belief about this ability. Karen also reported that when she could not immediately do a problem she would *not* assume she could not do it and give up on it, and she usually tried to understand the reasoning behind mathematics rules. Karen's negativity about her mathematics self, world, and future did not preclude an underlying hope in the possibility of change; she also had a view of herself not giving up when learning was difficult (see Appendix K, Figure K2).

Karen's responses indicated that she was more motivated towards learning than performance.^{iv} This surprised and encouraged me for Karen. Her focus was not just on results; she wanted to understand the material. She did believe mathematics to be more procedural than conceptual but her beliefs were not extreme (just below the midpoint) and with her expressed learning motivation and strategic support to find she could make the conceptual connections it seemed possible that her beliefs would improve (see Appendix H, Table H3).

Karen's responses over time on the *JMK Mathematics Affect Scales*, however, lent further weight to a diagnosis of entrenched negativity even depression. To monitor

her negativity/positivity Karen filled in the scales at every counseling session except the first. Karen's responses at the end of the second session were negative, all seven responses falling at or below the mid-point. She was very much discouraged about her problems with mathematics and she would withdraw from the current course if she could. She expressed moderate to severe negativity about her mathematics self (scales 1, 2, 6, and 7), about her current mathematics world—the class (scales 1, 2, and 4), and about her mathematics future (scales 3 and 4) the three spheres Beck (1977) found to be significant for people suffering from depression. As the course of counseling proceeded and Karen's responses on the *JMK Scales* did not improve in proportion to her improving grades, my awareness grew that it was mathematics situational depression (and related learned helplessness) rather than anxiety that Karen was struggling with (see Figure K3, Table K3, and Appendix B).

Identifying Karen's Central Relational Conflict

As we began *Session 1*, I was already drawn into Karen's anger and anxiety. I wanted simultaneously to rescue her from her plight and to defend Ann, the obvious target of her anger. So that she might not be angry with me too, I tried to be on her side, the fair, reasonable teacher she believed Ann wasn't. She kept her emotional distance from me too though as if I were on the side of the opposition. I did not want to believe her view that she was incapable of becoming more than a procedural mathematics learner, although I worried that the time-limited situation might force me to help her succeed only procedurally, thereby making her feel as if I agreed that she lacked the conceptual ability. Her view of herself as a mathematics learner seemed to be globally, diffusely negative, as if her mathematics self barely existed. Though she was trying to

contain it using external means (blaming Ann, formula sheet, last minute tutoring), Karen's sense seemed to be that this exam and this course were out of her control since there was little inside her to draw on.

Karen's responses on the *JMK Mathematics Affect Scales*, taken with her low indices on the *Learned Helpless/Mastery Oriented Beliefs* scale, her "cloudy" metaphor, and her defensive detached stance in relation to peers, Ann, and me, pointed to a diagnosis of moderate empty mathematics depression (cf. Kohut, 1977, and see chapter 3, pp. 91 ff.). This likely stemmed from Karen's deep sense of an underdeveloped mathematics self rooted in her poor mathematics preparation and low self-esteem. Her central relational conflict seemed to be between her strong desire and even need, to succeed in this course and her fear that there were powerful forces outside her control, including her own inadequacy and the instructor, which conspired to thwart that desire. Her significantly underdeveloped mathematics self seemed to be the chief conspirator. She seemed to be projecting her fear of her own inadequacy onto those around her.

*Karen and Me: Dealing with the Clouds Now:
Relational Counseling for Karen*

The Focus of Relational Counseling

I realized that, relationally, I had to provide myself as a guiding hand for Karen to safely negotiate her way out of the clouds that she had felt trapped in. To help Karen resolve her conflict I had to offer good-enough mathematics teacher-parenting to support the emergence and development of a firmer mathematics self that *could* succeed in the class. I planned to challenge her all-or-nothing thinking by mirroring her sound thinking and achievements and at the same time I would provide myself as a mathematics parent image that she could idealize and realistically incorporate into her increasingly competent

mathematics self almost like her internal mathematics guide. I expected that this development should go some way towards alleviating Karen's empty depression and underconfidence.

I would have to work at overcoming Karen's emotional distancing enough that she would accept my mirroring, though. To do this I had to resist agreeing with her about her mathematics hopelessness. Although her transference of past teacher relationships led me to believe that her low confidence was realistic, I had to resist that interpretation and instead see it as unrealistic underconfidence; Karen *was* capable of doing mathematics. It seemed crucial that Karen become free to avoid repeating her past experiences of doing poorly in mathematics classes and failing PSYC/STAT 104. Importantly, this would involve helping her recognize and take advantage of Ann as a secure mathematics teacher base, rather than a neglectful but demanding teacher from the past.

Although her angry anxiety was a potential focus I decided that it was a symptom rather than the root of her difficulties and could be ameliorated by helping Karen deal more effectively with her sense that external forces controlled her course outcome. I hoped that as her sense of her own competence grew, she would be increasingly able to take more responsibility for strategic exam preparation, she could seek help from Ann or me in a more timely manner, and she could make more effort to understand the mathematics conceptually.

The Focus of Mathematics Tutoring

Mathematically I would provide myself to Karen as a mathematical co-explorer with a flashlight and other tools that we could use to find our way through the cloudy terrain. Given Karen's multiple mathematical concerns and her evident course

management difficulties, I found that identifying a strategic mathematical focus initially overwhelming but I soon focused on Karen's underdeveloped mathematics self. I decided to mirror back to her what I saw as her strengths in mathematics and her positive approaches to the course. This was likely to help her begin to see her mathematics self differently. I also set out to nurture and coach that developing mathematics self, not only helping her to develop further mathematics understandings and competencies but also to *recognize* herself developing them. Then she might see herself finding her way through the clouds into the clear light to day.

It seemed that if I worked beside Karen as she mastered new procedures introduced in class and helped her link them to the concepts, and if she practiced she would be able to understand enough and do new problems; she needed to also recognize that she could. Karen's motivation for this deeper work could come from seeing her growing *ability* to grasp these links herself. Karen also needed to develop strategic structures (guide ropes to hold onto in the clouds) to compensate for her underdeveloped algebra, number, and operation sense.

Recognizing the usefulness of connecting the conceptual portion with the computational part would give her increased control. The primary focus needed to include her developing skill with letter symbols. Karen's mathematics self was affected by her poor facility with decimals and percents as well as her underdeveloped number and operation sense. Tackling this would, of course, depend on the time and emotional energy Karen was willing to invest.

Karen's Course of Counseling: Session by Session
(see Appendix K, Table K1 for Karen's schedule)

Karen's Session1

Karen came before her first appointment to drop-in and I observed her desperately trying to practice problems she had not yet gone over. It was at drop-in that she angrily denounced Ann for expecting the class to know all of the material in chapters 1 through 5 even though she had not covered it all in class. Karen had not gone to work that day because she was not feeling well and it seemed that she had spent some time scanning the chapters for material for her formula sheet (cheat sheet, she called it) and had become increasingly upset as she found unfamiliar formulae and concepts.

Karen felt that Ann had not been clear about what would be on the exam so studying the right material felt beyond her control. Karen did not interpret Ann's Exam #1 study guide, her presentation of all the material in her notes, and having the class work through specific problems from each chapter as likely cues that this was the material that would be tested. All of the students were feeling some anxiety about this first exam, but Karen seemed to be particularly misreading the situation. I wondered whether her failing experience in the previous PSYC/STAT 104 class was so prominent at this point that it was interfering with her ability to read the cues Ann was giving.

Her appointment with me was at 4:00 p.m. at Greenville campus and the exam was scheduled for 6:00 p.m. at Riverside campus. I had her continue with the problem she began in drop-in. I had reassured her that, based on the exam review guide Ann had distributed, and my sense that Ann had been careful to cover in class all that she would examine, that her she could safely ignore the other material in the chapters and erase that unfamiliar formulae from her sheet. She was already confident in the *direct* process of

finding the percentile rank of a given score procedure from chapter 3 of the text. This was the one she had done correctly for homework: “Percentile rank, I’ve never gotten one of those wrong.” But she was not confident of this *inverse* percentile point procedure, the one that we had done in class. After Karen did another of these to reassure herself, we checked the exam study guide for the list of symbols. We reviewed her understanding of the symbols to be tested for both name and meaning and she was quite confused.^v Karen was not aware of the Greek versus “English” (Roman) letters distinction between *population* parameters^{vi} and *sample* statistics,^{vii} which I showed her. Although pleased with this organizing idea for symbols and formulae, Karen was still somewhat overwhelmed with the discussion of the concepts the symbols and formulae represented.

As we proceeded I began to understand that not only was algebra cloudy to her, arithmetic was too. In finding 50% of 54^{viii} Karen was content with 1.08 (she had divided 54 by 50) as an answer; it became clear she did not understand percents, not even a benchmark generally known.^{ix} I wondered how pervasive were her arithmetic uncertainties and what effect that might have on the current course. With less than half an hour to go Karen announced “I have no clue on chapter 4 or 5.” I accepted that global statement on face value and anxiously joined in her desperate but seemingly impossible race against time to cover that material before the exam.

Karen’s Session 2

“Horrible!” was Karen’s response to her 62% on Exam #1. She was disgusted with getting the range wrong when she knew it; it was such a “simple concept!” Karen also reminded me that she hadn’t been feeling well. The next exam was to be in two days time so I felt some urgency to begin breaking down some of Karen’s negativity towards

Ann so she could take advantage of the structure and support she was offering. I also planned to help Karen recognize what she could do to begin to break into her global negativity about the rigid limits she placed on her mathematics self.

Right away I asked her:

- JK: How did you react to the exam itself? Better than you thought it was?
 {Yes} just what she [Ann] covered {right} rather than the whole book?
 K: Right↑

Karen agreed without hesitation that her fears before the first exam [that Ann would examine material not covered in class] were baseless but later in the session she brought up an assigned problem she had struggled with at home that she was pretty sure Ann had not covered in class.

- Karen: It's number 13 {number 13} right, and I don't remember doing that, using this formula.
 JK: Is this? Is this? No [hesitating]
 Karen: So we don't even need to do that one then?
 JK: Where is this? Is this on the list? Is this on our list to do? Do we have that on our list? {Yeah} you can do it but [the problem *was* expecting student to go beyond procedures taught in class]
 JK: Where's your list? I saw you had it before. You seem to be neat, keep your things in order.
 Karen: Chapter 6 one through 6 *. No, it's nooot (-). See, I don't pay attention (-) {laugh}
 JK: She's fairly careful which ones she picks [to assign as homework problems]. So I was thinking, "Why would she give us that?" So number 14, let's do number 14 (Session 2)

Karen had further good evidence that her fear that Ann might set her up with impossible tasks were ungrounded and I was able to take advantage of the situation to help Karen notice Ann's thoughtful planning designed to avoid such student frustration. Maybe Ann (and mathematics teachers) was more trustworthy than she thought. Maybe Karen could begin to *consider* trusting her.

Exam #1 Analysis

When we analyzed Karen's exam together she had done better than she had expected on conceptual section of the test on that section. She only missed one out of the 8 symbol questions^x and contrary to her expectation "If anything, I'll bomb the conceptual", was correct on 75% of the conceptual questions—it was the computational section she failed.

Questions involving decimals gave her trouble,^{xi} so I advised Karen to arrange an extra meeting to do decimal exploration,^{xii} since understanding and computing statistics involves a lot of work with decimals, it was likely that her anxiety and negativity were linked to her arithmetical uncertainties, and we were finding that Ann stressed arithmetical accuracy in her grading.

Karen had calculated all but one of the initial basic procedures accurately, and also succeeded on the direct, percentile rank procedure that she had practiced thoroughly. She was one of only four in the class to get this question entirely correct. Her response to the inverse find-the-percentile-point procedure showed her understanding of the concept but her anxious practice at drop-in and in the counseling session was not sufficient for her to reproduce the required procedure and she earned no points. She saw that what she failed was material she had not practiced at all from chapters 4 and 5.

This analysis revealed a mixed picture. Karen saw evidence that when she practiced sufficiently she could succeed and her global negativity seemed unjustified. Karen had not come to a help session early enough for Exam #1 but this session was two days before Exam #2. Karen had realized she had not focused strategically for the first exam so we planned to focus on problems like the ones done in class. Her defeatism

about her ability to do the mathematics and interference caused by repeating the class seemed to have contributed to her difficulties on Exam #1 so in this session I began to address these issues.

I asked Karen how her approach to the course work was different from what she did for course she had failed. She seemed taken aback by my question. Her response was “I think I just like, it just took me a while to get back into the [course] you know?” indicating that she realized that she had begun preparation for Exam #1 later than was wise. This seemed to contradict an earlier claim that she thought that she had been prepared for Exam #1. Now it sounded as if she might be revising her sense of what adequate preparation for an exam should entail for her. When I asked about her grades the first time she took PSYC/STAT 104, I discovered that Karen had all her course materials with her, including her test scores. The current test score 62% was considerably higher than her 47% on the first test then, she had succeeded in getting one of the most challenging questions correct, and she had overcome her confusion about the symbols, all of which began to break through my internalized negativity about Karen’s chances.

To prepare for Exam #2 we looked at the material that would be tested. When I asked her what Pearson’s r was, she responded, “Nothing (-)...See! That’s how I am. I just plug in the numbers ... That’s why I have so much trouble.” Her pronouncement indicated a significant change from her earlier defensive response that it might be *better* not to understand why. Now she conceded that not knowing was causing her difficulties.

Now I wanted to help Karen see that she could make her way through the clouds and see clearly for herself. I used a modified cognitive constructivist tutoring approach and kept alert for relational opportunities to mirror her competencies. I provided myself

as a model (a parent). The parallel modeling approach that we used looked much like best-practice traditional tutoring but had the added effectiveness of intentional relational attention.

As we individually set up and solved the problem side by side I talked through it.^{xiii} We began by constructing a scatter plot of the data^{xiv} and focused on identifying the independent and dependent variables. I waited for Karen's decisions before revealing mine. In the process, Karen found that her new understanding contrasted with her prior confusion in class when Ann had briefly demonstrated the scatter plot construction process, "Some of the points, either she [Ann] didn't do it right or I don't know where she got them from." Although she was implying that not understanding it in class could have been Ann's "fault" she also seemed to be conceding that it could also have been her own issue.

Karen graphed the coordinate points without difficulty, but she was in trouble once the scatter plots were drawn. As I questioned, coached, provided prompts, and worked the problem beside her, Karen explored the relationships among the symbols and their graphical representations and meanings. She gave no hint of her earlier anti-visual position. She even reluctantly revived her hazy knowledge of coordinate graphing of a straight line and explored that further, both graphically and algebraically. We did not have enough time to calculate a standard error of estimate but Karen seemed to feel less anxious about the upcoming exam. She had a much better idea of what to expect, she had understood material she did not think herself capable of, she had two more days to prepare, and she was a little more assured of Ann's care and good intentions. But just before we left, Karen filled in *JMK Mathematics Affect Scales* (see responses labeled 2 in

Figure K3) and her responses were very negative (see discussion of *Depression or learned helplessness* above).

Karen's Session 3

Before Karen's Session 3 I had interviewed Ann and Karen arrived just as she was leaving. Ann asked her how she was doing with her MINITAB computer module and Karen had some questions so Ann offered to go with her to the computer lab to resolve them. When they returned there was only half an hour left for our session. Ann resolved Karen's concerns about materials for her presentation and left after she told us of the research project she was launching the next day using an audiovisual presentation to help elderly nursing home residents become more alert and careful of their medications. This encounter provided a natural opportunity for Karen to experience Ann's positive support, an opportunity Karen would not have sought on her own.

Karen did better on the second exam but not as well as she hoped. When I commented that her 76% was a lot better than her 62% on Exam #1 she demurred, saying, "But they were so easy, the ones I missed." Her focus seemed to remain on the negative. Unlike the first exam when her formula sheet had adequate column prompts for formulae such as:

X	$X - \bar{X}$	$(X - \bar{X})^2$

to prompt the correct use of the sample standard deviation formula:

$$s = \sqrt{\frac{(X - \bar{X})^2}{n - 1}}$$

that she failed to take advantage of, this time her formula sheet did not have a necessary column prompt so she failed to compute a statistic correctly.^{xv} Karen interpreted this error as a procedural rather than conceptual failure on her part, but her failure to use her correct formula as a prompt for the missing column indicated that she had not explicitly linked the formula with each procedure she needed to follow—reasonable given her low level understanding of the algebraic variable and the fact that this was the problem we had not got to in Session 2 and that she had not therefore practiced. She had skipped another question because she had not understood what the question was asking. Karen's strategic preparation had improved but not sufficiently to compensate for her algebraic weaknesses and because of this lack of preparation, her formula sheet was inadequate.

Karen was reporting on her exam, as it was not available (students returned exams to Ann once they had looked at them—Karen had not arranged with Ann for her to give the exam to me for our session.). Karen did not mention the complex questions I saw later she had done and interpreted correctly on the exam. She had 78% of the computational section correct—a significant improvement on her 52% on this section in Exam #1 and on material that was mathematically and conceptually more complex.

At my suggestion we worked on an inference test problem worksheet Ann gave out for students to try on their own in Class 11 (Mann-Whitney U test of separation of two populations). The course focus had shifted from descriptive to inferential statistics and Karen had grasped the strategic importance of following the same hypothesis testing procedures for *each* test (using the step by step hypothesis testing procedure list Ann had distributed, see Appendix G). Again I used parallel tutoring and diagrams to aid

conceptualization and Karen struggled successfully creating a careful organizing scheme to provide structure.

I noticed that Karen was fast and accurate at addition of whole numbers, something I am not good at:

JK: ... they want you to add up the rank. [Karen circles the ranks that the c-group got] there you go, there you go; there you go; right now add those up

Karen: 16 ([immediately])

JK: Oh good on you! You did that boom! Wow quick! (Session 3)

And I found more and more opportunities to help her notice how she could move out of cloudy misunderstanding into the clear light. For example, after Karen had begun to add ranked scores instead of ranks we looked at them more closely and she saw it:

JK: so what is the difference? What is the difference between a score and a rank?

Karen: Well this is just how, um, chronologically where each one falls and that's the score that each one got {exactly} how many numbers, I mean how many words they actually remembered

JK: Right, right and they have them ranked there because you put them in order {right} but they are still each one are scores so the "rank" there is almost an adjective {right} okay? {right↑}

**Time (1-2 seconds) as K adds up

Karen: I can't subtract worth a dime //

JK: But you're adding is like whoosh, right? (Session 3)

Again she added a string of numbers almost instantaneously, again drawing my admiration despite her negative comment about her subtraction abilities, seemingly designed to keep my attention on her deficits. She seemed unused to receiving positive recognition for her mathematical work.

I continued to feel that Karen was maintaining emotional distance from me. In the following interchange she seemed to be carefully considering each word so as to reveal a

minimum of information and she ended the exchange by abruptly turning back to the statistics.

Karen: Usually I have this all done. I've been a little too harried

JK: oh! oh! Other things going on in your life?

Karen: Well, no, I've just been **, um, *** away.

JK: Away? (giggle) I don't know; do you find Fourth of July throws things off a little bit {yeah = yeah=} oh, vacation! {yep↑} and all of a sudden you remember it's summer {mmm} and you think summer we're really not meant to be doing this {right} is that what happens to you? {Yep} So where'd you go?

Karen: Um I just went to visit *some friends {yeah} but um* Okay, so the ranked score

Our interchanges about her former mathematics teachers and her family's

mathematics orientation were similarly limited. I was experiencing this as her not feeling secure enough to trust me further and that seemed to have been a long established pattern for her, at least with mathematics teachers.

At this session, because Karen seemed less stressed and more focused than in Session 2 and she had done better on her second exam, I expected her responses to the *JMK Scales*, to have become more positive (see Figure K3, responses labeled 3). However there were only small positive changes on items 1, 2, and 4, her sense of hopelessness about her mathematics future had increased, and the other responses remained the same. From Karen's point of view, it seemed her short-term future in this class was not assured. She had made gains on Exam #2, but her control of the material still felt uncertain. The last time she took this class, she had improved on the second exam, too, but it had not been enough to pass. A D would not be sufficient; for her major Karen needed, at minimum, a C. She was taking responsibility for doing more, but her focus remained on mastering procedures, rather than concepts. She was no longer blaming Ann, but now her self-criticism seemed, perhaps, too harsh.

Exam #3 was scheduled for the following Monday. Karen asked for an extra appointment before the exam. This request indicated to me her growing sense of responsibility and her ability to take some control by getting the help she needed *when* she needed it. I wondered if it also signified that the strategic urgency she felt was sufficient to override her defensive distancing from me at least with respect to the mathematics.

Karen's Session 4

Karen came to drop-in several hours before our scheduled appointment. I was anxious for the students, especially Karen, on Exam #3 because it was the first one on inferential statistics and hypothesis testing, and because of the number, variety, and complexity of the inference tests to be examined (see chapter 5, discussion of Class 13, p. 175 and Exam #3, p.179).

I offered Karen the materials I had prepared (see chapter 5, p. 179) and she decided to use the flow-chart template to create her formula sheet for the exam.^{xvi} Karen's other organizer was her "Steps for Hypothesis Testing" list. During this drop-in session, I was also working with other students, but I checked with Karen from time to time to discuss and help her work through her struggles in deciding which test to use for a particular situation. Deciding between the two-sample independent t test and the two-treatment correlated t test was difficult for her so we discussed ways to decide which test to use based on the situation described in the question.

At our scheduled meeting time. Karen had completed three of the six problems I had given her, with only occasional help from me. I commented that she was much calmer than she had been before the previous exams. She agreed to some extent but

qualified it. She seemed vulnerable to anything that went outside the structures she was carefully building for herself.

You know if she changed a word or the order of the words just one little bit, I wouldn't know what to do...that's what happened to me in the last test. She had worded it differently so I sat there and looked at it [what I'd done] and I took it out to her and she wouldn't say...I was so confused...I just left it because I had to go on to the next one. (Session 4)

I asked her what statistical test the next question called for. She decided on an independent t test and I asked about her reasons. She hesitated and asked for time to look at the question, then said firmly, "It is an independent t test because she has given us the two s^2 s." I affirmed her choice and her thinking. In retrospect my response surprised me. This was quite different from my response to similar reasoning by Mulder. When he used this reasoning I remonstrated, insisting that he link his decisions with the logic of the setting by determining whether they originated from two different groups of subjects or from one group of subjects tested twice.

Why did I not do that with Karen? I had had that discussion with her during drop-in earlier but I didn't bring it up again now. I think I was responding to my sense that she was carefully building up a fragile personal structure for negotiating the exam that I hesitated to challenge too forcibly.^{xvii} I did not want to risk upsetting the procedural control she seemed to be gaining over the material for the imminent exam by pushing her to make these logical links.

For the remainder of the session Karen worked problems while she carefully organized her formula sheet and I quizzed her on the definitions and the sample versus population categorizations of each letter symbol. By the end of our session Karen had completed at least one problem for each of the inference tests being examined.

I noticed out loud Karen's impressive grasp of the material and at first, she denied it, attributing her success to external factors such as being in mathematics counseling not in the exam room or "cheat[ing] in the book." I challenged that thinking pointing out that it was not me helping her; she was doing it herself. And she challenged herself noting that she *hadn't* used the book only her formula sheet that she could use in the exam.

Karen: So I just know that this and that are the same thing!

JK: Right {alright (+)}. You are doing quite sophisticated mathematical thinking!!

Karen: Yeah, but when it comes time for the test I'm not going to remember it (-). Maybe if I look over it just before.

JK: You will. You are able to do this, you know. I'm not helping you at all you're just doing it yourself.

Karen: See this is going to be my problem. This was already done you know what I mean? I cheated in the book. No, well I just looked at this [decision chart formula sheet that she had been adding to and could use during the exam] actually

JK: You really did and you'll have this on the test. Right? And you've become aware of how you might be tempted to choose one [statistical test] rather than another! Right? So I think you probably won't, right?

Karen: 'Cause I could probably get half way through the problem and realize {I would think so} that it wasn't right (Session 4)

Karen went on. She realized she had now internalized the material and ways of troubleshooting on the exam if she got into trouble. It was remarkable change from how she had experienced her confusion during Exam #2. Karen was going into Exam #3 a very different person from how she had gone into Exam #1. She had prepared strategically because she was comfortably aware of what would be tested; she could compensate for her algebraic and arithmetic deficits; she had a carefully prepared and strategic formula sheet, and now she had become aware of her own grasp of the material and her ability to monitor and troubleshoot if things went wrong.

Discussion of Sessions 3 and 4 and Exam #3

My Relational Focus in Counseling

Karen's responses to the *JMK Mathematics Affect Scales* in Session 4 before Exam #3 revealed that she felt considerably more able to make mathematical decisions and she was more positive about herself mathematically than she was for the previous exams (in Appendix K, Figure K3, compare responses labeled 4 with those labeled 2). On the other hand, the material was much more complex, and Karen seemed to be relying heavily on extrinsic clues and her formula sheet. Overall, her improved sense of mathematical efficacy (items 5, 7) seemed to be breaking up her sense of hopelessness (item 3), while the uncertainty of the imminent exam and her reliance on procedures and extrinsic artifacts seemed to have kept her discouragement (items 2, 4, 6) from dissipating.

Karen earned 85% on Exam #3 and she was *very* pleased. She lost only 2 points^{xviii} on the computational part, 12 points (6 questions) on the multiple-choice and 1 point on the symbol identification. As noted in chapter 5, Karen was one of only two students in the class whose grades improved on this test. With the in-class extra credit assignment where she earned 6 points, her overall exam grade was 91%.

Even though Karen was gaining control where she previously had felt helpless and discouraged, even depressed, and her mathematics depression appeared to be lifting somewhat, I was concerned that she had not made an extra appointment to address her underlying number and operation sense weaknesses. I realized that her teacher relational detachment issues would have made it very difficult to seek that appointment. It had likely been difficult enough for her to make the extra appointment before Exam #3.

Supervision

In presenting Karen at my own supervision session with Dr. P., I expressed a history of my thinking about the changes she had made that I realized, as Dr. P. heard, were considerable. “She’s doing better and she is growing into a competency she didn’t know she had.”

Dr P. suggested that Karen might have a new metaphor for herself doing mathematics, and that it could be helpful for her to assess her own change. At each session, I had been prompting Karen to look at what she was doing differently and seeing differently. He suggested that reflection should continue at the next meeting. I wondered how to help her reflect more deeply. Her underlying arithmetical and mathematics issues and our limited opportunity to explore and discover her real ability to think conceptually, dampened my enthusiasm about her current success. Was her success “good-enough?” For this class, perhaps! But I worried about its strength for restoring a healthy mathematics self. Karen’s opportunity for reflection would come when she did the final evaluations and posttests.

Karen’s Session 5

Again Karen came early to drop-in, this time with a little grin and feeling “good” about her grade on Exam # 3. She told me that her family were “waiting at the door” for her, obviously very pleased. Then she began doing homework problems from the text.

In the individual session, Karen tackled a question where she was asked to find the values missing from a one-way independent-groups ANOVA summary chart and answer questions about it. She was stuck on the question, “How many groups are there in the experiment?” (Pagano, 1998, p. 378). She said, “I don’t know what to do.” This was

the type of inverse reasoning question that Karen found difficult. I suggested some ways to think about how those numbers were derived and what they meant and I coached her to think backwards to find the number of groups and, in the next question, the number of people in each group (assuming equal numbers in each group).

Karen and I discussed strategy for preparing for the exams. In response to my query about completing homework problems prior to coming to the Learning Assistance Center, Karen replied, “Ah no, I did those tests [problems] that you gave me. They were way more helpful than doing all this homework.”

Karen had never directly attributed her difficulties in mathematics to lack of ability. In the *Beliefs Survey* she had agreed that her ability to do mathematic could improve but she also agreed that some people can do mathematics and other people can't. In addition she labeled herself (and her family) as a reading and writing type [not a mathematics type]. I took the opportunity in the following discussion to bring up the topic of ability.

Karen: Right, I was not interested in math at all. I don't like it; that's why I don't do as well

JK: Yeah that's got a lot to do with it probably, not much to do with ability↑

Karen: Probably not

JK: More to do with interest and CONFIDENCE

Karen: Right

JK: Because as you don't do as well, your confidence goes down {exactly}
You THINK you're not good at it (Session 5)

Karen's attributing her not doing “as well” to her lack of interest sounded defensive. When I mentioned ability as a possibility and dismissed it she gave only qualified assent “probably not.” That was when I told her of Liping Ma's (1999) work studying American elementary teachers' generally poor grasp of arithmetic. I speculated with her that she most likely had teachers who themselves had not understood any more

than how to do the procedures. She agreed. I could tell that it would take more than her good grade, my mirroring of her competencies, my logic, and research findings to convince her of her sound ability to do mathematics, but all of these were making rather large chinks in her armor.

Karen was pleased when I noted approvingly that she continued to follow the hypothesis testing procedure meticulously. She commented that, in class, the other students weren't doing it, but she was. Karen was beginning to recognize that more of her mathematical behaviors were positive, in dramatic contrast with her former almost exclusively negative evaluation of herself doing mathematics.

We worked on an ANOVA together. At one point I had a formula incorrect but Karen had it correct so our answers were different. I questioned her but she held her ground and then I realized that it was I who had it wrong.

Karen: I get more than that as the first one

JK: Why are you squaring that again?

Karen: I've just got the sum of x -one [x_1]

JK: Oh you've got the formula copied wrong [inspecting Karen's work]

JK: Oh no, you don't! I'm doing it wrong. You're doing it correctly!

Karen: They're really big numbers though. (Session 5)

Karen's caring attempt to reassure me that my mistake was understandable:

"They're really big numbers though," marked a reversal. Karen had experienced tolerable disappointment in me, the idealized teacher-parent, at the same time as she realized that *she* had it right. Her competent self was emerging and could care for me the parent. During class problem-working sessions, Karen continued to show no interest in working with anyone other than "experts" in the mathematics, in this case Ann or me. While the parent image was still prominent it was being modified by reality and incorporated into her mathematics values structure.

And Karen had changed her mind about Ann. Now she recognized Ann's efforts on her behalf and her defensive detachment had been replaced by a sense of secure attachment, as illustrated in the following exchange.

JK: Oh you'll plug all those into this. I can't imagine trying to do- I know it's- I hope she gives us one with a lot smaller numbers [on the exam], that would be better but no matter what

Karen: She usually does. {She does} Even when she gives us the practice problems she never has the [large number of large numbers]. I mean, the book is ridiculous sometimes like these aren't the biggest charts I've seen like way back when we were doing just frequency distributions like a whole page was writing; it was really long.

JK: Yeah, that's right it was wild. I think they do that because

Karen: She even made a comment too she said "I'll never make them as long as the book does." (Session 5)

It was in this session that I noted too a distinct change in Karen's emotional distancing from me. When I commented on different national views on mathematics ability and I mentioned Australia's, Karen talked at length of her girlfriend's visit to Australia.

As with Session 2 we had not covered all that would be on the exam. In fact, even more would be covered in class tonight that would be on the Wednesday exam. But I was confident that Karen had it well in hand and she was too. Although I did not know it at the time, this was my last meeting one-on-one with Karen.

Karen's Post Counseling Processes

Karen earned 88% on Exam #4. She was very satisfied. She lost no points on the computational and symbol identification sections. All her points were lost on the multiple-choice (6 out of 23 questions incorrect—a consistent result; see Appendix K, Table K2).

After Session #5 we had scheduled an appointment for the following week that Karen cancelled. I was concerned that we meet before Exam #5 because I knew the exam would require students to decide on an inferential test using a specific decision flow chart. I knew the questions on the exam would not contain the specific clues I thought Karen might be relying on such as the ' s^2 's for the independent samples t test so I suggested she come to drop-in on the day of the exam; she did not come.

In our final session her self-reliance had been remarkable. After that she felt she could handle the rest herself, and she did. I struggled with my countertransference parental concern. It was hard for *me* to let her go and trust that she was in a good-enough place, that she could do it on her own but I need not have been concerned. In fact, I should have been pleased at Karen's growth. She earned a 96% on that exam (although she did fail to correctly identify the independent samples t test!).

Optional Comprehensive Final

Karen decided to take the optional comprehensive final after class ended to replace her lowest exam grade. I offered an appointment by e-mail, but she declined, which made me quite anxious for her again. This was not my countertransference alone. Students' grades on comprehensive mathematics exams, even with review, are typically one-half to one whole grade lower than on their other tests. Karen earned a 57%, which was lower than her lowest test grade so it did not alter her final grade, a B. She had badly failed the conceptual multiple-choice part but on the computational part of this test Karen earned a 75%. Although this 75% was considerably lower than she had been getting on computational sections, given that the exam was comprehensive, and that she took it with no class or tutorial review, it was reasonable for her. Even her overall poor result was

relatively comparable with results of others who took the comprehensive final (see chapter 5, Table 5.1) and unlike her each of them had had a final review session with me. I wondered whether, however, without discussion of this overall low grade Karen might allow it to diminish in her mind the real gains she had made in her mathematical prowess (Appendix K, Table K2).

Evaluations

Karen's Evaluation of Her Changes

Karen said her initial “cloudy” metaphor for herself doing mathematics may have changed “a little” but she was not specific. She predicted she would not “ever *like* math” but that she was “more comfortable” with it. Karen attributed her own positive changes to “1 on 1” and to the “amount of time I put in outside the class” (*One-On-One Mathematics Counseling Evaluation*). She learned that she could “do a lot better than I thought” but still found the “conceptual” aspects of statistics puzzling and would “pay particular attention to the conceptual portions” of the next mathematics-related course she took.

My Evaluation of Karen's Changes

Karen took the *Feelings* and *Beliefs* posttests in class, and the *Algebra Test* and *Arithmetic for Statistics* assessment after she had taken the optional comprehensive final. These two tests confirmed my sense of Karen's weak arithmetical and algebraic understanding (see Tables 6.1 and 6.2, respectively) and they also confirmed her need for compensatory structures and strategies to achieve the success she did.

By the end of the course Karen's overall defensive and detached pattern of relationship in the classroom setting (or possibly the college setting) had eased. She engaged in conversation with other students during the problem-working sessions but still

she would not work with or check her work with anyone but Ann or me. Her initial angry negativity towards Ann had reversed. She had begun to forge secure attachments with trustworthy mathematics teachers—Ann and me.

Karen's sense of herself as a mathematics learner had become a little more positive. By the end of her last individual session Karen's discouragement responses (items 2 and 5) on the *JMK Mathematics Affect Scales* had lifted (see Figure K3, responses labeled 5). She also indicated that she was less likely to withdraw from the course. Until Session 4 (just before Exam # 3), Karen's responses on all scales were at, spanned, or fell below the mid-points. Now at Session 5, in 3 out of the 7 scales, her responses were above the midpoint (positive) and the others at least touched the midpoint.

Karen's moderate mathematics depression had lifted somewhat in the context of the mathematics counseling and the current course. However, her mathematics depression appeared to have developed over many years of school mathematics in an environment focused on procedural mathematics learning which she had little hope of understanding and where her developmental needs were neglected. This resulted in an underdeveloped mathematics self: she was underprepared mathematically and her mathematics self-esteem was therefore low. Ann's course forced her to tackle procedures on her own, and a formula sheet was allowed, so it was possible for Karen to gain control and succeed. She developed a more positive sense of her mathematics self and moved from an unhealthy detached independence to good-enough mathematical self-reliance. For a lasting improvement and success in a more conceptual mathematics course, I believe Karen would have to understand arithmetic better and develop her understanding of the

algebraic variable. She still had finite mathematics ahead of her, and she planned to take it in summer 2001. She said she would return to the Learning Resource Center for support “as long as Jillian is there” (*Follow-up e-mail Survey*). I worried that might not be enough.

The changes in Karen’s responses on the post-course *Mathematics Feelings* survey (see Appendix K, Figure K1) and *Mathematics Beliefs* survey (see Figure K2) surveys seemed largely consistent with her changes and her success in the course, although there were some apparent anomalies: Although her abstraction and number anxieties had decreased substantially, her testing anxiety had increased (see Appendix H, Table H3).

Evaluation of Counseling and My Changes

When I met Karen I was immediately drawn into her anxious, depressed, negativity. With her I saw her deficits and limits and heard her anger at Ann and despaired of her making it and of my being able to help her. But as I incorporated relational counseling assessments and approaches into best practice modified cognitive constructivist tutoring and course management counseling, I changed my mind about Karen and about me. As I helped her see Ann and herself differently I began to see *her* differently. My expectations of her rose, my role changed from motherly rescuing to guiding hand and co-explorer and she rose to the occasion. We found ways for her to compensate for her significant background deficits and my admiration of her grew. Going beyond tutoring to incorporate relational approaches led to her not only doing the mathematics but also to her recognizing herself doing the mathematics, and her underdeveloped mathematics self developed. I (and Ann) had provided the opportunity

for her to forge secure attachments to mathematics teachers and she had availed herself of that opportunity

Evaluation Summary

Karen's mathematical relationship patterns had begun to change. Her mathematics self was becoming firmer; she found she could gain control over the mathematical material to a greater extent than she had ever thought possible. I felt she was still quite dependent on teacher/tutor input and external judgment of her mathematical correctness rather than on her ability to judge the internal consistency and logic of the mathematics. But once she worked out how to use structure and strategic effort to compensate for her mathematical uncertainties, she did it on her own. Her final reflections indicated the movement she had made towards an improved sense of her mathematical self and mathematical self-reliance: She wrote, "I became more confident as the course went on and I came [to drop-in and individual mathematics counseling sessions] more for security in knowing I got the answers right" (*One-On-One Mathematics Counseling Evaluation*). She apparently felt she had enough of a mathematical self to do it herself; she no longer needed me except to check that she was on the right track.

Epilogue

Karen *did* enroll in Finite Math in the summer of 2001 and she *did* come to the Learning Assistance Center to get help from me. Following her pattern of summer 2000, she came first just before her first exam, overwhelmed with the amount of work, resentful that her transitional object—a formula and procedure sheet—was not allowed, and not having practiced each type of problem. The instructor allowed her an extra few days but she still did very badly. Karen regrouped and began to come regularly to Drop-In. She

did not like my going from person to person at Drop-In and not attending solely to her so she began to work on her own in the cafe and would come down to the Learning Assistance Center during Drop-In just to ask specific questions and then go away again. I suggested we meet to deal with her arithmetic issues, which surfaced again but she never made *that* appointment.

Karen's mathematical self-doubt remained a problem: Although she felt confident with Venn diagram counting questions, on a take-home quiz she erased and changed her answers when another student had different ones only to find out later that she had been correct. "I always assume that I am the one that is wrong." Her belief that the teacher was against her also returned though I did not feel included in that this time. As I suspected her mathematics depression had deepened again since the end of PSYC/STAT 104 but she persevered, and I continued to confront her globalizing self-negatives with proof of their fallacy from her own work. Again, her grades improved. She made and kept two individual appointments before the final, when I was more able to take a mathematics counseling approach with insights from our earlier counseling sessions. Karen was organized and knew what she needed to learn. She was allowed to use a restricted teacher-developed formula sheet and went on to earn a B⁻ on the cumulative final and a C⁺ on the course. This was quite an achievement because it was a more mathematically demanding course than PSYC/STAT 104.

For Karen this was a good-enough outcome. All the mathematics requirements for her degree were completed. She will probably not take up the challenge of dealing with her underlying operation sense, number sense and algebraic deficits, which are at the root of her mathematics depression.

JAMIE'S COURSE OF COUNSELING

Jamie needed help with her statistics course. She decided so herself. I know this because she signed up for mathematics counseling with me for once a week, not once every other week, which was an option, and later, in her end-of-the-course evaluation, she wrote that her initial motivation for signing up for counseling was “so that I could get a better grade in the course,” unlike other participants whose initial motivation was to help me with my research^{xix} (cf. Mulder, Robin, and Autumn). But if I had not crossed lines with Jamie that are generally drawn in the helping professions, it is unlikely that we would have worked together at all. As a helping professional I had learned that I should wait for the person seeking help to approach me; it is usually considered unacceptable to pursue the student in order to provide help, no matter how necessary that help seems to be. Jamie, however, despite signing up for weekly counseling sessions and despite an e-mail exchange between us about when, slipped quietly away after class night after night until finally I decided to sit beside her in class in order to arrange the appointment she had indicated she wanted.

Jamie was a tall, dark-haired, white, traditional-aged full-time student at State University who had just completed her sophomore year. Her father was an engineer and her mother was also college-educated. As a psychology major, Jamie needed PSYC/STAT104^{xx} but thought it might be easier to do it here at Brookwood State in the summer; the small class size and focus on only one course, she thought, should more than compensate for the course being faster than in a regular semester (ten weeks compared with 15 weeks to cover the same material). Jamie had withdrawn from Finite Math in the fall of 1999, without penalty because of illness, although she was failing at the time

(Session 1), so the last mathematics course she reported that she completed was pre-calculus in high school in which she earned a “C?”^{xxi} (*Pre-Test Mathematics History Survey*, see Appendix C). Ann, the instructor, thought she was “VERY quiet” and used the word “fragile” to describe her (Interview 2). Jamie wrote that she hoped for a B in PSYC/STAT 104 but expected a C (*Pre-Test Mathematics History Survey*). Her summer job was in a department store in a mall.

What struck me most about Jamie at the first class was her demeanor—she was sitting straight up with her eyes lowered. At times I wasn’t sure if she was asleep but her expression did not seem to change and she did not make eye contact or interact with anyone, except during the paired introductions interview when she told her interviewer that she was “not keen” on mathematics or doing this course.

I found out that Jamie was cognitively capable and well-enough prepared mathematically to succeed, yet in two attempts at mathematics courses in college she had *not* succeeded. Jamie’s personal and mathematical style and challenges induced her to accept my offer of help but dissuaded her from accessing it. And hers contrasted markedly with my personal and mathematics style and challenges. Mine induced me to cross accepted helper boundaries to give her the help she needed but caused me to struggle with helping her find her voice when mine was so loud and hers so quiet. How we understood and struggled with, negotiated, and made use of our differences together forms the substance of this account of Jamie’s and my growth as tutee and tutor over the summer of 2000. As I used the relational counseling approach that I delineated in chapters 2 and 3, I looked at her and at myself differently from how I would have in my former practice. Both Jamie and I benefited—she “realized it was more about my feelings

and confidence in my math ability, th[a]n any real problems with the math course work” and she earned a B⁺ and I learned how attending to our relationship helped me understand her and myself better and modify my approach with a student who was so different from me.

Jamie’s Metaphor: Mathematics as Stormy

By the time Jamie and I met for the first time in the fourth week of the course, she had received the results of the first exam and to her delight and surprise, had scored a 95%. Nevertheless, her metaphor for mathematics was a [violent] thunderstorm. She explained her choice: “stormy because it is usually very tough for me to do and understand math, even though I did good on the test I’m afraid the ‘storm’ will come back again” (archived *College Learning Metaphor*, see also Appendix B).

For a storm, Jamie said, she would, “prepare for it; before it comes, like, get your water or flashlights.” When I asked how she would handle the storm when it came, Jamie replied that she would “stay inside.” She saw how her storm preparation related to mathematics: “Well, you have to prepare for tests,” but she wondered “how staying inside does.” We did not initially explore what the storm itself was to Jamie—I assumed it was mathematics itself, in particular, mathematics tests. I did not pick up then on the connections between one of her other metaphors “shark,” her use of the word “afraid” in her “stormy” metaphor, and the link to my countertransference experience in the first study group: *my* experiencing being potentially dangerous to Jamie (see chapter 5, Study Group 1). I also didn’t attend to her wondering what “staying inside” out of the storm might have to do with her doing mathematics.

Over the course of the first three meetings Jamie told me of her stormy experiences with previous mathematics classes. Her most recent experience, she told me, was withdrawing failing from a finite mathematics class at State University^{xxii} and high school had been mixed. The storms began in elementary school, however.

Student-teacher Relationships as Stormy: Jamie's Internalized Teacher Presences and her Mathematics Self

Jamie's early elementary experience of mathematics sounded calm: "first grade and second grade and stuff, you know, I got 'A's in everything," and she remembered she'd liked her fourth grade teacher. Her experience of 5th grade had been different: Her 5th grade teacher "yelled" though not at her, and not particularly about mathematics. Jamie attributed the start of her doing poorly in mathematics and science to the frightening classroom situation this 5th grade teacher created, though her reading and writing achievement remained unscathed. She remembered:

But in 5th grade, my teacher kind of yelled a lot, and stuff, and I didn't do good [in mathematics]... Science, I think, too... I did good in writing and reading, that kind of stuff... It was from then on...I think she had a short temper, I guess.
(Session 1, June 20)

In Session 3, as I was asking Jamie about her shy, non-interactive demeanor in Ann's class, the effect of her 5th grade teacher came up again. Jamie explained further "You want to sit down and shut up so you don't bother her [the 5th grade teacher]." I was struck with how closely this described Jamie's current behavior that I observed in class. I was also aware of how much Ann, the instructor's, approach differed from Jamie's description of this 5th grade teacher.

In high school, to Jamie's surprise ("because I don't do good in math") she "did good" (a B or B⁺) in Algebra I. The storm hit again, though, in precalculus that she took with the same teacher she had for Algebra I. Her experience in precalculus was so "bad" that by the end she said she didn't understand anything and she found the teacher to be "stand-offish, like, 'You should know this.'" She remembered needing little help in Algebra I. It had gone smoothly ("I didn't do bad and good and I wasn't up and down"), but when she did need help in pre-calculus, she (and the other students, she said) found the teacher to be unavailable. Jamie conceded though, "Well, part of that not getting help is partly me." Jamie's unwillingness to seek help from the teacher (i.e., *Jamie's* unavailability), she believed, contributed to her problem of not getting the needed help. It seemed to me, however, from what she said that she had been inhibited, not only by her "stay inside" relational pattern, but also by her observations of other students' difficulty in getting a response from the teacher. She perceived this to constitute a negative change in the teacher from her Algebra I experience of her. Mathematics teachers had become potentially dangerous to her. It was as if she had internalized bad mathematics teacher presences through whom she saw Ann and me or any mathematics teacher. And all her difficulties she attributed to her own inability to do mathematics.

Student-mathematics Relationship as Stormy: Attachment and Jamie's Mathematics Self

I reflected on the probable effects of Jamie's stormy history on her sense of mathematics self. Her attachment to mathematics and to mathematics teachers had been secure through fourth grade. Then in fifth grade her expected secure teacher base was withdrawn: She could no longer safely explore and ask for or expect the support she needed. Thus began her sense of isolation, separation from a secure teacher base and,

from then on, from a secure base in mathematics—she could no longer be sure that she understood it, sometimes she did well, other times she did not, but she could not ask why because she was no longer sure of the availability of the teacher.

Jamie and Me—Dealing with Storms Now: Relational Counseling for Jamie

As I reflected on what “stormy” meant to Jamie, these understandings clarified for me the effects in this class of her current expectation and fear of these storms continuing. Our differences became more apparent but I also became more aware of what I needed to be for her. “Stormy” seemed to have multiple meanings to her, all negative, but the consistent theme was absence of calm—teacher “yelling” or “ups and downs” in understanding or grades. I, on the other hand, enjoy storms, especially the thunder and lightning, and calmness bores me. I would have liked to persuade Jamie that “stormy,” like mathematics, might have positive aspects— challenge, excitement, darkness lit up by the lightning, As—but I gradually realized that none of these (probably including ‘A’s) would feel agreeable to Jamie. If I could offer myself as a smooth, level path with no surprises around the corner, only more of the same, or perhaps a gradual ascent, nothing that would startle her or trip her up, that would be perfect.

Jamie’s Mathematics Relationality

Interpersonal Relationships and Self: Family and Personality Interacting

At Jamie’s second session, just after she had taken Exam #2 but before she learned her grade, I asked about her family’s reaction to her 95% on the first test. Jamie’s Dad was pleased and had expected it to continue; her grade was proof to him that she *could* do well in mathematics. Jamie saw it differently; this was not proof but rather an

anomaly, not likely to be replicated. She knew she had not done as well on Exam #2 and she was not surprised.

She had negotiated her panic on Exam #1 when she found and corrected an error so I presumed that that success and her high grade would result in reduced anxiety for the next exam. On the contrary, Jamie said she had higher anxiety on the second exam because of her family's (in particular, her father's) higher expectations. I realized that I had to navigate my own assumptions and expectations of Jamie. As I proposed a conjecture and learned to listen to Jamie's responses, including her hesitations, qualifications, and tone of voice, she changed my mind and revealed herself. Although she never contradicted my conjectures about her, Jamie's unconvinced "maybe"s contrasted with interested and curious "possibly"s; her hesitant "yeah↓"s contrasted with her somewhat "yeah="s and her firm, in-agreement "yeah↑"s, laughs, and "I know"s. Jamie usually qualified her own theories with "I guess"s and "maybe"s but I had to watch and listen to clues to how deeply she held these theories (See chapter 4, Table 4.3 for transcription coding conventions I use.).

JK: And the anxiety in the second was just to do with that confusion about the -- [Jamie had just told me that she had known how to do the various correlation and regression computations on the exam but had been confused about what each one were called.]

Jamie: Yeah=, that and I think I might have been more [anxiety] for the second, actually

JK: You were more anxious on the second one?

Jamie: Yeah, I think so.

JK: That's interesting. (surprised) Does that happen to you? Like for the first one in your course you're not quite as anxious?

Jamie: Maybe↓ (unconvinced)

JK: Why do you think you were more anxious for the second one?

Time [here I waited for Jamie to answer—several seconds]

Jamie: Umm, Well, I know this time why I was.

JK: Okay, why?

Jamie: It was because of my 95!!!

JK: Ahhh! That's interesting! That made you more nervous? Now why?

Jamie: Well, I guess 'cause my parents {Ahh!} were expecting it to be maybe a similar grade.

JK: Oh, so there was this high grade and it was really possible not to get that? {yeah ↑} put a lot of pressure on you? So do you think the actual level of confusion [also] contributed to your confusion?

Jamie: Possibly=

JK: Shouldn't tell them [your parents] your grades... {(laugh) I know↑} keep that to the end, but you were so excited it would be hard to keep that to the end...

Jamie: Yes. (Session 2, July 3)

When I offered the suggestion that Jamie might be relieved to get a lower grade on the second exam I didn't feel as if I was putting words in her mouth and her strong "Yeah↑"s confirmed this.

JK: So it actually may be a relief to get a little bit of a lower grade?

Jamie: Yeah ↑

JK: And then you won't feel so much pressure on you for the next one.

Jamie: Yeah ↑ (Session 2)

She seemed to have experienced the 95% grade as much as a storm as she might have a really low grade, an "up" that she seemed to dread as much as a "down"—the absence of calm. And I was surprised and curious. How could this be? It was hard for me to entertain the possibility that an A might constitute a burden for someone. When I considered where I stood in Jamie's world, I had been more with her parents than with her, not only in my own mind but also perhaps in hers. I heard her conflicting motivations—to "do better on the course" but also to maintain calm, that is, not to do *too* much better, not to raise hopes, not to elicit external pressure to maintain to her, an impossible standard. As she explained herself in contrast with her parents, I became more aware though, of how my expectations of her might differ not only from hers but also in some ways from her parent's. Could I hold high expectations of her *without* exerting the accompanying pressure that made her so anxious? Yes, I decided, because, unlike Jamie's parents, I was positioned to be able to help Jamie explore to what extent these expectations were realistic and to own them for herself if they were.

In the next session (3), I asked about her parent's reaction to the 74% she had earned on Exam #2.

Jamie: Um, I don't know. I guess my dad was just kind of like, "Why did you get a 74?" or something.

JK: Really? {Yeah} Especially when you got the 95, right? And what did you say?

Jamie: ...I just kind of said, "To me it was more surprising that I got the 95 than the 74." You know? (Session 3, July11).

Jamie seemed calmer; as her parent's expectations had been reduced so also was the pressure. Jamie and I could continue to explore and challenge *her* expectations with evidence of her prowess and achievements.

I used the word "quiet" for her when we discussed her reaction to the 5th grade teacher and her demeanor in class; it was held to be self-evident in our discussions. Jamie agreed that she was quiet, like her Dad. During the course, she never used words like "shy" about herself although in the *Follow-up E-mail Survey* she did. "I'm kind of shy," she said, "and don't really like to ask for help, even when I need it, (especially from someone I don't know)." She reported her sister to be "the exact opposite," like her Mom. Jamie saw herself as not so "bad" now, particularly in smaller groups of people she knows. Giving presentations used to be hard but is doable now.

Jamie: Yeah, I've grown a lot since then, believe it or not ... Like, I used to be worse.

JK: Really, a lot?

Jamie: Yeah.

JK: Oh dear. You say, 'worse,' as if this is a bad thing.

Jamie: Yeah.↑

JK: Like, if people say, 'This is bad, you need to speak more', or?

Jamie: Well, like, whenever I had to do oral reports and stuff, it was very traumatic.

JK: Oh, dear.

Jamie: Whereas now I'd be able to get by. (Session 1)

In Session 3, I raised the question of whether Jamie felt I assumed that her quiet, non-participatory style was all “bad” as she labeled it. I asked her what she saw as advantages of her style and she immediately responded that she was “able to listen more. ’Cause some people don’t listen; they’re just talking all the time.” I experienced this as illustrative of how Jamie might experience me at times and I began to explain my own efforts as an outgoing extrovert with shy, introverted family members to modify my behavior and listen. I told Jamie (and reminded myself) how difficult I found it to listen to quiet people, to wait long enough for them to form their thoughts and answer; I was aware of how important in the recovery of her mathematics self it was for Jamie to find and express her voice and I had to allow that to happen.

This led Jamie to discuss her mathematics ability in relation to family beliefs. She reported that her mother often said she passed her own “not good” mathematics genes to Jamie and her sister. “My dad, he’s very good at math. My mom always said that unfortunately, me and my sister got our math genes from her ’cause she’s not good” (Session 3). I questioned her mother’s theory and reminded Jamie that we were gathering evidence that refuted that claim.

Jamie used the words “good,” “not bad,” “bad,” “not good” or “worse” to classify how she and her family did mathematics, to describe her progress in dealing with her shyness in school, and to describe her feelings. I wondered whether Jamie meant them as polarized judgments and if so how much they might be locking her into particular positions—if she (and the females in the family) was “not good” at mathematics or “do[es]n’t do good at math,” if storms were all “bad,” if only others like her Dad were

“good” at mathematics, even contradictory evidence such as her Algebra I experience or her 95% could be discounted as anomalies.

Jamie’s Attachment to Mathematics

My first impression was that Jamie’s mathematics cognition functioning level was very different from and considerably higher than Karen’s. Her 95% grade and the story she told of how she achieved it spoke of a firm mathematical knowledge base, good trouble-shooting skills, and an ability to perform under pressure. Because we didn’t have Jamie’s Exam #1 with us,^{xxiii} she had to recount her experience from memory.

Jamie: Well, there was one part... I started doing it, and then I was like, ‘Wait. That’s not right!’ So I went back and I changed it. Like, it was one of the ones that had to do with some of the earlier problems too... So I went back and I had to change everything, because I was getting all in my brain, like I was ...how to do the wrong ... wrong equation. Like I was doing the right one for a different one,^{xxiv} but not-- ... So I was getting them mixed up. ... But then I realized it, and I went back and fixed them all. ... ‘Cause I was having problems, and I was, like, ‘Why is this not coming out right?’ ... And then I figured it out.

JK: Great! So did you feel good when you went back? And you were like, “Yeah!”?

Jamie: Yeah,[↑] because I wasn’t really sure at first; I was confused if it was right or not. ... It didn’t really look—you know... So—but then after I fixed it, I was confident...

JK: And so what about it made you feel it was not right?

Jamie: Um, I think it was the answer I got. ... Like, I think it was the ‘z’ score [standard normal deviate score—a transformed score indicating how many standard deviations a score is from the mean] or something....And I got a really high number that was...not even on the chart.... so I figured it was probably wrong, if it wasn’t even on the chart.

JK: Right. Cause zs only go up to, like 3 something--

Jamie: Yeah, so then I went back and I was like, ‘Oh no,’ and I was all panicky, and then ... I realized what I did. So it was okay... I think I was doing the wrong thing for that [the sample standard deviation].... But then I noticed, so I fixed it. (Session 1)

In this interchange, there was clear evidence of Jamie’s robust number sense, her understanding of the statistical concepts, her use of letter symbols, her self-monitoring,

and her problem-solving strategies under stress. When we did look at her first test (Session 5 just before the optional comprehensive final), I saw what she had done (see Appendix L, Figures L1 and L2). Jamie had remembered accurately. Jamie's number sense was illustrated by her realization that a z score that was too large was caused by a standard deviation s that was too small since the s is in the denominator of the z formula: $Z = \frac{X - \bar{X}}{s}$. Jamie was clearly pleased with herself that she tracked down and corrected her error, especially because the realization of her error pushed her from her customary anxiety into a panic. As I listened to her I affirmed her masterful handling of the situation.

Unlike Karen's sense of "always" having struggled with mathematics, Jamie's variable history, including positives such as getting As through fourth grade and doing "good" in Algebra I, and her less categorical "usually" very tough pointed to the probability that her mathematics self had developed soundly-enough—arithmetically and algebraically—despite the storms.

This evidence of good-enough mathematics functioning was tempered, however, by Jamie's high *Abstraction Anxiety* score^{xxv} on the *Feelings Survey* pretest and her repeated declaration that "I don't do good at math." I wondered whether there could be a cognitive base for her high abstraction anxiety. In Session 2, I suggested Jamie complete the *Algebra Test* (see Appendix C) to see if her concept of variables was indeed related to her high abstraction anxiety. The results of the test showed that she was comfortably at Level 4, the highest level identified by the compilers of the test, but not at Level 5 the highest level postulated by Sokolowski (1997) whose adaptation of the test I used (p.97) (see Table 6.2 and Appendix L, Figure L3). Jamie thus began the course with an

understanding of the variable that I expected should be more than adequate for the task.^{xxvi} I interpreted the results of the *Algebra Test* to her: “You’re a powerhouse, woman! This discounts my theory that your abstraction anxiety might be related to poor understanding of the variable. You’re very sound right through level 4! Amazing! Not really amazing!...so it’s just this issue, learning to ask for the help you need when you need it...” Jamie had more than a good-enough concept of the variable to negotiate this course successfully; she grinned. Perhaps her parent’s expectations raised by Exam #1 were not so unfounded!

Jamie had some uncertainties about operations with the variable, but she was able to problem solve and check herself as she had on the first exam. Even on Test # 2, where she earned 74%, she had tried an inventive (though incorrect) strategy, on a problem dealing with the probability of success of .7 to solve a binomial probability question using the table that gave probabilities through only .5.

It seemed that it was her stormy experiences with algebra, not an actual inability to do algebra that caused her abstraction anxiety to be so high. I hoped that these results might help allay her uncertainty about her mathematical ability, confirm that her algebra base was secure, alleviate much of her abstraction anxiety, and give her more confidence that she *could* do well in PSYC/STAT 104. I had also seen strong indications (e.g., Exam #1, see Figures L1 and L2) that Jamie’s arithmetical understanding (including her operation and small number sense) were sound. This was confirmed when she took the *Arithmetic for Statistics* assessment with the posttests in class (see Table 6.1 below). I saw the main cognitive focus of our meetings then to be continued efforts to reconnect

her with her good-enough mathematics self, a process already begun with our Exam #1 discussion and her *Algebra Test* results.

Emotional Conditions: Anxiety, Learned Helplessness, or Depression?

Although her *Testing* anxiety score on the *Mathematics Feelings* survey was high, Jamie had successfully used mathematical trouble shooting in a crisis in Exam #1, even though the crisis put her into a state of panic. This was not the type of testing anxiety that interrupts or derails cognition. Rather, it seemed that it was a type of mathematics social anxiety confounded with mathematics and mathematics teacher separation anxiety (see chapter 3) that prevented Jamie from clarifying what she understood and from getting the help she needed, even when it was readily accessible. In class, during problem-working sessions, I had observed that Ann generally spent considerably less time with Jamie than with other students, although she checked over her shoulder almost as often as with others. During these sessions, Ann used a combination of roving checking over shoulders (and offering help if she saw trouble) and responding to cues from students: a raised hand, a head up as she went by, a verbal plea. Jamie gave such cues less frequently than other students. Ann's sense that Jamie was "fragile," seemed to inhibit her from offering Jamie more help (Interview 2).

Jamie's beliefs about mathematics were slightly more procedural than conceptual on the continuum (a 2.7 on the 1 through 5 scale) and were more towards the toxic/negative rather than healthy/positive (a 2.5 on a 1 through 5 scale; a 3 is middle of the scale). On learned helplessness versus mastery orientation, Jamie had the most learned helpless score of the class (a 2 on a scale of 1 (learned helpless) through 5 (mastery oriented) see Figure K4). However, her noticing and troubleshooting her error

on Exam #1, despite her panic, indicated a more mastery oriented than learned helplessness approach in that situation. Again it seemed that it was not so much cognitive but a kind of social learned helplessness that was impeding her ability to take the initiative to get help she needed when she needed it.

I wondered if Jamie's reported learned helplessness was indicative of the often linked situational depression but examination of Jamie's responses on the *JMK Mathematics Affect Scales* seemed to rule that out (see Appendix L, Figure L5). After the first session Jamie's responses had been largely positive. This was not surprising to me since she had just found out about her 95% on her first exam, although her responses during this second session while lower were still at or above the midpoint of the scale (even though she knew she had done worse on the second test). The only responses that were of some concern because of the level of negativity expressed (average of 46.5% positivity; five responses at or below the 50% mark) were Jamie's responses at Session 3 by which time her expected low grade on Test #2 had been confirmed—a 74%. After that her responses bounced back and remained largely positive. Taking this positive affect with the strong indications that her anxiety was more central suggested that mathematics depression was not a real concern for Jamie.

Identifying Jamie's Central Relational Conflict

My experience of Jamie's transference was that she saw me as no less dangerous than the teacher who had first sent her into hiding. If I had reacted to this transference as Ann did by staying away in order not to hurt Jamie, I would not have pursued Jamie to begin counseling. Her insights into her own shyness and introversion pointed to a conclusion that her central mathematics relational difficulty was multi-faceted anxiety

and should be the focus of the counseling. This anxiety not only increased in mathematics testing situations, it kept people at arm's length and stopped her from getting the help she needed and therefore frustrated her goal of "doing good" (but not *too* good) in mathematics. My interest was in the domains, triggers, and origins of the anxiety. Jamie spoke of her social anxiety and her success in overcoming it in public speaking. But this social anxiety influenced her behavior in other domains, specifically the mathematics classroom, and in relating to mathematics teachers. Her lack of interaction with anyone in class, effectively hiding while we faced each other around one rectangular table, her failure to make a follow-up appointment with me, her discomfort when I asked her a question in Study Group 1, and my worry (influenced by Jamie) about asking her a question in front of her peers, all spoke of her anxiety in relating to people, her wish to avoid them, and the demeanor that dissuaded them from interacting with her (at least in a mathematics setting).

In the one-on-one setting I found Jamie more open and willing to connect with me than Karen had been, although at times, especially in the beginning, she exhibited discomfort (In my notes written immediately after Jamie's first session I wrote "At times Jamie seemed close to tears."). Jamie had done well in mathematics and related positively to her teachers through 4th grade, secure in a mathematical learning base. The mathematics separation anxiety that was connected to her 5th grade experience seemed to have been exacerbated by the subsequent experience of finding then losing a secure base in her Algebra I teacher, who she perceived as unavailable when she needed her later. This precalculus class experience also seemed to have made her attachment to the mathematics itself, particularly algebra, feel insecure. At this point Jamie did not know if

or how she might do well or poorly and she did not perceive mathematics teachers (or tutors) to be safe enough to ask for input and support.

In addition to her mathematics social anxiety and her separation anxieties she seemed to have experienced a debilitating performance/fear-of-success anxiety on her second exam related to a combination of shyness and her family dynamics—her father’s expectations and pressure versus her mother’s acceptance of Jamie’s lack of mathematics ability.

The central conflict that was keeping her stuck seemed to be between her desire to succeed in the course, her uncertainty about her ability to succeed, and her sense that becoming conspicuous might endanger her in some way. She seemed trapped far from home, separated from mathematics teachers and her mathematics self and she saw no means to reconnect safely and inconspicuously.

Central Counseling Focus

I realized that if I stayed where Jamie’s transference put me (leaving her alone so as not to endanger her) she would not become aware of her issues in a way that would allow her to change. In counseling therefore my focus was to reach past the protective shield Jamie had built up for herself, to disempower the objects of her anxiety. I had to provide myself as a secure mathematics teacher base, a smooth path with few surprises around the corners, and I had to help her find a way to assess the level of safety of the class and the instructor so that she could choose to access her as a safe support base. Ann’s non-intrusive, respectful approach made the present class a good-enough secure base where this could happen for Jamie, if she were able to see it and was willing to take advantage of it. I also had to help Jamie reconnect with her own sound mathematics

cognition so that she could proceed with her mathematics learning, secure in her mathematics (arithmetic and algebraic) base.

Jamie's Course of Counseling: The Process of Brief Relational
Mathematics Counseling, Session by Session
(see Appendix L, Table L1 for Jamie's schedule)

Jamie's Session1

I found out a lot about Jamie's mathematics experiences and her current orientation at our first meeting. Because she had done so well on her first test and the next test was not imminent, there seemed to be the necessary leisure. Telling her story did not seem easy for Jamie—at times her color deepened and she seemed uncomfortable, sometimes close to tears—but that did not impede her or me from our exploration. My curiosity about her story, the connections, and the apparent contradictions seemed to help her become conscious of it in a piece for the first time—no one had asked about it before. Themes of Jamie's shyness and social anxieties evident in class and in study group, her variable success, the impact of teachers on her success and sense of mathematics self, and her personal preference for calm rather than storms—the absence of “up and down,”—emerged.

When we worked on linear regression problems I noticed that Jamie's arithmetic and use of algebra seemed adequate and she did not seem anxious as she worked. At the end of the session, as we worked some of the assigned homework problems in parallel, another student came for his appointment. Jamie finished the problem she was on and left without arranging our next meeting.

Jamie's Session 2

Again, as for the first meeting, although there was ample opportunity, Jamie did not approach me to make an appointment for a second meeting. I eventually approached her and we arranged to meet during the July 3 cancelled class time. Our second meeting took place during the sixth week of the course when she knew that she had not done as well on the second test but didn't know her grade.

At this meeting we discussed a picture I thought seemed to be emerging. I showed Jamie her *Survey Profile Summary* (see Appendix L, Figure L4) where her testing anxiety and helplessness scores were the most extreme in the class and her abstraction anxiety high, exceeded only by that of another student. Discussion of learned helplessness (and its counterpart mastery orientation) and Jamie's extreme score led us away from the cognitive domain (Jamie's mastery recovery on Exam #1 seemed to discount helplessness in that domain. See p. 251.) towards the relational domain.

I asked if Jamie would have eventually approached me for an appointment if I hadn't approached her. She didn't know but from past experience it seemed unlikely, even though she had not done as well on Exam #2. I suggested that her learned helplessness might be more about this apparent inability to access help even when she knew she needed it—a type of relational helplessness—and that this was perhaps also illustrated by her behavior in class. When I asked Jamie why in class, for example, she didn't ask the questions she had in her mind, she replied, "That's my fear that I'll be wrong." When she was not convinced by my response: "*Asking* a question, you don't have to be right!" I realized that to safely even *ask* a question, for her, it would have to be a "right" question, one that fit logically and unobtrusively into the context. Even during

problem-working sessions when students were working in pairs or alone and Jamie always alone, she would not ask Ann a question though Ann made herself available by circling the room and checking on each student's progress. Neither did she answer any questions during lecture-discussion sessions even when several students responded together and she knew the answer. The unconscious subtext seemed to be that she would draw attention to herself if the question or answer were "wrong." Why was I so concerned that Jamie speak in class? Perhaps it was just her style and of no consequence? I asked Jamie what she did if she did not understand what was going on or how to proceed and she replied, "Nothing." That concerned me and I believed it should have concerned her. We discussed a possible relationship between my asking her questions in study group in order to get her to verbalize and clarify her thinking (even when I knew it made her uncomfortable) and her exam results.

I told Jamie that I would no longer approach her to make appointments. To do so would allow her to continue in her pattern of getting help only if she was required or if people like me pursued her (even though her "don't hurt me" demeanor made that unlikely). I wondered out loud if she might be able to practice getting the help she needed, and I suggested two homework assignments for her: (a) to set herself an assignment to ask or answer a question in class. (b) to approach me to make the appointment for our next meeting if she wanted one. I confessed to Jamie that I found it hard to let go of my practice of making appointments with her, and allow her to choose to do it or not:

That's risky to me {giggle}'cause ...I have the sense that you have so much potential and I have a sense that here are some of the clues as to why you don't do as well as you could and that's exciting to me {laugh}. I think, "We could really

get her over this hump,” you know, doing math, so I’d really, I’m like, “I want to go get Jamie!” and that’s my mother thing. (Session 2)

When Jamie then took the *Algebra Test* to explore possible connections between Jamie’s high *Abstraction Anxiety* score and her understanding of the algebraic variable, her sound level 4 pleased and surprised her and moved my “sense” of her mathematical potential to conviction of it. Jamie left after rehearsing her two relational assignments.

During the next class after Session 2, Jamie asked Ann some questions during the problem-working session and she was pleased with herself. At the end of the class, she came round the table, beaming, calendar in hand, to arrange an appointment with me. Jamie was making a move to throw off the hold of the bad internalized teacher presences from the past, overcome her social anxiety, and alleviate her separation anxiety, both in Ann’s class and with me.

Jamie’s Session 3

Jamie wanted to go over her second test and Ann had given it to me so we could. After we had briefly discussed Jamie’s significant achievement in asking Ann questions and making an appointment with me, that is much of what we spent the session on—analyzing her work in relation to her preparation and affect. This was the test for which she had been so anxious because of pressure she felt from her family to get the unlikely 95% she had on the first test. Although her anxiety was elevated, Jamie again did not seem to have been cognitively derailed by it. Her difficulties, she realized lay mainly in insufficient preparation—“like I knew how to do the things but I didn’t know what they were called” and she had only put formulas on her formula sheet, not what they referred to. That she lacked understanding of what it was she was (accurately) computing affected or was affected by her poor preparation for the letter symbol understanding section. We

had both made the same error in the multiple choice section and I shared with Jamie a significant computational error I had struggled with and corrected just in time. I was attempting to challenge the lines Jamie had drawn between “good” and “bad” at mathematics; I, like her, struggle and make errors.

We began talking about Jamie’s now feeling able to ask Ann some questions in class. She attributed some that to Ann: “That’s partly because it was just her,” which led to a discussion of how Jamie’s “staying inside” behavior in mathematics class seemed to have begun with the distress of her 5th grade classroom experience and its effect on her shy personality. I suggested that as a 10-year-old, she had to survive what to her was a frightening situation, so she did what she could—“sit down and shut up so you don’t bother her [the teacher].” But I wondered out loud with her whether, as a young adult, now Jamie might have more choices. Maybe she could now assess the safety of the classroom situation and decide whether she could participate. Jamie agreed that the small classroom and the positive, supportive emotional atmosphere Ann had created made the PSYC/STAT 104 classroom was such a situation and she had chosen to participate.

Jamie had been filling out a *JMK Mathematics Affect Scales* at the end of each session (see Appendix L, Figure L5). At the end of Session 3, despite her having made such strides in personal interaction in relation to the class and getting help, her scores were the lowest yet, three of her seven responses (Items 2, 5, and 6) falling below the midpoint towards the negative end for the first time and another (Item 3) remaining there from before. Overall her responses seemed to indicate the presence of mild mathematics depression that had not been evident earlier.

Supervision

On July 20, I met with Dr. P. for supervision. By then Jamie had taken Exam #3 and earned an 84%, improving a whole letter grade over her Exam #2 grade (see Appendix L, Table L2). She and Karen were the only student participants whose grades did not drop by at least a half letter grade. I expressed my struggle as an out-going extrovert to be quiet and listen to Jamie, a shy introvert who preferred not to speak. Dr. P. encouraged me to invite Jamie to reflect on the changes she had made. “Commend her, give her a bouquet. Have her write a new metaphor.” And he encouraged me to continue to my struggle to listen more and talk less, allowing, more, encouraging Jamie to express her voice (Dr. P., Supervision).

Jamie’s Session 4

Jamie again approached me for an appointment for a fourth session. And she did compose a new metaphor:

JK: So your metaphor was a storm; what would it be now?

Jamie: I kind of see it like it would be different for this class ...not necessarily math in general ...maybe partially sunny ... maybe bring an umbrella in case it turns to rain but it’s okay to go outside, maybe, more, you know, because it’s sunny I can go out in it, but I would still take my umbrella.
(Session 4)

This metaphor shows significant changes from Jamie’s prior sense of endangerment in the mathematics class. Jamie’s behavioral changes in the classroom situation—her little smiles, making eye contact, asking Ann questions in problem-working sessions, and her continuing to make appointments with me—were all outward indications of the changes she experienced. I observed myself doing better at waiting and listening for Jamie and she was now receiving Ann’s offer of respect and safety. She felt safe to “go outside.” She seemed to be resolving her conflict between fear of being

noticed and humiliated, and wanting to succeed in the course. She was finding that by going “outside” she had reconnected with mathematics teachers who were available and had not caused her damage, and had helped her reconnect with her mathematics self that she found to be good-enough for success in the course.

Jamie’s additional 6 points on the extra credit in-class power assignment brought her grade on Exam #3 to a 90%, an A⁻. She reported that her father was very pleased. She did not show the increased anxiety she had in response to her high score on the first test, however. Jamie herself seemed encouraged and quietly determined, I surmise, because she was feeling more firmly attached to and was drawing on her own good-enough mathematics self. Following Session 3, she had been more active and strategic in her preparation and found that *she* could change the outcome, so her father’s expectations were no longer felt as external pressure to pull off another flukish feat, but rather were now more in line with her own realistic expectations, given what she now knew of her sound mathematical base and the importance and possibilities of strategic preparation.

We worked together on questions Jamie might encounter on the Exam #4 scheduled for that evening. When Jamie filled out the *JMK Mathematic Affect Scales* there was no longer any of the mild mathematics depression that she seemed to be experiencing at our last meeting (see Appendix L, Figure L5).

Study Groups and the Final Exam (Exam #5)

I did not meet again with Jamie one-to-one until after the final exam. She was at the study group just before Exam #4 with Ann and me, but she kept to herself; Jamie earned a 76%. Her symbol identification was perfect as for Exam #3 but her score on the multiple-choice had not improved from the previous exam and this time she also lost 12

points on the computational section. Only one third of these errors were from lack of preparation or conceptual understanding of an analysis. So this score though similar to her score on Exam #2 meant something quite different about Jamie's grasp of the material and the process.

Jamie came to the study group immediately before Exam #5. I asked individuals in turn to name the appropriate statistical test for scenarios I compiled from the text, and then we discussed the responses. Jamie responded incorrectly about a scenario requiring a two-way ANOVA when it was her turn but through discussion she understood the solution. Each student was involved in the others' questions. Jamie went on to earn a 100% on Exam #5. I felt less anxious about causing Jamie trauma by asking her questions at that final study group than I had at the first study group, and again the outcome was good. It gave her the opportunity to express and evaluate her thinking whereas merely thinking about it might have left misconceptions unchallenged.

She had earned a 100% on her MINITAB presentation with Robin where she was poised and showed no signs of embarrassment. Thus, with her 100% on the final exam Jamie was getting a B⁺. With her father's encouragement she decided to take the comprehensive final to replace her lowest test score of 74%, hoping to bring her final grade up to an A⁻. She asked me if we could meet once again to review all her exams as preparation.

Jamie's Session 5

Session 5 was a marathon at a coffee shop on the Sunday evening before the comprehensive final exam. We reviewed each of her exams. Because the grade earned on each test did not necessarily reflect the level or quality of her mathematical thinking, I

decided it was important for Jamie's growing mathematics prowess to identify where she had thought well and to reduce the role of the grade as sole measure of her ability. It was also important, however, for Jamie to realize that issues other than mathematical understanding, such as clear communication and correct solutions, can be so important in real life application, that instructors use severe point penalties to emphasize this on an exam. Twice on Exam #4 Jamie had made the logical decision based on her (incorrect) calculations of statistics, but had 4 points deducted on each because these were incorrect decisions for the problem. Ann had also deducted points for Jamie's technically accurate but poorly communicated definitions of symbols. Jamie was able to see that her grade on Exam #4 undervalued her actual mathematical thinking and ability; I nevertheless emphasized that the grade *did* enforce the importance of her improving accuracy and clarity for her chosen field of psychology.

We also discussed changes in Jamie's responses to the *Mathematics Feelings* and *Mathematics Beliefs* surveys. She had made substantial changes on each of the anxiety scales. By the end of the course her score on testing anxiety had gone down from high to moderate (a 17% decrease,^{xxvii} see Jamie's post-scores on Figure L2, Appendix L). My anxiety about Jamie and her mathematical learning and my inclination to control and mother had also decreased as she took more control and internalized realistic expectations of herself. Jamie's beliefs on the *Learned Helpless* versus *Mastery Oriented* scale had changed very little but her social learned helplessness, at least in this setting, had abated considerably. Jamie earned a 71% on the comprehensive final exam (80% on the computational part), and could not replace any of her earlier test grades, so she ended the class with a B⁺.

Jamie's and My Final Evaluations

On Jamie's *One-On-One Mathematics Counseling Evaluation* she described her initial motivation for signing up to meet with me was "so that I could get a better grade in the course." Since I only learned after the course from Jamie that she had been repeating PSYC/STAT 104, I speculate that initially she had not disclosed this because of her "fear that I'll be wrong" and thus conspicuous and censured by a mathematics teacher whom she did not know (me) and who was not to be trusted to do anything but humiliate and abandon her, as past teachers had done. Her end-of-course written comment about "a better grade" may have been an indication of her now feeling safe to let me know, perhaps also affirming her trust that I could know that she had not done well in the course before without rejecting her.

Asked whether her motivation had changed during the course, Jamie indicated that she recognized the focus was primarily relational, "Kind of, I realized it was more about my feelings and confidence in my math ability, th[a]n any real problems with the math course work" (*One-on-One Counseling Evaluation*, archived).

Jamie had learned how to ask for help in this course but it seemed this experience was not enough for her to do it in a new class. Unless the class was structured like her English composition class with *required* meetings with peers and instructor, or had a resident class-link tutor who initiated the contact, I concluded Jamie would probably continue to be an involuntary loner. Although it had for this class, for other mathematics classes, her conflict between wanting to do well (and knowing that likely means getting help from and working with strangers) and fear of being conspicuous had not been resolved and the latter would probably predominate. She *had* however, become aware

through counseling that *she* could judge the safety of the situation and the instructors and helpers and not feel compelled to hide no matter what.

Jamie had done well in the PSYC/STAT 104 by reattaching to her sound-enough mathematics self and to safe mathematics teachers/classroom. I had learned to wait, listen, and affirm her strengths as well as challenge her to confront her fears. I (and Ann, once I had helped Jamie see) had provided her with good-enough objects to replace the bad 5th grade teacher internalized object (presence). I had become a secure enough base—a smooth level path, with a gradual incline—from which she could experience this class, not as a storm any longer but now as a “partially sunny [day]” where she could go “outside.” It is not possible to say what grade Jamie would have earned in PSYC/STAT 104 without counseling, but she almost certainly would have remained hidden, the instructor would have tiptoed around her, she would have remained isolated from her peers except for the required contact over the computer module presentation, and her questions and comments would have remained unspoken. Most importantly Jamie’s sense of her mathematics self would likely not have changed. If I had not examined her transference and my countertransference reactions I might not have pursued counseling with Jamie at all. If I had only gone by Jamie’s responses on my traditional anxiety and belief surveys and not delved with her for the underlying meanings they signaled, if I had not explored her metaphors and tracked her progress with the *JMK Scales*, in other words if I had taken a traditional approach instead of brief relational mathematics counseling with Jamie, she is likely to have remained an under-confident involuntary loner achieving variable results over which she felt little control—always afraid of the storms of incomprehension, anxiety, and unwanted attention.

Epilogue

Jamie decided not to try Finite Math again to fulfill her quantitative reasoning general education requirement, but to take a logic course instead. She has not let me know how it went. Jamie now knows she could assess the relational safety of the instructor and the class to see if she might go “outside,” ask questions, and ask for help, and she has a budding understanding that she was in fact quite capable of doing mathematics. If Jamie perceives a new situation as benign enough so that she does not regress and go back into hiding with “you are dangerous to me; don’t come near me” transference, an attentive instructor or class-link tutor might feel less reluctant to approach her to offer help and she is more likely to accept such offers. If the mathematics counselor or tutor waits for Jamie to come to the Learning Assistance Center or make contact with the instructor or even the class link tutor, it is likely they will wait in vain and Jamie will not receive the help she needs.

MULDER’S COURSE OF COUNSELING

Mulder^{xxviii} exuded an outgoing social energy. During the first class lecture discussion on the scientific method, he was actively involved^{xxix}, telling classmates of his research project on centipedes’ attacking postures. During problem-working sessions he always worked with any neighbor willing to engage. Ann thought he seemed “smart” and “on the ball [with respect to] his research experience into caterpillar aggression”—but not likely to put in the effort needed to succeed in the class and not very committed (Interview 2, Class 1).

When he found out that I was available as a tutor, Mulder was enthusiastic. He didn’t think he would need much help with statistics (He had used some statistics for his

biology research projects.), but he thought he might for his finite math class—that seemed more challenging to him. So Mulder signed up for mathematics counseling once a week. He struck me as a charming scallywag. In fact, I called him that once. He seemed busy, mischievous, stubborn, and somewhat of an opportunist but he was confident that he could handle PSYC/STAT 104 fine, so I believed we would focus mostly on finite math.

However, Mulder and I soon found reason to suspect that his confidence was perhaps overconfidence. He earned a ‘D’ on his first exam and thus began a quest unlike any either of us had been on before. I found I could deal with anxious and underconfident students like Jamie, using relational approaches to get at the roots of her anxieties; I could even overcome the despair that depressed and underconfident students like Karen threw me into because both of these students knew that they needed help. But how was I to use relational approaches to recognize that a student with a social, confident, and up-beat demeanor might actually be *overconfident* and that he might then be drawing me into believing he was less needy than he really was? Then once I recognized this, how could I help when he seemed to have all the answers? And for Mulder this seemed to be new too. It turned out that he had never really tested his “I know I can do math” theory by actually *trying* to do it well and he had not worked with someone who was trying to support him in that endeavor.

Sometimes Mulder developed what appeared to be indirect and to me illogical schemes to improve his achievement; at other times he stubbornly resisted the mathematics he found did not yield to these devices. He *did* improve his computational grade and then his grade on symbol identification improved, but on exam after exam he

failed to improve his conceptual section score. The narrative that follows chronicles how we struggled and how relational counseling insights and approaches I used not only helped Mulder resolve the conflicts that hindered his success but also helped me grow as a mathematics counselor.

Mulder was a 20 year-old white man who was a biology major at a small university in the Midwest. He was home for the summer, taking Finite Mathematics—MATH 120—in addition to PSYC/STAT104 at Brookwood State. The finite mathematics course was required for his major but not statistics. He had the option of transferring his statistics credits for elective credit if he did well enough.

Mulder was an only child. He was short, muscular, and fit—participating in both soccer and track (Class 1). The last mathematics course he took was Algebra II as a junior in high school; he reported that he earned Cs in mathematics classes then. He indicated that he hoped for a B and expected a B in the PSYC/STAT course (*Pre-Test Mathematics History Survey*), both overestimates perhaps, given his history.

Mulder's Metaphor: Fox Mulder Searching for Aliens

Mulder asked if he could think about choosing a metaphor “because I really—I don’t know that I could say for a while” so I suggested we come back to it later. When we did come back to it I asked him if he would rather do a drawing of himself doing mathematics, but a metaphor came to him, “For me math is like Mulder searching for aliens. I am searching why I make math so difficult for myself.” He referred to Fox Mulder from *The X-Files*,^{xxx} a popular science fiction television program. Mulder explained further, “I have confidence in everything else I do. It’s not that I don’t have confidence [in my ability to do mathematics], but it’s just like—I know what I’m doing,

but I can't explain it to other people." I wondered how this related to his "mak[ing] math so difficult" for himself. And what if anything did the metaphor tell us about how Mulder saw mathematics?

Student-Mathematics Relationships: Mathematics as a Search for Aliens

When I asked Mulder about how mathematics had been for him, he responded "It's never been my favorite." Later, "It's, like, it's the only thing that ever gives me any problems." It was from freshman year of high school that mathematics seemed to have become an issue for him. His theory was that it was his lack of effort rather than low ability that accounted for his difficulties, yet he had not tested his theory by putting in that effort even after he "realized" that was his problem. He avoided mathematics altogether his senior year because he knew he would not do well in it: He wanted to "save [his] grade point average".

I formed this attitude in high school, you know, high school, if I had really, really tried in high school, I could have done really, really well [in mathematics]. It wasn't 'till the end of my freshman year I realized and I still don't think I try as much as I should, you know ... It's just a matter of applying it and taking advantage of it ... study skills in high school weren't that great [I did] three sports a year. I did real well my senior year because I took no math ...I was enrolled in trigonometry and precalc but I dropped it because I wanted to save my grade point average. (Session 3)

Mulder had done better in Geometry than in Algebra II even though he had "thoroughly slept through it" and he put that down to the difference between the teachers rather than differential ability or a preference for that type of mathematics.

I got 'C's in all these [mathematics] classes. This one [pointing to Geometry on the list] I thoroughly slept through; I'm not lying...ironically enough I did better in this class [Geometry] than in this one [Algebra II]. It was the teachers. (Session 3)

He could attribute his low performance to not trying or to sleeping, so his belief in his potential ability to do it was preserved especially since “my uncle is getting his masters, great student and my dad’s really smart so it’s kind of like a thing I know I have” (Session 3).

I found my initial reactions to Mulder and his prospects in the class were quite different from my reactions to Karen, even though her grade on the first test was almost the same (62% compared with Mulder’s 63%). His metaphor was active, if somewhat self-defeating, and he seemed willing to engage. Mulder was positive about his mathematical potential. I was drawn into his confidence and considered then neither that his knowledge base might be weak nor his underlying mathematics self-esteem low. Because he had earned Cs through Algebra II with lots of sleeping, and not really trying, the result may have been a relatively underdeveloped mathematics self. I eventually found considerable evidence to support this conjecture.

After the course it was confirmed that Mulder did have a minimal algebra background for college though this was not obvious to me during the course. When he took the *Algebra Test* after the conclusion of the course, he tested at a low level 2, indicating that he, like Karen, had not yet developed an understanding of letter symbols at least as specific unknowns or generalized numbers (and in some cases as true variables) nor could he coordinate operations using them (see Table 6.2). That perhaps explains why his formula sheet for the first test had been so inadequate for his needs: he, like Karen, needed detailed formula sheets for exams that interpreted formulae into columns and step by step procedures. Unlike Karen though, he seemed relatively arithmetically

sound, with accuracy and confidence in his number and operation sense (see Session 1 and Table 6.1).^{xxxi}

Initially I was taken in by Mulder's sound-enough arithmetic and confident take-charge approach. I did not become conscious of his real deficits with respect to the algebraic variable and related concepts until later in the course. I now see that Mulder's low understanding of the algebraic variable, his more procedural than conceptual beliefs about mathematics (2.9 on a scale of 1 through 5 on *Beliefs Survey*, see Appendix M, Figure M1), and his poor high school preparation seemed to have combined to make the statistics almost alien to him, especially the conceptual aspects that required him to understand and communicate in earthly rather than alien terms. Unconsciously at least, these factors were almost certainly calling into question for him his own ability. Maybe it was not just about effort. Maybe he really could not do it.

Student-Teacher-Self Relationships:

Mulder complained about Ann's lecturing style (She "jumps around a lot.") Later in that first session he said of Ann's lecturing: "It's not that she goes through it so fast; it's just I have a hard time following her" (Session 1). His references to past teachers were in a similar vein. He attributed doing better in Geometry than Algebra to his teacher's different approaches.

Mulder: It was the teachers.

JK: You seem to react to teachers

Mulder: Yeah, I do

JK: And it seems to affect how you do in class?

Mulder: Yeah. (Session 3)

Present struggles with teachers seemed to be closely linked to Mulder's struggles to understand the course content and gave me clues to the nature of his past struggles. He

strongly preferred his finite teacher's direct approach, "[S]he tells us how to do it and she tells us why and how to use it" (Session 3). This teacher made explicit the links among concepts, procedures, and applications and she demonstrated the procedures. By contrast Ann had students work out how to do the problems for themselves during problem-working sessions, *after* she presented concepts involved. It seemed hard for Mulder to see how the concepts discussed in the lecture related to the problems worked later, even *that* they were related. And the struggle seemed to be exacerbated by Mulder's auditory processing difficulties and his compensatory visual memorization strategies.

Learning/processing style and Mulder-teacher relationships. A pattern of Mulder's difficulties in understanding and expressing new knowledge through his auditory and verbal channels emerged. This was evidenced in his relative difficulty with finding a metaphor, following Ann's lectures, and in his linking his "Mulder" metaphor to difficulties he made for himself in mathematics, especially in explaining what he understood. Initially my realization of this difficulty was masked by Mulder's outgoing social learning style and I speculated that other mathematics teachers may have been similarly misled. I began to wonder whether Mulder was making it hard for himself or if it was a processing difficulty that he blamed on himself. Perhaps it was a combination of factors. Perhaps what he labeled as his laziness was, in part, avoidance of these primarily verbal study tasks he found difficult.

The seemingly illogical visual memorization strategies I observed him using perhaps served to compensate for his auditory struggles, and his perception was that he did better on assessments that required visual recognition of material. For example, as Mulder tried to understand why he did not do as well as he expected on certain exams he

cited testing anxiety as a factor for all exams except “practicals,” (those requiring identification of visually discriminated materials: “Like, a bone practical [where he had to identify and describe the function of bones of humans or other animals]” He always got As on those [Session 3].) Did Mulder’s have a global, visual-pictorial, mathematics learning style II (Davidson, 1983; Krutetskii, 1976)? Or was his approach the result of continued use of strategies he had developed to compensate for auditory/verbal processing difficulties? Or some combination?^{xxxii} It seemed that he was not easily classifiable but I began to wonder if his atypical approaches to mathematics learning might not only have negatively affected his level of mathematics understanding, but also how he was perceived by his teachers.

I needed to explore with him what effect these approaches had on his mathematics self development. I needed to know how teachers had reacted to him and what effect that had on him. I myself reacted with amazement and sometimes horror to what seemed to me to be a lack of observable logic in some of his tactics (see Sessions 4 and 5). Mulder’s approach seemed consistent with his metaphor at least; he did indeed seem to be using alien methods to search for his aliens.

But these methods looked enough like attempts to avoid hard work that I speculated that his mathematics teachers had not only perceived him as capable (because of his confident upbeat demeanor) but lazy, but also labeled him thus. Indeed this *was* how Ann saw him: smart but not likely to put in the effort. Mulder’s constant concern that he might be perceived to be lazy (“I hate doing this ... It’s just—it makes me feel lazy” when admitting to putting work off ‘a lot’ when he filled in the *JMK Affect Scales*, Session 1) and his repeated description of himself as lazy about doing mathematics

supported this conjecture that he was used to being labeled thus and had taken it on himself. Perhaps the “capable but lazy” label had become a shield for his possibly incapable self and, if so, it may also have functioned as a trap, hindering him from doing what he needed to do to develop his capabilities and deterring teachers, whose help he needed, from helping him.

Emotional Conditions: Anxiety, Learned Helplessness, Depression, or Grandiosity?

Anxiety. Mulder certainly didn’t strike me as anxious. But after his poor showing on Exam #1, he brought up testing anxiety as one of the factors he believed was operating against him, especially on tests like mathematics tests that were not visual memory oriented “practicals.” I had not highlighted any of his average anxiety scores on the *Feelings Survey* for discussion with him (see Appendix M, Figure M1) because each fell in the middle of the class range and was not extreme compared with the class. However, his testing anxiety averaged at a little above moderate (3.1 on the 1 through 5 scale) and could be considered high for a physical science-oriented student and even for a social science student if compared with means Suinn (1972) reported on the *Mathematics Anxiety Rating Scale* (MARS).^{xxxiii}

Was Mulder’s anxiety a normal reaction to a challenge he was not adequately prepared for or something more than that? It seemed feasible that it was linked with his history of not having done well on mathematics exams and an underlying belief that he may not be able to do it. That combined with lack of strategic preparation for this exam to compensate for his mathematics deficits (e.g., well constructed formula sheet, strategic practice of target problems) would give good reason for considerable but normal anxiety.

Depression. And Mulder gave me the impression of being anything but negative or depressed. This observation seemed to be confirmed when he completed the *JMK Affect Scales* during his first session. Apart from his extreme negative response (a lot) to putting work off” all other responses were at or above the mid point (see Appendix M, Figure M2, responses marked 1). His average positivity on the scales was approximately 55% or 64% if the “putting off work” item were removed (see Appendix M, Table M3). Given Mulder’s poor showing on Exam #1, rather than indicating mathematics depression, his responses perhaps showed the opposite, mathematics optimism.^{xxxiv} .

Learned helplessness. Although depression was not an issue for Mulder, learned helplessness did seem to be. When I showed him his low learned helpless score average, Mulder responded, “I think it’s math. Any other thing I’d be up here [pointing to the *Mastery Oriented* end of the scale]” (see Appendix M, Figure M1). Perhaps this was a chink in his up beat armor that I initially did not explore. Although Mulder saw mathematics as somewhat more procedural than conceptual and his approaches seemed procedural, his achievement motivation became more for learning than for performance over the summer (Items 4, 7, 9, and 10, Part I, *Beliefs Survey* and Appendix H, Table H3). Perhaps like Karen, he wanted to understand the material but used procedural approaches both by habit and also because he did not feel capable of achieving that understanding.

Grandiosity. Mulder’s emotional response to his mathematics challenges did not seem to be marked by anxiety or depression that could be considered unrealistic. The evidence seemed to be pointing to grandiosity. It seemed that Mulder might have developed an overconfident demeanor combined with relative lack of effort and indirect approaches in order to protect an underdeveloped mathematics self that was

compromised by his learning style challenges: “I know I can do really, really well ... [but] I don’t really try/I thoroughly sleep [through class].”

Identifying Mulder’s Central Relational Conflict

Mulder now faced a dilemma. He wanted a B in the two summer mathematics classes he was taking to make it worth transferring the grades. His high school tactics would not work but if he actually tried he risked being found out. On the other hand he did not want to be considered lazy. When I asked him about how much work he did for a finite exam, at first he denied doing any work. This seemed to express his grandiose stance (I can do well; I don’t need to work at it.). But then he conceded that he *had* practiced but only *some* of the problems (perhaps his “I don’t want you to think I’m lazy” stance)

Mulder: And I don’t—I’ve never really sat down and done practice problems before a test.

...

JK: And you didn’t do practice problems?

Mulder: No.

JK: Was it stuff you were already familiar with?

Mulder: No.

JK: No? But you got it from the class?

Mulder: Yeah. I knew how to do it. I did—I did some of the homework; I don’t do all of the homework but I do some of it.

JK: Just pick a few things?

Mulder: That I need—that I need to work on. (Session 1)

Confounding his difficulties were the very real challenges that his auditory processing difficulties, his compensatory visual strategies, and his poor mathematics preparation posed, especially as Mulder did not seem to be aware of them or their potential for sabotaging his success.

*Mulder and Me Dealing Jointly with Aliens:
Relational Counseling for Mulder*

The Focus of Relational Counseling

I realized that relational counseling should focus on helping Mulder and me become aware of the conflict between his competing goals—to do well in the course but also to protect his underdeveloped self. Perhaps the very defenses he was using to protect his mathematics self were what were “mak[ing] mathematics so difficult for [him]self.” I would have to recognize that Mulder’s grandiosity might be masking an underdeveloped and vulnerable mathematics self. He was so convincing and I found myself believing his grandiose view and not attending to his real challenges. I was likely falling into a pattern of former teachers—believing him, being disappointed, getting frustrated, scolding and pushing him, and even giving up—and not offering him what he really needed. Not only would Mulder need to become aware and change, but I would also have to change *my* approach in order for him to feel safe enough to drop his counterproductive defenses. And he might need me to change before he could. I realized that we were unlikely to resolve his conflict unless I could work out how I should change.

The Focus of Mathematics Counseling

Because my preferred learning style is strongly auditory, I had to be aware of the risks of devaluing Mulder’s mathematical learning approach, simply because it was different from mine. Instead I needed to accept and try to understand how his visual-memorization, his procedural mastery, and his social style both facilitated and impeded his mathematics learning. How could I help him use his strengths and preferences to help rather than stand in the way of his grasp of the mathematics? It became clear to me that

the strategic mathematics tutoring focus should be on Mulder's finding a way of seeing this alien mathematics in a more accessible, logical, earthly form.^{xxxv}

Mulder's Course of Counseling Session by Session
(see Appendix M, Table M1 for Mulder's schedule)

Mulder's Session 1

Mulder was doing well in his finite math course, but he had done poorly on the first PSYC/STAT exam (63%). He had failed the multiple-choice conceptual section, with the lowest score in the class (see chapter 5, Figure 5.2). At times he had failed to follow directions^{xxxvi} and at others he did not know the information adequately so he guessed rather than trying to work them out from the context. His computational score was less extreme but still only 65% correct. He had lost only one point on the symbol identification part, but he had only *named* the symbols and not defined them as was required. Ann said she had been lenient in her grading on this section because it was the first exam.

When we examined how Mulder had prepared for the first test where he had done so poorly he identified the fact that his formula sheet was not adequate and he had no direct information (e.g., a quiz) to guide him to work out how Ann tested so he had not prepared sufficiently or strategically enough. These factors seemed to give good reason for the testing anxiety he said he had suffered.

Although he had not done well on the computational part of the exam,^{xxxvii} Mulder seemed to have solid number sense and no problems with decimals so I was not alarmed as I was for Karen about his prospects. The questions involving number or decimal sense

(e.g., deciding on real limits for the weight of 0.35 grams of cheese) were answered correctly.

As we discussed how to prepare for Exam #2, Mulder seemed to be astutely assessing the mathematical tasks required for him in the computational section. “They’re not really word problems, you know. The information’s there and the equation’s there and she shows us how to set everything up, and I understand all that.” Although perhaps globally positive rather than realistic, he did seem to have pinpointed a crucial problem with his first exam, that is, he had not set up his formula sheet adequately. If he had, “It’s easy to write down the equation, say what the ground rules are, and then plug the numbers in.” He saw the mathematical tasks as procedural and felt that he could manage that. Mulder seemed confident that he could remedy the situation in Exam #2 by improving his formula sheet and studying the procedures now he knew how Ann tested.

Mulder seemed to consider the computational and conceptual sections of the exams as separate, requiring different types of preparation and despite his low score on the conceptual he commented, “I did all right on the conceptual part” and for the next test it was the computational part he was going to focus on. Later in the session Mulder did concede though, “Obviously, I need to spend more time on the conceptual.”

Although Mulder’s decision on how to improve his computational preparation did not include understanding and linking the concepts, I did not pursue it, thinking maybe his plan would work. I was concerned about his conceptual understanding of symbols (on Exam #2, Ann would require that) but when I made a suggestion, he was defensive and claimed he had already done what I suggested. I also suggested ways of tackling his multiple-choice challenges but felt some resistance to my recommendations.

When we turned to working on the statistical procedures to be tested on Exam #2 using both notes and diagrams, Mulder had some questions about the statistics but seemed to have control of straight line equations needed for regression.

It was in this first session that I became aware that Mulder might have verbal and auditory processing problems. He talked about his struggles with following Ann's lecturing approach, he had more trouble than any other participant in composing his metaphor, and he had failed the conceptual portion on Exam #1. I decided that at our next session I should try to help Mulder work out ways to compensate for his processing difficulties so he could make the conceptual connections with the procedures that he seemed to think he was capable of mastering. At this point in our relationship I thought that this would simply involve beginning with his procedural competence and working in parallel as I modeled finding conceptual connections.

Mulder's Session 2 and Session 3: The Conflict Emerges

Mulder's Session 2

Although the focus of this session was on exam preparation for a finite mathematics exam in an hour,^{xxcviii} Mulder also tried to make sense of what was happening in PSYC/STAT 104 in terms of how he had previously done in mathematics courses. He had taken Exam #2 the night before and he knew that he had done badly again on the conceptual part of Exam #2. He had lost 19 points out of 50, including 5 of the 6 points for symbol identification—a D; Ann had shown him his score when he handed in his computational section. However, he was fairly certain that he had the computation 100% correct. Mulder had conflicting theories about whether historically he was better at the “math part” or the “conceptual.” Now he was irritated by the fact that he

had done ‘A⁺’ work on what he considered the mathematics but was being denied recognition for that because of this “other stuff.”

I shared my ideas about what might be happening and proposed that he had a theory about what was and what was not mathematics and that, according to that theory, the conceptual part of PSYC/STAT was not mathematics. In addition, I told him that I saw him as a strong-minded person who acted on his theories, and in this case he was rejecting the conceptual aspects and concentrating on what he saw as real mathematics—the computation. Mulder agreed that I was accurate but explained, “I’ve always thought math was the harder part for me so that’s what I’ve been concentrating on in the lecture. Everything seems to be centered around that formula, so I concentrate on that formula, on how to do that formula rather than taking it all in.” With his poor result on the computational part of Exam #1, concentrating on that formula was indeed an important element of his recovery strategy. After all, it had led to success on the computation section of Exam #2. I was concerned about his rejection of “taking it all in” because it seemed tantamount to his deciding to dismiss the conceptual aspects of the course and not make the conceptual link to the computational. Did Mulder think that he was not *able* to do both or that he should not *have* to do both? It seemed that unconsciously he felt he was not able; consciously he insisted he should not have to.

We were at this point still living in the initial transference-countertransference relationship. I had assumed Mulder’s transference of past teachers so I had higher expectations of him that were reasonable and I was becoming frustrated when he would not or could not deliver on his confident plans. My countertransference reaction was to

push him, to accuse him of avoiding tasks (e.g., the conceptual), and to nag him with direct advice, expecting that he could get his act together.

Mulder's Session 3

At this point Mulder was frustrated, "I don't know what to do! ... I don't know what to expect on the multiple-choice. ... We've had two exams now and I can't work out what it'll be." I also felt at a bit of a loss. I recalled from Session 1 that Mulder was ambivalent about doing the homework problems from the text because he was confident about the computational part of the exam, so I asked if he had done the homework this time. I also asked about the first text problems from each chapter set that were conceptual questions like: "What is the range of values that a correlation coefficient may take?" and "From each scatter plot in the accompanying figure (parts a-f, on page 124) determine whether the relationship is ...positive or negative ...perfect or imperfect" (Pagano, p. 123). Mulder replied somewhat indignantly, "Those are the ones I *did* ... and I wrote them down [the answers]." But when we looked at his conceptual Exam #2 errors, though, it seemed that he had not linked the concepts from this homework to their numerical meaning. For example, he had responded with 0.75 as the correlation coefficient that indicated the greatest strength on one question where the correct response was -0.80 because its magnitude is greater. Mulder insisted that he had read his class notes and the book a few times to prepare for the conceptual section. I suggested that only reading was probably too passive to be helpful. Further I suggested that he might even be he was even stopping himself from really learning it in some ways, resisting it because he did not believe he should have to learn it. His sheepish reaction seemed to confirm my supposition.

As we continued to discuss what might be hindering Mulder's success I brought up his low learned helpless score on the *Beliefs* survey. When he said it was only with math that he was like that, I suggested that he could change it even in mathematics: He seemed to be doing well in his finite class, and could do so even in this statistics course. Mulder seemed skeptical until he remembered that he *had* recently experienced not giving up on a math problem. On the computational part of Exam #2 he had initially made an error that led to what seemed to him to be anomalous results. "I sat there a long time [looking at it] and I realized my standard error was wrong. It was way too big." He went back to find and correct the initial error in his calculations and then to fix all the computations affected: a mastery oriented response he now recognized. But could he do that with the conceptual section on the test?

I recommended that he use study guide multiple-choice questions to prepare better for the conceptual section. That we focused on the contents of the previous test rather than on material for the upcoming test was not strategic but tackling these multiple-choice questions highlighted Mulder's misconceptions about material that would continue to be needed^{xxxix} and his ineffective study methods, especially on symbol definitions and their links to the calculations that would be on the next test. Mulder agreed that this new tack of working on multiple-choice questions should help. I gave him copies of sets of multiple-choice practice questions for each chapter to be covered on the next exam for homework. He emphasized as he left, however, that at our next session just before Exam #3 we should review all the symbols because he had done so badly on the symbol section of Exam #2.

Mulder's Session 4: The Central Relational Conflict Becomes Clearer

Session 4 took place on the day of Exam #3. Mulder was tired, grumpy, and oppositional. I asked about what he had done to prepare for the exam. I had prepared practice materials using the problems we had done in class but I had removed any reference to the type of statistical test required to solve them so students could practice also identifying the test. I had also prepared an empty flow chart template for the statistical tests that would be on the exam for students to fill in as their formula sheet if they desired. Mulder was taken aback that he might be required to identify which statistical test was appropriate because he had been certain Ann had said that she would tell us what statistical test to use with each problem and we would not be expected to identify what test was necessary until chapter 19 and Exam #5. I told him that my e-mail exchange with Ann on the subject left the question open. He was not happy, grumbling about curve balls.

Mulder grabbed one of the problem sheets, declaring that by *looking* at a question he could identify the statistical test required. Rather than analyzing the problem statement, he tried to remember by the look of the problem and the order it had been presented in class, but he remembered incorrectly. Another strategy was to identify the type by whether the data were presented in columns or as an already computed statistic. Again, he was incorrect. He made little attempt to read the questions and understand the situation or experimental design. I remonstrated and insisted that as a “bright man” he could and should *think* about the questions.

Mulder ducked my comments and moved to a discussion of symbols. I set up a divided page to sort the population symbols from sample symbols and we began to discuss how to make decisions on tests. The single sample tests went smoothly but when

we returned to the two sample tests Mulder again tried to use his memory of the class when students worked them. He seemed to enjoy my frustration with his approach. I realized Mulder had not understood a central concept—that the words “independent” and “correlated” described the groups or samples not the data numbers. However, he insisted that to do a problem correctly on the exam he did not need to understand such distinctions, saying, “I would have figured it out because I would have looked at my equations and I would have figured out what went where.” He then predicted that Ann would give the alternative type (SS_1 and SS_2) on the exam because one type of already computed statistic (s_1^2 and s_2^2) had already been given on a class problem. This speculation seemed illogical and risky to me. I was quite alarmed by how Mulder was orienting himself to the exam and he seemed to be enjoying my alarm.

Next we engaged in a much-needed discussion of symbols; I drew a reluctant Mulder into making links with symbols he already knew. We discussed how to identify sample mean symbols^{xl} and looked at what might be a logical value for the population mean of difference scores, μ_D ; because of the null concept of *no* difference in the problems the class was mastering, μ_D should be zero in the null hypothesis statement and therefore in the formula. Mulder ultimately got that wrong on the test (see discussion in Session 5). I coached Mulder as he applied this logical classification and linking process to the definitions he had prepared, modifying them in ways that made more sense to him or that were necessary to be accurate. I encouraged him to link this process with the use of the statistics they represented in the computation but he seemed bent on keeping the sections separate in his mind.

We briefly discussed his plans to complete his test preparation during the day, and he left to do his chores and go to work.

In this session, I pushed Mulder to make logical decisions and connections and he quite vigorously resisted, using visual memory and pattern finding of generally extraneous details as benchmarks for decision making rather than exploring the logic of the material.

The only thing Mulder had done differently to prepare for the multiple-choice section had been what we did in Session 3; he had not used the practice multiple-choice questions I had given him to do at home. He had written out definitions for the symbols but his resistance to changing his approach or to doing more than memorize patterns and procedures remained entrenched. The resistance may even have grown and Mulder seemed to use considerable amounts of energy for this resistance. The more I reacted the more he resisted. I urgently needed to understand this resistance and help Mulder find a way to put his energy and intelligence into preparing for and taking his exams.

I was getting frustrated! I experienced sessions with Mulder as enjoyable. Even when he was tired and grumpy, I found him quick and funny. I tried laughing at his outrageous strategies, appealing to his intelligence, scolding, and cajoling him but seemingly to no avail. I was acting out of my countertransferential role of mother of a rebellious teenaged boy. I tried but could not manage to get him engaged in anything other than sparring with me. He was certainly not interested in addressing his issues seriously while I was trying so hard to change him.

Supervision

By the time I presented this case for supervision the results of Exam #3 were known. Mulder had earned a 76% but with the in-class open-book extra credit assignment he earned 5 of a possible 6 points his final grade for the Exam was 81%. He had significantly improved his symbol identification score but had lost 18 points on the multiple-choice section, 59% correct (on Exam #2 he had lost 14 points). In addition Mulder had lost 6 points on the computational section, most because he had failed to set his $\mu_D = 0$, despite our discussion of this in Session 4 the morning of the exam.

I presented Mulder to Dr. P. and realized that what I wanted to do was to talk with Mulder about how he stands in his own way and to propose to him that he had the choice, that he might be able to make the choice to stop doing it. Dr. P. suggested instead a strategy established by Alfred Adler (Mosak, 1995) and called paradoxical intention by Victor Frankl (1963) that might help Mulder make that choice. The theory suggests that, “The symptomatic patient unwittingly reinforces symptoms by fighting them...to halt this fight, the patient is instructed to intend and even increase that which he or she is fighting against” (Mosak, 1995, p.74). Dr. P suggested that in the next session I have Mulder experiment with truly resisting on an exam. His directions were clear:

Ask him how he resists; suggest that as an expert in resisting that he let me know what strategies he uses to do that, so that he paradoxically really exaggerates this thing; it's his life but as long as he is into resisting he might as well do it really well. (Dr. P., July 20, 2000)

In presenting other cases for supervision (cf. Brad and Autumn) I had revealed my tendency to tell rather than ask participants how they might make helpful changes using the insights we uncovered. Dr. P. gently but firmly helped me recognize how counterproductive that was. I needed to see how by my telling Mulder I was likely

exacerbating his resistance—he was not only resisting a conceptual understanding of the mathematics, he was resisting me as a “teacher” and perhaps even a “mother.”

Dr. P. also suggested that I have Mulder explore the implications of the metaphor that he was standing in his own way. “What’s he doing and then what’s the part of him that’s standing in his way doing? What gesture? What sound? What stance? Is he tripping himself over? Is he holding himself back?” (Dr. P., July 20, 2000)

Finally he suggested I compliment Mulder on his insight into how he gets in his own way, “Insight saves you a lot of trouble, not having to say, ‘I don’t know what’s the problem here’” (Dr. P., July 20, 2000). I puzzled over this. What exactly did Dr. P. mean? In some ways I felt that Mulder might be expending too much energy struggling for insight (or was it an excuse?), so I had to concede achieving insight should certainly free that energy for actually doing the coursework. And the importance of congratulating him, mirroring his achievements in insight and cognition was becoming clearer to me.

Session 5: Honing Resistance Strategies

I had videotaped the class on the evening before Mulder’s and my Session 5. Mulder sat beside and around the corner from me and was very interested in my research activities. He noticed that I was observing students and taking notes when no one else was writing. He “acted up” during the problem-working session in class to the extent that, at one point, I called him a “scallywag.” He seemed pleased.

At the start of tutoring I told Mulder I had found out more about Fox Mulder and he gave an appreciative laugh. I said that Fox Mulder seemed to me the kind of antihero who does things opposite to how others think they should be done. I asked Mulder whether that was the characteristic that appealed to him and he agreed that Fox Mulder

was like that, doing everything in a way that was “definitely indirect” but he denied that was the element he identified with. He insisted what appealed to him was Fox Mulder’s constant effort in looking for the truth. It seemed to me that he was denying a reality that was largely unconscious (opposition) while consciously espousing a desire that was not yet a reality (finding truth).

I suggested Mulder’s own search for “why I make mathematics so difficult for myself” might entail exploring his underlying and seemingly growing resistance to mastering the conceptual part of the course (that Ann tested using multiple-choice questions) that seemed to be making it harder for him to succeed. Paradoxical intention theory suggests that getting Mulder to consciously and vigorously resist as he answered multiple-choice conceptual questions (i.e., enacting the very behavior he needed to stop) should result in his overcoming his resistance.

I proposed to Mulder that he take a mini-test of conceptual multiple choice questions from the textbook’s study guide^{xi} that I would give him, but that he should strongly resist while talking aloud about his resistance. He seemed intrigued but initially challenged my instruction to really resist with, “What, not do it? I can do that!” Mulder began working silently so I asked how he was doing. Rather defiantly he replied, “You tell me!” but in a few more minutes he said, “I don’t like this one” and to my query, “Because I don’t know if it’s really, really easy or if I’ll have to do some work to find it.”

I began talking Mulder through the problem by tapping into his existing knowledge of the process of finding critical values using tables in the appendices in the back of the book. As we did this I probed a little, “Is that part of your resistance because you think about ‘is that easy or is that hard?’” Mulder conceded, “Probably.”

He decided the answer to his original query (is it easy or will it require work?) was, “Too much work for me!” so we talked about how much work he was doing comparatively for the finite mathematics class. When I asked if it was about the same as for the statistics class he demurred, “I don’t know; next to ‘bout none.” I pursued this further since he had clearly been doing better in the finite class from the beginning

Mulder insisted it wasn’t because he was doing more work or because it was coming more easily to him but rather that, “I just don’t have to do conceptual questions!” I wondered, “Maybe then, [for] your resistance you say, ‘This is conceptual. I don’t have to do that.’ Maybe if you could say, ‘Ah this is *not* conceptual.’ Rename it: this is just mathematical...” Mulder blurted out “Pain in the butt!” I was prescribing and telling so his resistant reaction to me should not have surprised me yet I was startled and asked if he was calling *me* a pain in the butt but no, he insisted, it was “that section of the test.” I responded, “Well you are doing a nice job of resisting, which is good!” I could not consciously acknowledge to myself that it was almost certainly me he was calling a “pain in the butt” and resisting.

Mulder went on with his multiple-choice mini-test. He grumbled as he went, at one point exclaiming “Crap!” when he picked the wrong value for N . I guided him to interpret the table using the given α value of .01 and when he did it correctly I

commended him. He interpreted that as condescension on my part declaring indignantly, “I can *read*!” I came back with, “You can also think!”

I gave him the package of class problems with the test name whited out, intending him to use those after he was done with the multiple-choice, but he immediately grabbed them and began naming the test using the same type of pattern memory “logic” that he had exhibited in Session 4. As he expected, I scolded him reminding him of all the multiple-choice questions he was now avoiding and commenting again on how well he was resisting.

At that point, I interpreted what I saw happening between us from my point of view, “I’m thinking this guy is so smart he could do so well and the mother in me comes out and it’s like ‘If I could only persuade him’” I spoke from my side of our power struggle. Mulder continued with the next question, appearing to ignore me. In a few minutes as he began to work on new questions, I commented,

Every now and then you stand up against your own resistance by speaking [the resistance, but then you stop resisting and do the work]...First of all you say I don’t want to do this, I don’t want to come back here, she’s a pain in the neck ...but then [you see] what happens when you try them [the questions]. (Session 5)

After this interchange I felt a distinct change in Mulder’s demeanor. It felt like a turning point. He began to ask “why” questions. For example, “I have a question: Why do we find the total of the degrees of freedom?” Mulder became actively engaged in the discussion of my responses. When I admired his thinking he was spurred on, but when I drew his attention to this apparent change and put it in terms of his forgetting about his resistance he became defensive and seemed to return to it. It would have been better if I had asked him “Does this feel different to you?” rather than telling him of the change I felt. By not doing so I was pulling him back into our power struggle.

I wondered out loud if his resisting was really letting the instructor win, a form of “I can’t work this through...I’m wimping out here.” Mulder ignored me and went on working, even suggesting that he do some from chapter 18 and triumphant when he got those correct.

I went further with the Mulder-has-two-parts theory. I reminded him of his reaction to losing points only on the conceptual/symbol definition section in Exam #2. I suggested that he had seemed almost proud that he had proved the point that he couldn’t do the multiple-choice conceptual part,” but he disagreed, “Not proud of that just pleased I got all the calculations right ...I got a B!” He sounded indignant that I didn’t seem to appreciate that fact and admire his competence. I wanted to give him a little more feedback.

There’s a part of you that is really listening and engaging in it and there’s another part that’s like “uugh” [pushing away with my hand]. It’s almost like you’ve got this little battle going on [Mulder chuckles]. You don’t think that’s happening? (Session 5)

Mulder continued with his mini-test as I looked at what he had done on Exam #3. I continued to alternate between exploration of Mulder’s mathematics approaches and his resistance. I interrupted him to ask him about an error on Exam #3. He said that forgotten some details that Ann had told him so he made something up. I had also gone over this fact and the logic of it with him in Session 4 the morning before the exam, but he was at the time resisting my push to link symbol definition to the computation. He clearly didn’t recall that we had discussed it. Mulder had however successfully used his visual memorizing of in-class worksheets on parts of the exam He described the process, “That one I just sat there forever and ever until I came up with it. ...I kept thinking of the worksheet in my head until I came up with it.” I wondered about what might have

happened if he had sat for ever thinking about the multiple choice questions. I suggested, “It’s like a cooperative part that does this [the computational part] and a resistant part [that doesn’t do the multiple choice]”

Mulder continued working, talking out loud as he did liberally sprinkling his reactions with “crap” and “turd,” and at one point calling himself an idiot for not dividing accurately. He seemed intrigued with the concept of his two conflicting parts doing battle even though he said, “Let’s not remind me!” when I noticed that he was doing lots of good thinking so “the cooperative side of Mulder is working.” At one point when he called himself an idiot I was wondering out loud about his sense of his own competence, which he thought was fairly robust, and I hesitated thinking I was interrupting his work. But I was surprised when he said, “Keep going, I’m listening,” so I did:

JK: In the test when you’re doing this part is there anything that your I-want-to-do-better; I-can-think-about-this; This-may-be-logical part of you can say to the resistant part that says “turd” or “I’m screwed!” or “I don’t like this!” or “this is ridiculous!”

Mulder: You quoting me still? You’re going to write your dissertation and say this person said all this stuff about math.

JK That could be part of it but the main thing is what you’re saying to yourself. (Session 5)

Mulder sidestepped my question by drawing me into a difficulty he was having with χ^2 , but I felt that I was really being invited to witness his internal dialogue as he found his error and mastered the procedure *and* the concept—that was his answer to my question. Mulder was *demonstrating* his answer—that his smart, achieving self was in charge now.

I became aware that in my counseling role, even my tutoring role, I should have taken a neutral stance with respect to Mulder’s warring parts. I was truly moving closer to that but I was so used to taking sides that I continued to betray my partiality for Mulder’s

“good” side, risking as I did, propelling him towards his “bad” side and getting us back into our power struggle.

After Mulder successfully negotiated more questions on χ^2 , I moved in again as tutor. I wanted to help him make more visual and logical connections:

- JK: Have you seen a picture of a χ^2 ? [JK draws a graph]
 Mulder: So it's going to be positively skewed. . .
 She didn't show us this [resisting what he saw as additional material].
 JK: [chuckles] That's true But it's not going to do *her* in if you get this wrong; it's going to do *you* in!
 Mulder: Yes.
 JK: Yes and you want to be done in to prove your point.
 Mulder: Yes that's my goal in this test: to fail miserably on the multiple-choice.
 JK: To prove your point.
 Mulder: So I *have* to take that final.
 JK: Ah, there you go!
 Mulder: I'm not going to take it if I don't have to... if I could do well on these next two tests I wouldn't have to take the [optional] final.
 JK: There you go! This is that logical sensible person. Don't speak to me!
 Speak to that resistant part that *has* to get a bad grade on the multiple-choice. (Session 5)

Mulder continued, alternating between resistant grumbling and engaged cooperating. But the focus of his grumbling was changing. It was less about *her* [Ann] and *having* to do the conceptual work, and more about the cognitive demands of the conceptual work. He grumbled about a change in how to identify significance required in a new test, about a question we both agreed was badly worded in the study guide, and especially about the practice problems I had prepared for the exam. He tackled the rest of the multiple-choice questions in an engaged, positive way. And I stayed out of the battle.

At the end of our session Mulder's finite math instructor came in and Mulder told her in discussion “I have a resistant side of me.” To her dismissive “Don't we all,” he insisted, “I lost 18 points on the multiple-choice.” This was the first time that Mulder had verbalized the theory of his two battling sides and owned it.

Final Discussion: Mulder and Mathematics Counseling

Mulder completed the practice problems, spent time on his formula sheet, and even came to study group before the Exam #4 the next evening. And he earned a 91% overall and an 82.6% on the multiple choice section! He went on to earn an A on Exam #5, an A⁻ on the MINITAB presentation, and a B in the class. He was satisfied with that and decided not to take the optional comprehensive final. He dropped in a couple of times at the Learning Center, once to have me check over his MINITAB presentation paper (I suggested corrections that he did not make because he did not have time.) and once to help him prepare for his finite math final.

Session 5 was a pivotal session both for Mulder and for me. At the start of the session, I believed it was primarily his stubborn resistance against the assessment that prevented him from improving although I suspected that, in some ways, I was exacerbating that resistance by my mother-of-a-teenager countertransference. Though I planned to use Dr. P.'s suggestion to try paradoxical intention, I found myself telling him what I saw of his resistance and suggesting how to fix it. Ah, that was it, I realized. When I tried to get him to do or know or believe something, he resisted me. His resistance to Ann and the conceptual part of the test was confounded with his resistance to me, and that in turn made me push harder. It was when Mulder and I assumed the same stance in looking at his approaches that the change in his self-awareness began. When I implied that he might be betraying or letting down his resistant self by the intelligent engagement, things didn't go well—I was taking sides. When I couched it in terms of two legitimate parts of himself that were engaged in battle, Mulder went with that and I was able to withdraw myself from the battle. I no longer had to battle Mulder's resistant side trying to

persuade him to capitulate and cooperate. He could fight the battle himself, using his intelligent engaged self, and I could let him go. It felt good (but a little scary) to pull myself out of the fight and let Mulder battle himself and fight his own demons. He did this successfully, and I congratulated him.

Evaluations

On the post-surveys, Mulder's learned helpless beliefs changed significantly towards mastery orientation (see Appendix M, Figure M1 and Appendix H, Table H3). He said his motivation for coming to mathematics counseling changed from helping me with my research to getting help with his strategies because the help with the statistics was "great," but his metaphor had not changed much: the Truth (mathematics?) was still "out there." Mulder had found that despite a relatively underdeveloped, vulnerable mathematics self, he could do well in a mathematics course if he got out of his own way and tried to think strategically and conceptually. That very success could contribute to the development of that self. There was still room for growth but now Mulder might draw on this experience and risk trying to understand rather than using illogical alien approaches or overconfidently avoid trying.

I had learned to attend to Mulder's transference of past relationships with teachers into our relationship as he alternated among confessions of laziness, pronouncements of his potential, and theories about what mathematics was that precluded the parts he was struggling with. I learned to attend to my countertransference reactions: I was the frustrated, cajoling mother of a young man who seemed to be his own worst enemy, and he sparred with me and appropriately resisted my efforts to fix him. Mulder's own metaphor was the key to resolving the conflict. In supervision with Dr. P., I began to

understand better what was going on as we examined Mulder's metaphor and I shared my transference-countertransference insights. In our last counseling session Mulder resolved his central conflict when I withdrew from my countertransference stance. I truly used a relational counseling approach, the outcome for Mulder was good, and I learned how powerful and counterproductive countertransference reactions can be even when one theoretically knows about their reality. I learned that although it is difficult, examining my countertransference reactions and choosing consciously to do things differently is crucial.

REFLECTING ON THE COUNSELING CASES

Crossing previously drawn lines—that seemed to be a common thread through the course of mathematics counseling with Karen, Jamie, and Mulder. Indeed a relational approach required it. I crossed lines and so did they, and we crossed lines together. Although mathematics was our primary activity, my persistent curiosity into how they did well and why they struggled led us to new ground. Of the three Mulder was the most willing on the former and the most resistant on the latter. Karen put strict limits on her responses to what she perceived to be non-mathematical discussions but she crossed her own previously drawn lines in mathematics effort. Jamie was willing to cross lines with me after I crossed lines to draw her into counseling in the first place. But we stayed within boundaries acceptable in the Learning Assistance Center context.

Each of these students made progress academically. Each achieved a grade as high as or higher than they had hoped (see Appendix H, Table H1.). Mulder and Karen who earned D's on the first exam went on to earn Bs in the class. Both did so despite significant deficits in their mathematics preparation (see Tables 6.1 and 6.2). Jamie found

Table 6.1

Focus Participants' Levels of Understanding of the Variable on the Algebra Test
(Sokolowski, 1997; Brown et al., 1985, p. 17; see Appendix C)

Participant	Number Correct (of 53)	Level 1	Level 2	Level 3	Level 4	Level 5	Level of Understanding
Jamie	42	6/6	5/7	8/8	6/9	1/3	Level 4
Karen	30	6/6	5/7	3/8	2/9	1/3	Level 2
Mulder	25	6/6	7/7	3/8	3/9	0/3	Level 2

that, contrary to her belief, she *was* adequately prepared mathematically (see Tables 6.1 and 6.2). She and Karen were repeating the class and they saw and did things differently and did well this time.

And each of these students gained new insights into themselves as mathematics learners. Jamie realized that her difficulties with mathematics were not to do with her ability but rather with relational issues; Karen found that she could achieve well in mathematics despite her considerable arithmetical and algebraic deficits and lingering doubts; and Mulder overcame his resistance to aspects of mathematics he found difficult because of his auditory processing difficulties and in defense of his vulnerable mathematics self and found that he *could* do well.

I crossed lines and found a new way of looking at myself and them and us that gave me new power to reflect, monitor, and change my approach and steer the counseling. At the same time, this new way of looking gave me new ways of listening, observing, and responding to them so that they could and did choose their way and modify mine. The counselor-student dyad indeed was the key to the changes we all made.

Table 6.2

Focus Participants' Understanding of Arithmetic on the Arithmetic for Statistics assessment (Appendix C and chapter 8 discussion)

Participant	Class work	Small (<1000) Integer Number sense	Large Integer Number sense	Fractional number Sense	Place Value/ Decimal Sense	Operation Sense	Open Ended Arithmetical thinking/ problem-solving	Statistical Sense
Jamie	adequate	100%	43%	67%	$ a < 5$:	80%	95%	88%
		adequate	inadequate	~adequate	97%	adequate	adequate	adequate
					adequate			
					$ a > 5$: 100% adequate			
Karen	marginal	45% of	17% of	33% of	$ a < 5$:	20% of	35% of	45% of
		total; 100%	total; 30% of	total;	65%	total; 33%	total; 42%	total;
		of attempts	attempts	75% of	~adequate	of attempts	of attempts	62.5% of
		inadequate ?	inadequate	attempts adequate?	$ a > 5$: 40% inadequate	inadequate	inadequate	attempts ~adequate?
Mulder	adequate	100%	56%	66%	$ a < 5$: 90%	100%	76%	56%
		adequate	marginal	~adequate	adequate	adequate	adequate	marginal
					$ a > 5$: 69%			
					adequate			

good

X ≥ 85%

adequate: 70% ≤ X < 85%

~adequate: 60% ≤ X ≤ 69%

marginal: 50% ≤ X ≤ 59%

inadequate: X < 50%

inadequate? X > 50% but < 50% attempted

adequate?: X ≥ 70% of attempts

~adequate?: 60% ≤ X ≤ 69% of attempts

In the next chapter I briefly profile the remaining nine students in the class and discuss the developing theory that emerges from this pilot study. In particular I propose criteria for a new way of categorizing students as mathematics learners that surfaced from

analysis of participant profiles. I then analyze and present what I see as the essentials of the brief relational mathematics counseling approach that emerged.

ⁱ Because this is an even numbered chapter I use “she,” “her,” and “hers” as the third person singular generic pronouns.

ⁱⁱ This pattern continued except for two occasions later in the course before exams when she did allow herself to be drawn in to some of the study group’s discussion.

ⁱⁱⁱ I saw the impact of this limited understanding in the first exam when she used the deviation of only one score from the mean instead of the required deviations of all values of the variable (the scores), to find the standard deviation of all the scores from the mean.

^{iv} Performance vs. learning achievement motivation questions were numbers 4, 7, 9, and 10 of Part I of the *Beliefs Survey* (see Appendix C)

^v For example, she mistakenly thought σ (sigma, the standard deviation of the population.) represented the mean of a population.

^{vi} For example, μ = population mean, and σ = population standard deviation, both constant identifiers for a particular population distribution.

^{vii} For example, X *sample* mean, and s sample standard deviation, each constant identifiers for a particular sample.

^{viii} She had to find the median, P_{50} , of a set (distribution) of scores.

^{ix} She agreed with prompting that her answer didn’t make sense but when I instructed her in the use of her calculator to get the correct answer she remained baffled, “I don’t really know why though. I just plugged in what you told me.” In this session there was no time to teach Karen the concept of percent.

^x Despite her emotional state in Session 1, Karen had managed to successfully correct, learn, and retain these symbol designations during the session.

^{xi} For example, for the real limits of the weight of a slice of cheese of 0.35 grams, Karen had answered

- d. may be anywhere in the range of 0.34 – 0.36, instead of the correct
- b. may be anywhere in the range of 0.345 – 0.355.

Karen’s answer to this and similar questions showed an understanding of the concept being tested but anything beyond the first place (tenths) of decimals confused her. Karen was able to give 8.5 and 9.5 as the real limits for 9 and other whole numbers but not for 0.9 or 2.9. She misnamed decimal places, calling hundredths tenths and vice versa. She did not seem to have a firm sense of the relative size represented by the places nor the places’ relationships with each other.

^{xii} In reality, when Karen took the *Arithmetic for Statistics* assessment after the course ended, she showed that she *did* understand relative sizes represented by the places, on the number line graphing questions though not by using numerals alone. If Karen had made that extra appointment, I would have given her the *Arithmetic for Statistics* assessment then and coached the exploration beginning with her number line understanding.

^{xiii} We each copied the data from the class question onto our own sheets of graph paper. Next we copied the formula as I read it out loud and constructed columns beside the X and the Y columns with column headings corresponding to pertinent elements of the formula for r , namely X^2 , Y^2 , and XY .

^{xiv} I suggested we use the questions for the assigned problem in the text (Pagano, 1998, chapter 6, Problem 14, p. 124) and the data from the in-class question because the questions were more delving and the data were less complex.

^{xv} Karen had not computed the standard error of estimate of Y accounted for by X correctly because she had not constructed the squared deviations $(Y - Y')^2$ column. She had the $(Y - Y')$ but not the $(Y - Y')^2$ column on her formula sheet, so she used the $(Y - Y')$. This should have summed to zero prompting her to check her formula (which correctly included the squaring) and create the $(Y - Y')^2$ column. Instead of adding Karen tried multiplying the $(Y - Y')$ s to get 0.000151 which she then wrote as 1.51 to make it more reasonable. She knew this was not correct, however, as she wrote on her exam “still can’t figure out where I screwed up.”

^{xvi} I had checked with Ann about student use of the blank flow chart and she was agreeable.

^{xvii} Karen knew from class that being given s^2 s or the SS s (sums of squared deviations of scores from the mean) would indicate an independent samples t test in contrast with two sets of data that would indicate the correlated groups t test. I knew, however, that in the real world of data gathering and analysis, students could be given two sets of scores for *either* situation, independent or correlated, and *could* calculate SS s and s^2 s from that data for either.

^{xviii} Karen made an error in one inference test that did not result in her losing points. On the normal deviate z test, Karen had compared the magnitude of the p value she obtained (0.0013) with the z score she had obtained (-3.01) instead of with the critical alpha level of 0.05. She came to the correct conclusion though so Ann did not deduct any points.

^{xix} Still later, a semester after the end of summer PSYC/STAT 104, however, Jamie revised her stated motivation to helping me: “...I’m much more of a helper, which is why I think I signed up to do this with Jillian, cause I saw it as helping her with her project. If it had been just for my benefit I don’t really know if I would have approached her or not.”

^{xx} Jamie was, in fact, repeating this course, but I did not find that out until after the course was over. With appropriate permission, I obtained the printouts, without names, of the grades of all students of the PSYC/STAT 104 for the 5 years before the summer of 2000. It included the data from this class with some that suggested that Jamie was repeating the class. When I sent a post-study e-mail survey to check that and other data I was unsure of, Jamie replied to my assumption that she was not repeating the course, that in fact she was repeating it because she got a D^+ the first time and that was not adequate for her psychology major.

^{xxi} Jamie’s “?” indicated her own uncertainty about her exact grade.

^{xxii} She did not tell me of her D^+ in her first attempt at PSYC/STAT 104 at State University.

^{xxiii} See Chapter 5 for a discussion of Ann’s policy regarding students’ tests.

^{xxiv} Here Jamie indicates her belief that her error lay in using “the right one [equation] for a different one” perhaps thinking of the different formula for σ , the standard deviation for a population for which the denominator is N rather than the $n - 1$ for the s , the standard deviation of a sample. In fact, the class had learned no formula for which the denominator is $\Sigma X - 1$ the one she had initially used.

^{xxv} 3.7 on a 1 through 5 scale—close to the highest in the class (see Figure L4, Appendix L)

^{xxvi} Sokolowski’s three college student subjects who achieved a level 4 of the algebraic variable, had each succeeded in at least one college level mathematics course, was at the time of her study an A/B mathematics student, and succeeded in combinatoric-/probability-/statistics-related mathematics (p. 70, 98).

^{xxvii} Since a score of 1 represents zero anxiety, the drop of 0.5 in Jamie’s Mathematics Testing Anxiety form 4.1 represents a 0.5/3.1 that is a 17% decrease.

^{xxxviii} This participant chose this pseudonym for himself when the question arose during his mathematics counseling Session 5 on July 25, 2000. Fox Mulder was also his metaphor for how he approached mathematics.

^{xxxix} Over the course he talked during the lecture portions of the class in every class but one, averaging three interactions—answers or corrections—per class. This placed him as the third most involved in these lecture discussions, after Robin and Lee (see chapter 5, Table 5.2).

^{xxx} A science fiction television series featuring FBI paranormal detective Fox Mulder (and his partner Scully) in search of the aliens who he believed had abducted his sister.

^{xxxi} I was able to check on this more formally when I gave Mulder my *Arithmetic for Statistics* assessment (Appendix C) as a posttest on July 31, 2000. He asked if he could fill it in later and eventually sent it to me in March 2001. See Table 6.2 for Mulder's results, all of which were adequate except for his statistical number sense and large integer number sense which were marginal. These last areas (tested on this assessment) were not tapped during the course.

^{xxxii} His approach lacked a number of the identifying features of learning style II; he did not seem to grasp the gestalt of a situation or use an inductive (rather than deductive) reasoning approach nor did he have difficulty with details and step by step procedures. In these areas he seemed more analytically procedural (like Davidson's mathematics learning style I) though he used visual memorization rather than verbal tactics. On the other hand his finding solutions without being able to satisfactorily explain how and his sense of appropriate sized solutions supported a learning style II conjecture. It was also not clear to what extent he had adapted his approach to handle mathematical tasks that seemed beyond him.

^{xxxiii} Suinn (1972) found on his 98 item Mathematics Anxiety Rating Scale (MARS) from which all the testing and number anxiety items of my *Mathematics Feelings* survey are drawn, that mean scores were as low as 1.47 for physical sciences majors ($sd \approx 0.4$), and 1.7 for social sciences students ($sd \approx 0.6$) which would seem to imply that Mulder's 3.1 shows high anxiety (more than 2 standard deviations above the mean). But because Suinn's scale was found to confound testing and number anxiety factors (see Rounds and Hendel, 1980) and students' number anxiety scores were on average 0.75 points lower than their testing anxiety scores when separated here in the *Mathematics Feelings* survey, I would suggest a higher average for testing anxiety and a lower average for number anxiety than Suinn's should be considered moderate on my *Feelings* survey. Given this consideration Mulder's 3.1 testing anxiety score could still be considered well above moderate even for a social sciences student.

^{xxxiv} The fact that he was 75% satisfied with his mathematics achievement (Item 6, responses marked 1) and 75% confident about his mathematics future (Item 3, responses marked 1), and 75% positive about the course he was taking now (Item 4, responses marked 1) indicated a positivity that did not seem justified by his history or his performance on Exam #1 (see Appendix M, Figure M2 and Table M3).

^{xxxv} That is, to have him discover that developing a conceptual understanding of the procedures he had mastered by Exam #2 would help ensure continued success in the computational part and mastery of the conceptual part of the test.

^{xxxvi} He pointed to question 3.

3. The 20 subjects constitute a _____
- population
 - sample
 - parameter
 - variable

Mulder asked me who the 20 subjects were; what was that about? I found that he had not realized that the first 5 questions were referring to an experiment described and bolded at the top of the page. He got three of these five questions wrong (and lost 6 points).

^{xxxvii} Ann mislaid the computational section of Mulder's first exam so we were unable to analyze his errors on that section.

^{xxxviii} Mulder wanted to "memorize rules for doing a SIMPLEX problem." He felt he had the material under his control and was really just checking that he had it correct. He knew the material procedurally but was not able to explain to me nor did he want to know why he had to do what he was doing. He was using the SIMPLEX method to maximize profit given a system of linear constraints (Rolf, 1998, chapter 4). He knew that an equation had to be changed to two inequalities, in particular, inequalities in which the variable sum was less than the constant, before slack variables could be added, but he did not know why, probably because he did not understand the meaning and use of the slack variables. He was able to perform the necessary procedures.

^{xxxix} For example, Mulder thought s stood for "sample" (rather than sample standard deviation).

^{xl} For example, we worked out what \bar{D} meant knowing already that \bar{X} was the mean of scores for a sample. I pointed out the links: the bar conveys the idea of mean and the D represents the list of data being analyzed (in this case the differences between pairs of before and after scores).

^{xli} The chapters covered were: Chapter 15: *Introduction to Analysis of Variance*, chapter 16: *Multiple Comparisons* and chapter 18: *Chi-Square and Other Nonparametric Tests* in *Understanding Statistics in the Behavioral Sciences* (Pagano, 1998).

CHAPTER VII

DEVELOPING THEORY: STUDENT CATEGORIES AND WAYS OF COUNSELING

In this study I gathered mathematics cognitive and affective data from 12 of the students of PSYC/STAT 104 and I counseled ten of them using cognitive constructivist tutoring and relational and cognitive counseling approaches described in chapters 2 and 3. The results of the study are described in chapters 5 and 6. As I analyzed these results I noticed a number of interesting interlocking patterns that I discuss in this chapter. I will demonstrate how this analysis supports a categorization scheme of mathematics learners that emerged from this research. I will then present my analysis of this brief relational counseling approach as I found it relates to students thus categorized.

When I analyzed the three in-depth cases and the briefer profiles of the other nine students in the class I found that categories of mathematics self development emerged from interactions between two dimensions—mathematics preparation and relational experience. These interacting factors produced relatively well-defined categories that can be compared and contrasted with Tobias's tiers described at the end of chapter 4. These categories although similar to Tobias' tiers are distinct in important ways.

This result was of particular interest because I also found that different relational mathematics counseling approaches and the relative balance among its components (degree of cognitive constructivism, amount and kind of mathematics tutoring, amount of course management counseling, and cognitive and relational counseling) were differently applicable to specific categories of student.

Mathematics Preparation

Students in the class fell into three broad categories according to the adequacy of their mathematical preparation for the class: *well prepared*, *adequately prepared*, and *underprepared* (see Table 7.1).

Table 7.1

Criteria for Determining Level of Mathematical Preparedness of PSYC/STAT 104 Participants

	Well Prepared	Adequately Prepared	Underprepared
Course grades:			
Exam #1	B+ through A	D through A	F through C ⁻
Final Course Grade	A ⁻ (B ⁺) through A	B ⁻ through A ⁻	AF through B
Algebra: <i>Algebra Test</i> and class, exam and counseling session work	Level 4 or 5	Level 4	Level 1 or 2 (or 3?)
Arithmetic: <i>Arithmetic for Statistics Assessment</i> and class, exam, counseling session work	Good ($\geq 85\%$) in all 8 categories	Adequate or above ($\geq 70\%$) in all but one or two number or operation sense categories; variable in other sections	Ranges from adequate or above ($\geq 70\%$) on at most 6 categories to inadequate ($< 50\%$) or marginal ($50\% \leq$ $X \leq 59\%$) on three or more categories

Evidence for how participants placed in these categories was gathered throughout the course. Not all participants took the *Algebra Test* (Robin, Brad, and Kelly did not.) or the *Arithmetic for Statistics Assessment* (Robin, Mitch, Brad, and Kelly did not.) but in these cases there was sufficient evidence from their exams and work in class and counseling by the end of the course to place them with reasonable confidence. The three criteria that served best to categorize students in this sample were (a) understanding of the algebraic variable (measured on the *Algebra Test*, see Appendix C), (b) understanding

of and facility with arithmetic (measured on the *Arithmetic for Statistics Assessment*, see Appendix C), and (c) performance on the first exam of PSYC/STAT 104. When I considered students' high school and college course-taking and grades as an additional criterion for this sample, there was not enough consistency for this to be useful (although with larger groups of students this might be found to be a factor). The students who were *well prepared* mathematically had a high level understanding of the algebraic variable (see also Appendix H, Table H1), were arithmetically confident and competent, had always done well in mathematics, and did well on the first exam in the course (see Table 7.1). Those who were *adequately* prepared had a high-enough level understanding of the algebraic variable. However, while their arithmetic was generally sound they had some deficit areas, they had each had variable success in previous mathematics courses, and their performance on Exam #1 ranged widely from D⁻ through A. Those who were *underprepared* had a low level understanding of the algebraic variable, deficits in arithmetic that ranged from significant to mild, and they did poorly on the first exam of PSYC/STAT 104.

It is possible that using this approach to classifying students with larger groups might result in the *underprepared* group's being split into more categories. With this small group, placing the student/s who were weak in both arithmetic and algebra in the same category with student/s who were weak in algebra but sound in arithmetic makes sense given other identifying criteria. Further evidence may suggest otherwise.

Mathematics Self: Mathematics Preparation and Self-Esteem

After I sorted students according to their mathematics preparation (see Table 7.1), further analysis revealed that students' level of mathematics self-esteem roughly matched

the preparation categories and that these taken together gave a measure of students' mathematics self development. As noted in chapter 2, self psychologist, Kohut (1977) proposes that healthy self development leads to internalized values and ideals that provide structure and boundaries as the person's own competence develops. When this process proceeds appropriately the internal self-structure is consolidated and it provides what Kohut calls "a storehouse of self confidence and basic self-esteem that sustains a person throughout life" (p. 188, footnote 8). From this study I found that it was a student's mathematics competence (preparation) taken with his level of his self-esteem that indicated that self development level: Category I (sound), II (undermined), or III (underdeveloped) (see Table 7.2). I found that his level of *confidence*: realistic, under, or overconfidence, was an initial cause of confusion in assessing a student's category of

Table 7.2

Emerging Categories of Mathematics Self Development

	Mathematics Preparation		
	Well Prepared	Adequately Prepared	Underprepared
Level of self-esteem			
Sound self-esteem	Category I students with sound mathematics selves		
Compromised self-esteem		Category II students with undermined mathematics selves	
Low self-esteem			Category III students with underdeveloped mathematics selves

mathematics self (see *Jamie, Karen, and Mulder* in chapter 6). I found that a student's level of *self-esteem*, however, was directly related to his mathematics preparation

(competence) level. The levels of self-esteem I found in students in this sample were: sound, compromised, or low (see the student profiles below for discussion of how I discerned these levels). The shaded cells in Table 7.2 indicate that I found, as I expected, no student whose level of self-esteem was not directly related to his level of mathematics preparation.

Mathematics Self Category and Relational Malleability

I found that students in the second and third categories of mathematics self could be further sorted according to the extent of malleability (willingness to change beliefs and behaviors) versus inflexibility (resistance to changing beliefs and behaviors) in their mathematics relational patterns. This malleability versus inflexibility seemed to stem from personal characteristics interacting with past mathematics experiences in the current course environment. Students in Category II fell into these two subcategories according to how they had handled their *compromised* self-esteem: they had developed mathematics relational patterns that were either *malleable* or *inflexible* for the brief semester timeframe. Students in Category II of mathematics self similarly fell into these two subcategories according to how they handled their *low* self-esteem (see Table 7.3 for criteria I used to gauge malleability). This classification became important from early in the course because a student's willingness to engage in the struggle early in the course and to change if heⁱ was persuaded that he needed to was, not surprisingly, a pivotal factor in his success. This was especially important for underprepared students with low self-esteem (i.e., students with an underdeveloped mathematics self).

Interestingly, students I found to be inflexible seemed to fit Tobias' categorization of students as "utilitarian" (see chapter 4) and Mercedes McGowen's categorization of

Table 7.3

Criteria for Determining Malleability of PSYC/STAT 104 Participants

	Malleable Relational Patterns	Inflexible/unstable Relational Patterns
Achievement motivation ^a	Stated learning achievement motivation /learning motivation when he believes he is capable	Stated performance achievement motivation
Learned Helpless versus Mastery Oriented beliefs ^b /behaviors	Mastery orientated in beliefs and/or behaviors	Learned helpless in beliefs and behaviors
Procedural versus Conceptual Mathematics beliefs ^b /behaviors	Conceptual mathematics beliefs and/or behaviors and/or change towards conceptual	Procedural mathematics beliefs and behaviors
Problem-solving/trouble shooting beliefs/behaviors	Engagement in problem-solving practices/behaviors	Avoidance of problem-solving practices/behaviors
Changes over course		
1. Behaviors	Willingness to change/ resistance to change that changes to willingness to change during course	Resistance to change over course
2. Beliefs:		
a) Mathematics ^c	a) Substantial change in behaviors/beliefs over course—some positive (especially focal beliefs or emotions), some negative	a)Limited positive changes in beliefs/behaviors over course
b) Fixed trait beliefs about personality, limitations, and/or mathematics potential ^c	b) little need for change or positive change for this situation at least	b) limited change
3. Attachment Patterns ^d	Secure; avoidant to secure; dependent to secure	Remains detached or dependent or ranging between both

Note: ^a Achievement motivation beliefs were gauged initially by averaging 1 through 5 responses on a subscale of the *Beliefs* survey: Part 1, Questions 4, 7, 9, and 10 (see Appendix C). Achievement motivation behaviors and further explanation of beliefs were gauged through observation and conversation in counseling.

^b These beliefs were gauged initially through responses on the *Beliefs* subscale; behaviors and further explanation of beliefs were gauged through observation and conversation in counseling.

^c Changes were gauged through posttesting of *Beliefs* and *Feelings* surveys, by conversation, and by observation of responses (verbal and behavioral) to counseling interventions and in course achievement.

^d Changes in attachment patterns gauged in counseling through transference/countertransference etc.

students as “rigid” (personal communication, April 11, 2000). However, in addition, some malleable Category III students (e.g., Karen and Mulder) initially presented as inflexible (utilitarian/rigid) but their apparent inflexibility turned out to be defensive in a way that was adjustable with appropriate relational counseling.

I am very aware that my own personality and behaviors might have been a factor in the extent to which a participant exhibited malleable or inflexible behaviors in the counseling situation. This may not be entirely intrinsic to him. My particular challenge as a counselor may be to develop ways of helping students I perceive to be relatively inflexible to bend. Effective ways to achieve that are certainly not by advising, lecturing, or scolding. A relational understanding led me to see that inflexibility may be at least in part in defense of compromised or low self-esteem. It was in students’ responses to the surveys and their interactions in the classroom, with the course material, and with me that a malleable or inflexible profile emerged. Whether and how students changed over the course also helped confirm such a malleable or inflexible profile (see Table 7.3 for details of criteria and individual characteristics and Table H3 in Appendix H for student changes). No one met all the criteria identified for a profile but each participant had a predominance of characteristics of one type with relatively fewer of the other.

I will now present brief profiles of the students in the study showing how they led me to develop the categories and sub-categories I have identified.

*Category I Students with a Sound Mathematics Self: Mathematically
Well-Prepared with Sound Mathematics Self-Esteem*

In this class there were only two students who fit the Category I mathematics self profile and both had malleable relational patterns. It is possible that Category I students could exhibit inflexible relational patterns but I believe that to be unlikely unless such

students are faced with mathematics challenges well beyond what they are prepared for.

Then the inflexible/malleable distinction might surface.

Sound Mathematics Self Students with Malleable Mathematics Relationship Patterns

Two students in the class (Catherine and Robin) each had a constellation of characteristics that identified them as mathematically well-prepared students *with* sound self-esteem: Each earned a good grade on Exam #1, had experienced prior steady success in mathematics and had no crucial knowledge base gaps in arithmetic or algebra, had more conceptual than procedural beliefs, showed mastery orientation to mathematics learning, and had low to moderate anxiety. They had learning (rather than performance) motivation for taking the course, and each was realistically confident and exerted a realistic amount of effort towards mastery in the current class. In other words, each had a well-developed mathematics self, no toxic internalized mathematics presences, and current patterns of mathematics relationship that were flexible and constructive.

Although Lee had the highest conceptual beliefs score in the class on the pre-beliefs survey and had recently succeeded in a finite mathematics course in college, her high anxiety scores, low confidence about her mathematics (related to struggle and variable success in prior mathematics courses), relatively low Exam #1 score, and underdeveloped arithmetic operation sense ruled her out of this group. Autumn's performance motivation, procedural beliefs, learned helpless orientation, and history of uneven mathematics course performance also ruled her out of this group despite her high expectations and good grade on Exam #1. Robin signed up for mathematics counseling to help me with my research, but Catherine declined the offer.

Catherine. Catherine,ⁱⁱ a non-traditional biology major who had just completed Calculus I with an A, was confident but quiet in class. Her high conceptual belief score, low anxiety, and high course outcome expectations seemed congruent with her presence in class and she was not considered to be at-risk in a statistics class. Initially, the instructor and I both thought she would do well without help. I wondered about how she would handle her own expressed need for conceptual understanding of the mathematical procedures (*Beliefs* survey) because these links were not generally made in class, but she did enough work on her own (5 hours per week) to make the conceptual links to the mathematical procedures that she needed in order to master the material. She did not ask for nor seem to need mathematics counseling.

Robin. Robin's mathematics successes were much more distant in time, and she obviously struggled in class in both lecture discussions and during problem-working sessions. She was not initially recognizable as a Category I student. Her membership in one of the groups traditionally at risk for Brookwood (older, female, nursingⁱⁱⁱ students) and her classroom presence initially raised questions for both the instructor and me about her prognosis in the class. I had more early information about Robin than the instructor because of my pre-course surveys, but it was in such contrast to how she presented herself in class that I questioned its reasonableness. She seemed to need mathematics counseling so I was not surprised that she chose to participate—I thought she would need considerable emotional and cognitive help. It was in the counseling setting that I observed Robin's competence and confidence. I discovered then that she volunteered for the study to help with my research, and not because she believed she needed help.

Robin revealed her positive mathematical self-esteem and history through her metaphor for herself—Belle (from Walt Disney’s animated movie *Beauty and the Beast*), an intellectually curious and competent feminine woman. She explained this in terms of her family’s identifying her with a mathematical grandmother. Her success in school mathematics was tempered by her parochial school teachers’ censure when she knew an answer but could not explain how she arrived at it. Robin seemed to be a global learner^{iv} with some auditory processing difficulties. She had not taken a mathematics course for 25 years.

My counseling support consisted of helping Robin become conscious of her positive mathematics self-concept by interpreting her Belle metaphor, inviting her to tell her story, and affirming her achievements and her current approach to mathematics course material. I chose parallel conceptual-to-procedural link tutoring to help her feel more grounded in her competent, conceptually oriented mathematics self. Given how long it was since she had taken a mathematics course, Robin expended a realistic 10 hours per week on homework. She struggled successfully to compensate for her learning style challenges and make the necessary conceptual-procedural connections. Although the instructor’s perception of her as a struggling nursing student never changed, Robin’s confidence improved, she mastered the material to her satisfaction, and she earned an A⁻. Even without the mathematics counseling, it is probable that Robin would have done well but her mathematics base for further mathematical study became much more secure because of the affirmation of her good mathematical abilities by a mathematics expert.

Category II Students with Undermined Mathematics Selves: Adequately Mathematically Prepared with Compromised Mathematics Self-Esteem

I found that more than one third of the students were adequately prepared to succeed in PSYC/STAT 104 but because of the interactions of past experiences with personal characteristics, they had developed relational patterns that could compromise their mathematics success in this class. They had relatively sound mathematics selves that had been undermined. Within this group there seemed to be two subtypes that I characterized as: a) students with malleable mathematics relational patterns and b) students with inflexible mathematics relational patterns (see Table 7.3). Students from these subtypes seem to have reacted differently to similar assaults on their developing mathematics selves.

Undermined Mathematics Self Students with Malleable Mathematics Relationship Patterns

The students who fell into this group were Lee, Pierre, and Jamie. They saw themselves as successful students in all but mathematics (and perhaps the sciences). They either underestimated or were ambivalent about their mathematical ability because of mixed mathematics success in the past. This caused moderate to severe affective problems in the Ann's mathematics class, particularly anxiety (for the women) and an expectation that they might do worse than they hoped. Generally, they had sound algebraic and arithmetical conceptual understanding but each had important gaps. Their beliefs about mathematics ranged from slightly more procedural than conceptual to conceptual, and they responded with positive mastery orientation to the challenge of developing a conceptual understanding once they believed they could. In other words,

each had an underlying sound-enough but undermined mathematics self from which he felt separated.

These 3 students signed up for mathematics counseling with an initial motivation of getting help to negotiate the course. Although it became clear that each had a good-enough mathematical knowledge base^v to succeed in this course (despite variable Exam #1 results, see chapter 5, Table 5.1), each had secondary problems that could have jeopardized this success. The women had developed anxiety problems expressed in their *Feelings* survey responses that were confirmed by observation and in discussion; the man had developed over-inclusive study practices that were counterproductive. Whatever the complexity, these students were willing to change their course approach in order to understand the concepts and achieve good grades.

Lee. From the beginning, Lee was the most mathematically insightful of the participants. She was interested in how different elements of statistical analysis related to each other (see chapter 5, discussion of Study Group 1, pp. 175-176). She was the second most verbally responsive student in the class, with an average of 3.36 questions or answers per lecture discussion. Most of her questions were about exam strategy and concepts. Lee initially had the most conceptual beliefs in the class and was significantly more mastery oriented than learned helpless (on the *Beliefs* survey), but all three of her anxiety scores were high; her testing anxiety and number anxiety were each the second highest in the class and her abstraction anxiety the third highest on the *Feelings* survey. She signed up for mathematics counseling the day before the first exam because of anxiety, but we could not meet until after the exam. She had blossomed in a mathematics environment where she was required to think and explore deeply. She was driven to the

point of anxiety in classes where the conceptual connections to the procedures were not explored and where she felt that only mathematics procedures were being taught. However, she was convinced that, because the mathematics was not immediately clear to her and she had to work hard to understand, she was not good at mathematics.

Lee found the PSYC/STAT class difficult because of a lack of in-class guidance linking concepts to procedures. She did well with the instructor's problem-working approach because it forced her to explore and master the procedures herself. It seemed that she did not feel secure in her relatively sound mathematics self because of variable past success in her past and her self-comparisons with peers who "just got it" without having to work hard at it as she did. Lee's strong performance on the *Algebra Test* and *Arithmetic for Statistics* assessment helped allay her concerns somewhat (see Appendix H, Table H1), but she performed poorly ($< 50\%$) on the operation sense section of the *Arithmetic* assessment. This significant gap seemed to affect her mathematics self and probably contributed to her anxiety.

In our sessions I focused on affirming Lee's conceptual problem-solving orientation and providing a secure base for her to explore the concepts and the connections that she did not experience in class. Lee relied on these sessions perhaps too much. She reported at the end of the course that she did only about 20 minutes homework a week. That was likely a factor in the high testing anxiety that increasing over the course.

Another issue in Lee's anxiety may have been linked to the fact that she valued the conceptual understanding of the mathematics but may have undervalued the importance of thoroughly mastering the procedures. Her grades fluctuated, apparently

linked to whether she and I practiced the mathematical procedures or not, but she finished the course with an A⁻ after taking the optional comprehensive final to replace a lower test grade.

Mathematics counseling was beneficial for Lee. It provided a secure conceptual base so she could repair her undermined attachments to mathematics and supported her in making the conceptual links to the procedures. In mathematics counseling I should have given more attention to providing bridges of understanding between her and her instructor (given their different priorities). I did continue to affirm her sound ability, learning motivation, and mastery orientation to achievement tasks, and Lee became more mathematically self-reliant.

Pierre. Pierre had been in the U.S. for only two years and his English was difficult to understand. He had earned a D in the calculus course he had just completed so he signed up for individual counseling once a week but we did not meet until the end of the fourth week of class because of miscommunication. He reported no difficulties with mathematics in his early schooling. His anxiety scores were low and his *Belief* survey results indicated a mastery-oriented approach to mathematics learning although his beliefs were somewhat more procedural than conceptual (2.5 on the 1 to 5 scale).

We first met after Exam #1 where Pierre earned only a 68%. He put this down to having to take the exam early because of a prior obligation but his C⁻ on Exam #2 seemed to point to something more. Pierre was in the B⁻/B⁺ range on the conceptual multiple-choice and symbol section but in the D⁻/F⁺ range in the computational section. It did not seem that he had any fundamental problem with his arithmetic or algebra, although his operation sense (like Lee's), was inadequate (Appendix H, Table H1). He

seemed to have an over-inclusive approach to his learning. In his reported 17 hours per week of homework he surveyed and studied the greatest amount of material possible including extra material he asked Ann for and Pierre met with Ann and with me often.

Because Pierre gathered and worked on so much, he was not mastering the mathematical computational material focused on in class, and he at times confused the extra material for material he *was* meant to use. In addition he approached the mathematical computation in a very procedural way, separate from its conceptual base. For the third exam I suggested that he focus on the course material. When he did not and earned a D⁻, I forcefully confronted him before the fourth exam with the likelihood that if he did not change his approach he would get another D. He seemed a little shocked by my forthrightness but this time he listened. On Exam #4 Pierre earned a 91%, losing only one point on the computational section! When he came to tell me, he was very pleased and a little surprised at how much difference this strategy change had made.

Pierre's English language difficulties contributed the most telling perspective on his performance in the classroom. It was clear that he had to use much of his energy to comprehend the material and to understand the organizational decisions. He did not collaborate with other students during problem-working sessions. Pierre did contribute a little in class (an average of once per lecture discussion) but his English continued to be a challenge for him and his peers. Although it dominated his class presence, it was not the main issue in his struggle to get a good grade; rather that issue was whether he was willing to give up his over-inclusive strategy to take a strategic approach.

With much improved grades on Exam #5 and the MINITAB projects and a reasonable score on the optional comprehensive final to replace his lowest test grade,

Pierre went on to earn a B⁻ in the course, much better than the D he was earning through the third exam. He retook Calculus I in the spring of 2001 and with this new approach earned a B⁺ to replace his original D.

Jamie. Since Jamie is a focal student (see chapter 6) I will review her profile only briefly, chiefly to explain why I believe she falls in this category. As with Lee and Pierre, once we had ruled out arithmetic and algebra knowledge base issues as a central concern and began to reconnect Jamie to her secure mathematics base, counseling could focus on her central affective issues, which in Jamie's case was her severe anxiety as revealed in her *Feelings* survey, metaphor, and presence in class. Her shy personality had interacted with classroom teachers and family theories, and caused her to question her ability in mathematics. Work on repairing damaged mathematics and mathematics teacher attachments, replacing her negative internalized teacher presences with positive ones, and supporting healthier interactions with the mathematics classroom personnel resulted in significant reduction in her anxiety, an improved sense of her mathematics self, and a B⁺ in the course. However, her slightly more procedural than conceptual beliefs did not change and her performance orientation remained (see chapter 6 for a detailed account of Jamie's course of counseling).

Students with Undermined Mathematics Self and Inflexible Mathematics Relationship Patterns

Autumn and Mitch fell into this group. Like Lee, Jamie and Pierre, they had sound-enough mathematics preparation and compromised self-esteem emanating from an undermined mathematics self but unlike Lee, Jamie, and Pierre they did not seem willing to change their counterproductive ways of protecting their undermined mathematics selves. Their primary achievement motivation was for performance (certain grades)

rather than learning. They had achieved quite well in mathematics at times in the past but had also gotten disappointing results. They saw themselves as capable procedural mathematics students, but feared and resisted both problem-solving and the conceptual demands that were made on them. They did not want to risk exploring conceptual links. This approach resulted in a learned helpless orientation in conceptually demanding or problem-solving situations. Their underlying understanding of the algebraic variable was good-enough to support some conceptual exploration and their facility with arithmetical processes was adequate, although there was some question in my mind about operation sense. They tended to avoid open-ended questions (cf. Autumn's efforts on *Arithmetic for Statistics* assessment, archived). Both had an overall negative attitude to themselves doing mathematics that could be classified as mild to moderate mathematics depression. They maintained detached distance from mathematics teachers and peers.

These students seemed to have the most difficulty of all students in the class with any change of approach in how a class was taught and managed; their strong conservative impulse (cf. Marris, 1974) led to strong resistance against change. It seemed that painful or disappointing experiences with mathematics in the past had led to their building defensive barriers around their relatively sound but fearful mathematics selves to guard against scrutiny or further assault. They seemed inflexible and unwilling to give up their defensiveness in order to risk growth in understanding and achievement.

Autumn. Autumn said she signed up for mathematics counseling to help me with my research. Although she reported disappointment with herself for not pursuing and succeeding in the algebra through calculus sequence, she was confident of success in PSYC/STAT 104 and did not want to explore conceptual connections or try to develop

her admittedly poor problem-solving abilities to become more mastery-oriented. She wanted a good grade rather than a conceptual understanding of the material and she maintained a performance motivation to learning statistics (see Table H3). She was a voluntary loner in class and maintained a detached distance from both the instructor and me.

From her middle and high school mathematics history it became clear that Autumn's performance motivation had prompted her to take an easier class in order to earn an A. Her detached independence prevented her from getting the help she needed when she did try a harder class, particularly her advanced Algebra II class where she had a poor background because of the easier Algebra I class she had taken to get her A. Her low grade in advanced Algebra II had in turn contributed to her disappointment with herself, her compromised mathematics self-esteem, and mild to moderate mathematics depression that was evidenced in her *Metaphor* and responses on the *JMK Affect Scales*. Autumn's depression was not allayed by her consistently high grades in the course.

If Autumn had participated in counseling designed to help her understand these connections and also supported her in exploring conceptual links and problem-solving, the current course experience might have developed her self-reliance and sense of mathematics self and perhaps even broken up her mathematics negativity. As it was, in counseling Autumn was willing to report her mathematics history, discuss her survey responses, and take the *Algebra Test* (a sound level 4) and *Arithmetic for Statistics* assessment (see Appendix H, Table H1), but she resisted doing exam analysis or exploring statistical procedures and concepts. Over the course, she remained relatively inflexible. Her procedural beliefs and learned helpless orientation changed little and her

abstraction anxiety score increased from 2.9 to 3.3 on the 1 to 5 scale although she reported that her confidence in her mathematics ability had improved (see Appendix H, Table H3).

Mitch. Mitch signed up for mathematics counseling because he needed to erase an F from his GPA. That goal was admittedly limited but his self-reported rigidity and resistance to change jeopardized his achieving even such a limited goal. He did not want to explore his affective problems with mathematics although he alluded to them. If he had been willing to explore his metaphor of Inspector Javert^{vi} as mathematics relentlessly chasing him through the years, he might have felt less beleaguered. Since he was not willing, what we did in the mathematics counseling was to work on the statistical problems at hand as I affirmed Mitch's sound mathematics self (e.g., his level 4 understanding of the algebraic variable on the *Algebra Test*) and tried to help him reconnect with it. I helped him notice that not changing his approach from his failed attempt at the course was negatively impacting his attempt to do better this time. Through the third exam he used a formula sheet of the type his former teacher had allowed despite my pointing out this instructor's more generous criteria that allowed the inclusion of more information. His extreme negativity on the *JMK Mathematics Affect Scales* at the first session did abate somewhat but only two responses were on the positive end of the scale by his last session (6: mathematics achievement, and 7: making mathematical decisions). He made good-enough adjustments, earned a B to replace the F, and he is finally safe from Inspector Javert's pursuit; he never has to take another mathematics course at least as an undergraduate.

Category III Students with Underdeveloped Mathematics Selves: Mathematically Underprepared with Low Mathematics Self-Esteem

The same number of students in the class had underdeveloped mathematics selves as had undermined mathematics selves. Those with underdeveloped selves fell into similar sub-types as those with undermined mathematics selves, that is, malleable and flexible/disorganized.

Underdeveloped Mathematics Self Students with Malleable Relationship Patterns

Karen, Mulder, Brad, and possibly Floyd were students in this study with underdeveloped mathematics selves who evidenced malleable mathematics relationship patterns. They had a history of struggling and/or not trying, poor mathematics achievement, and little (if any) feeling that they had ever understood. Like the adequately prepared students, they experienced relatively more success in other subjects. They were interested in understanding mathematics but felt capable of learning it only procedurally, if at all. They were more learning- than performance-motivated and were open to developing conceptual understanding once they believed they could, but all (particularly the men) seemed to fear risking the effort to understand, in case they found that they were incapable.

Karen, Mulder, Brad and Floyd each had mathematics knowledge gaps evidenced in a low understanding of the algebraic variable and possibly also in arithmetical number and operation sense deficits. In their attempts to deal with the discomfort engendered by being in a setting where they felt lost and incompetent, these students had developed compensatory procedures and approaches that included avoidance, busy work, memorization techniques, under or overconfidence, external blame, and hostility. Each

had an underdeveloped and shaky mathematics self that produced negativity and empty depression on the one hand, or unrealistic bravado and resistance on the other.

These underprepared, malleable students differed from the inflexible, adequately prepared students because they retained their learning motivation and an openness to learning conceptually despite their having experienced mathematical neglect.^{vii} In contrast, the inflexible adequately prepared students who also presented with mathematics depression, had experienced some mathematical success and had developed a good-enough knowledge base. Nevertheless they exhibited independent detachment and personal rigidity, performance motivation and resistance to problem-solving and conceptual learning.

Karen. Since Karen is a focal student I briefly review her profile in terms of her mathematics preparedness and self characteristics. Karen's negativity about herself, the class, and mathematics, along with her hostile detachment relational pattern with teachers and peers and her knowledge base gaps were evident early. A picture of her moderate empty mathematics depression emerged as relating to an underdeveloped mathematics self (see chapter 2, *Self Psychology*). She was learning- rather than performance-motivated but took a procedural approach to mathematics because she did not believe she *could* understand conceptually (although she wanted to). At the start of the course Karen consciously attributed bad outcomes to external sources, and in the counseling setting I had to overcome my countertransference reaction of feeling hopeless and depressed on her behalf. I challenged her external control beliefs with evidence, provided mirroring of her tentative self, helped her see the instructor as on her side rather than against her, and offered enough structured guidance and course management that she was willing to

consider changing her external control to internal and to take responsibility for what she realistically could do. Her mathematics depression abated somewhat, she became more self-reliant, her grades improved, and her beliefs about mathematics became significantly more conceptual. Her overall confidence in her own ability to understand conceptually grew only slightly, however. Since the course was taught with manageable limits that she could handle with strategy and effort, what we did was good-enough (see chapter 6 for a detailed account of Karen's course of counseling).

Mulder. Because Mulder is a focal student I highlight his characteristics briefly. Mulder had not really experienced success in mathematics, at least in high school. He “knew,” based on his theory of family genes that he could succeed but he had not really tried. When he did try after Exam #1 in the class, he found that he could handle the mathematics computations but he struggled with the conceptual multiple-choice questions. Rather than mirroring his emerging prowess and supporting its application to the difficult multiple-choice, I was somewhat dismissive of that success. I pushed him on the multiple-choice and he resisted.

It was not until we tried a counseling intervention suggested to me at my supervision session and I withdrew my counterproductive countertransference stance that he was able to overcome and succeed (see chapter 6 for a detailed account of Mulder's course of counseling).

Brad. Although Brad's bravado was more extreme and more unrealistic than Mulder's, it seemed to have stemmed from a similar source—his underdeveloped mathematics self. It brought forth a similar but more extreme countertransference reaction in me. I bristled at his we're-the-adults-here way of relating to Ann and me in

class. Unlike Mulder, Brad had tried PSYC/STAT 104 once and failed it, a fact that he seemed to almost inadvertently let slip in study group. He had, in contrast, written on his survey that he expected an A in this class and had earned a B in his last mathematics class, Algebra. He wrote on his metaphor survey that “anyone can do well” if he allows enough time and energy, yet he seemed ambivalent about doing that himself. He was surrounded by women at work, and had a woman as his superior. His motivation for doing this class was to get a degree that would allow him to change to a more male-favored position. His conflict seemed to be around a fearful sense of not being capable of doing the mathematics, combined with a desperate need to be able to do it. He was taking a risk enrolling again, and my scolding and pushing him rather than supporting him in this effort was not helpful to him. Unlike Mulder he did not stand up to me but oscillated between avoidance and non-strategic effort in a way that did not achieve any more than marginal results.

Floyd. The data I gathered on Floyd (from class surveys, the *Statistics Reasoning Assessment* and his Exam #1) revealed a similar bravado and resistance to getting the help he needed that Brad and to some extent Mulder exhibited. Like the other men in this group (it seemed) his grade hopes and expectations (both As) were unrealistically high, especially in light of his 42% failing grade on Exam #1 (see Appendix H, Table H1). He exuded confidence in class and declined the offer of mathematics counseling. However, like the other malleable underprepared students, Floyd’s achievement motivation was more learning- than performance-oriented and his sound understandings on the *Statistics Reasoning Assessment* were the fourth highest in the class (10 of the 20) (see Appendix H, Table H2).

Analysis of Floyd's Exam #1 efforts^{viii} revealed what appeared to be minimal if any prior study or practice, a somewhat surprising ignorance of basic statistical concepts such as median and mode (the only student in the class to show such ignorance), and probably a poorly constructed formula sheet. He did not make errors that indicated arithmetical gaps or misconceptions but there was too little data to assess that accurately or to assess his understanding of the algebraic variable. He overcame his resistance to getting help too late. He asked me for an appointment (just before Exam #2) but he did not come and then stopped attending the class.

Underdeveloped Mathematics Self Students with Inflexible or Disorganized Mathematics Relational Patterns

Kelly. Kelly had a history of poor mathematics achievement. She had deficits in number sense, operation sense, and understanding the algebraic variable. She had performance motivation and procedural beliefs, high levels of anxiety on all scales, and a learned helpless orientation to mathematics learning. Kelly had belief and anxiety scores similar to Karen's (a malleable student with an underdeveloped mathematics self) except that Karen was significantly more learning-motivated (3.5 compared with Kelly's 2.5). In addition, Kelly's externalized surprising-to-her "sudden storm" metaphor for mathematics, her and her mother's blaming her mathematics difficulties on something she felt was out of her control (a learning disability), and her relational pattern of dependence on both the instructor and me filled out a picture of her periodic sense of mathematical self disintegration. I allowed myself to be drawn into this vortex and was not able to help Kelly avoid another failing experience. My suggestions for counseling that may help such a student avert failure are discussed below.

Summary of Mathematics Self Categories

From this analysis of characteristics, behaviors, and responses to counseling the three student categories emerged according to how students' mathematics selves had developed and what that implied about their present approaches to mathematics learning. These categories are summarized in Figure 7.1.

Categories of Students According to Mathematics Self Development

Category I Students with Sound Mathematics Selves: Mathematically Well-Prepared with Sound Mathematics Self-Esteem: Defined by soundness of arithmetical and algebraic knowledge base and absence of any experience of assault or questioning of mathematics ability or achievement, resulting in a sound current mathematics self.

Type A: Sound Mathematics Self and Productive Relationship Patterns: e.g., Catherine and Robin

Category II Students with Undermined Mathematics Selves: Mathematically Adequately Prepared but with Compromised Self-Esteem: Defined by sound-enough arithmetical and algebraic knowledge base and a variable experience of achievement with or without outside assault on student's mathematics self concept, resulting in a relatively sound but undermined and vulnerable mathematics self.

Type A: Undermined Mathematics Self and Malleable Relating Patterns: e.g., Jamie, Lee, and Pierre

Type B: Undermined Mathematics Self and Inflexible Relating Patterns: e.g., Autumn and Mitch

Types A and B are differentiated by their affect and relational patterns developed around vulnerable and ambivalent mathematics selves.

Category III Students with Underdeveloped Mathematics Selves: Mathematically Underprepared with Low Self-Esteem: Defined by serious algebraic and/or arithmetic deficits or underdevelopment and a history of poor achievement resulting in an underdeveloped mathematics self.

Type A: Underdeveloped Mathematics Self and Malleable Relating Patterns: e.g., Karen, Mulder, Brad, and possibly Floyd

Type B: Underdeveloped Mathematics Self and Inflexible or Unstable Relating Patterns: e.g., Kelly

Figure 7.1. Mathematics self development categories of PSYC/STAT 104 participants.

After I analyzed the student participants from PSYC/STAT 104 into these categories, I was able to sort the counseling approaches I found to be appropriate and

helpful against these categories. I found that approaches indicated were closely related to the categories and also to students' affected dimension of mathematics relationality.

Integrating Relational Mathematics Counseling with Mathematics Tutoring: An Analysis

Dimensions of participants' mathematics relationality were interdependent, but some students had more pronounced difficulties in one dimension than the others. The categories of participants' mathematics functioning identified in this chapter (see Figure 7.1) seemed to be related to the problematic dimension (particularly the self dimension) and the depth and type of the relational difficulty.

As I have demonstrated, mathematics knowledge base deficits interacted predictably with students' mathematics self development and it was these two factors and their interaction that pinpoint a student's profile type. Past negative teacher-student experiences formed internalized presences that interfered in the present and had caused damage to mathematics selves. These negative teacher-student experiences had also caused damage to mathematics and mathematics teacher attachments (cf. Jamie).

Difficulties with establishing or maintaining secure attachments to mathematics and/or mathematics teachers also strongly affected their present relational patterns and mathematics functioning (cf. Jamie and Karen). Mathematical and counseling instruments and techniques for diagnosing and treating difficulties in one dimension at times resulted in improvements in another; in other cases they proved inappropriate and even counterproductive in dealing with another dimension.

In the following three sections I present my analysis of mathematics relational counseling for each of the three relational dimensions that Mitchell (1998) identified, which form the basis for my approach. In this analysis I show how a student's category of

mathematics self identified above affected the counseling needed and its possible outcomes.

Mathematics Counseling and the First Dimension: Self

Some students with pronounced defects in their mathematics selves presented as either unrealistically negative (underconfident) or positive (overconfident; in either case, unrealistic) about their mathematics self (Category III, Type A students, cf. Karen, Mulder). Others clung to the counselor with little or no sense of having an independent mathematical existence (the Category III, Type B student, Kelly). Students whose mathematics self was relatively sound but had been undermined so that they were no longer confident in it were likely to present with inappropriately severe anxiety (Category II, Type A women, cf. Jamie, Lee) or with a rigid resistance to change or risk (Category II, Type B students, cf. Autumn, Mitch) depending on their attachment patterns. I was able to use the following means to explore disordered self relational patterns:

1. *Investigation of the mathematics knowledge base:* using diagnostic assessments of arithmetic and algebra, class exams, and/or learning modality and style checklists, and
2. *Investigation of self relational patterns* by:
 - (a) Investigation of pronounced *ongoing negativity/depression* on the *JMK Mathematics Affect Scales*, *Learned Helpless-Mastery Oriented Scale*, generalized negativity, and underestimate of the mathematics self (underconfident). Clues lie in *Metaphor*, in self-statements in counseling, and in my countertransference feeling of depression or despair for the student's prognosis (cf. Karen);

(b) Investigation of pronounced *discrepancies* between a student's elevated *perception of his mathematics self* when compared with a realistic assessment of mathematics self (overconfident). Clues lie in *Metaphor*, in self statements in counseling, and in my countertransference of first believing and then wanting to dispute inflated and unrealistic self assessment and to deflate it (cf. Brad, Mulder);

(c) Investigation of *anxieties* that seemed *disproportionate* with measured levels of mathematics competency on the algebra and arithmetic assessments and/or exams. Clues lay in *Feelings* survey scores [and possibly *Metaphor*] relative to sound mathematics diagnostic scores [and possibly class exam scores] (cf. Jamie);

(d) Investigation of *inappropriate dependence* on counselor combined with lack of focus, willingness, or belief in ability to engage cognitively in the mathematics. Clues lie in *Metaphor*, in self statements in counseling, and in my countertransference feeling of being sucked into a bottomless pit (cf. Kelly);

(e) Investigation of a marked *discrepancy between personas* in different settings, for example, in class compared with the counseling setting (cf. Robin).

Students with moderately to severely underdeveloped mathematics selves were underprepared mathematically and their self-esteem was consequently low. They had inadequate scores on the algebra diagnostic (and some also on the arithmetic diagnostic) and low scores on the first exam in the course. Where I treated these problems

effectively, I mirrored sound mathematics thinking and course strategy practices so students' sense of their own competence would become both realistic and hopeful. I provided myself as a good-enough mathematics parent image for students to idealize and model themselves on but I subsequently provided manageable frustrations and disappointments so they could withdraw dependence and grow into their own competence. I found this easier to accomplish with Karen, who presented with symptoms described in 2. (a) above.

I did not deal as well with the students described in 2. (b) Brad, or 2. (d) Kelly because I did not discern soon enough that their root problem also lay with their underdeveloped and vulnerable mathematics selves. With Mulder and Brad, for example, instead of mirroring areas of real competence I tended to act out my countertransference reaction to deflate their overly positive opinions of themselves. My inappropriate approach tended to increase their overt grandiosity and their resistance to or avoidance of the task but with Mulder, supervision advice and my becoming aware in time of the part I played in his resistance, a positive outcome was achieved. With Kelly, I was drawn into her vortex and tried frantically to give her all she thought she needed instead of mirroring her evidenced competencies and providing bounds she could not establish for herself.

Category II students who had suffered some short-term and/or long-term blows to their mathematics selves did have adequate underlying mathematics selves (and knowledge bases) but they had been undermined. They needed not so much to develop their mathematics selves through mirroring and permission to idealize, but needed rather the offer of a secure base and help with repairing damaged attachments (i.e., techniques of the interpersonal attachment dimension, see below).

*Mathematics Counseling and the Second Dimension: Internalized Object
(Internalized Presences from the Past).*

All students' behaviors and expectations are influenced by their prior mathematics learning experiences. But students suffering from the undue negative influence of their internalized mathematical presences (internalized objects) typically behaved towards the instructor or peers or tutor in ways that were incongruent with present realities. In this study Jamie was most affected this way, but it does not appear that a problem in this dimension is restricted to or indicative of a particular student category. I found that certain learning style differences and learning modality preferences could be confounded with a problem with this dimension, however, so assessment needs to be careful. The most important diagnostic data came from:

1. *Investigation of the mathematics knowledge base:* using diagnostic assessments of arithmetic and algebra, class exams, and/or learning modality and style checklists, and
2. *Investigation of internalized mathematics relational patterns* by:

Investigation of *observed behaviors* in class or study group or counseling that seem *incongruent* with the way class members related to each other, the teacher or the counselor/tutor. Clues lie in *Metaphor* and *History Profile*; in my sense of student's transference that was very different from reality [e.g., dangerous to Jamie]; in my countertransference feelings that I should act differently from what I believed would be appropriate [e.g., stay away and not ask questions so as not to cause damage].

I found that discussion of metaphor and mathematics history quickly uncovered bad internalized teacher presences that interacted with personality and caused present

mathematics teachers to seem dangerous, for example. Counseling involved support in close analysis of the instructor and the mathematics counselor to see if they could displace the bad object (presence), and devising relational assignments (cf. Jamie's assignment to ask Ann a question and make an appointment with me) based on a new more realistic evaluation. If a student's life were constrained by extremely bad internalized mathematics presences, however, such a straightforward process would likely not be possible. In that case, the mathematics counselor should not proceed except as a team member with a mental health counselor.

*Mathematics Counseling and the Third Dimension:
Interpersonal Attachments*

Students in the study who had developed insecure attachment patterns with teachers presented as avoidant, overly dependent, ambivalent, or fearful of the teacher or counselor. Certain personality styles seemed to be conflated with this dimension, however, so diagnosis has to be careful. Some students suffered from an insecure attachment to the mathematics; they presented most often as procedural in their mathematics (cf. Autumn), with associated uncertainty about their ability to do mathematics, and separation anxiety in exams. The most important diagnostic data for insecure attachment to teacher or mathematics came from the following:

1. *Investigation of the mathematics knowledge base* using diagnostic assessments of arithmetic and algebra, class exams, and
2. *Investigation of interpersonal mathematics relational patterns* by:
 - (a) Investigation of observed *avoidant or dependent attachment behaviors* in class or study group or counseling. Clues lie in *Metaphor* and *History Profile*; in my sense that a student's transference was keeping me at a distance

personally (cf. Karen), or that she was excessively needy for my presence; or that she was ambivalent; in my countertransference that I should try to gain her approval or that I wanted to escape, or that I was confused and continually moving between the two reactions;

(b) Investigation of a student's apparently *unwarranted insecurity in her ability* to do the mathematics at hand. Clues lie in responses to the *Beliefs Survey* and *Procedural/Conceptual* and *Learned Helplessness* sub-scales; history of intermittent success and relative failure in mathematics; more anxiety on tests than seems appropriate given preparation and mastery of the material. This lack of a secure base in the mathematics seemed to be the result of a history of procedural transmission teaching and never having truly understood the mathematics or a history of having been suddenly separated from a secure mathematics base.

When students evidenced an insecure *teacher* attachment pattern, my counseling role was to provide a secure teacher base where they felt mathematically accepted and safe so that they could begin to explore on their own and risk taking paths that might be wrong, so as to eventually become self reliant. I also needed to help the participant reevaluate the present instructor and her approaches and begin to receive rather than reject her good offerings (cf. Karen^{ix}). With students showing an insecure *mathematics* attachment pattern, my counseling role was to help them rediscover the existing sound basis in mathematics from which they had been separated (cf. Category II students, Jamie, Mitch). These students' mathematics separation anxiety dissipated as their security in the mathematics grew. When I helped insecure procedural learners link their

procedures with the underlying concepts they began to establish their own secure mathematics base.

Summary of Brief Relational Counseling Analysis

I have analyzed here the application of brief relational mathematics counseling according to the dimension of student relationality: self, internalized presences, and interpersonal attachments. I have also shown the interactions among the categories of students according to mathematics self development, which I identified earlier in the chapter, and the counseling approaches that are applicable. It is not possible to establish a causal link between student outcomes and the counseling because of the many variables at play. However, in the process of the counseling, the participants and I did identify relational conflicts by attending to patterns of relational episodes and we attempted to resolve them. As could be expected counseling Category II students required less emphasis on the mathematics itself than for Category III students because their greater level of mathematics preparation enabled them to proceed without as much mathematical support once they were reassured of their competence. Category III students needed more mathematical support throughout and each category of participants benefited from relational counseling to help them resolve the relational conflicts that had arisen over their learning histories. In most cases, counseling worked well enough that focal participants and others felt their originally questionable course prognosis changed and they succeeded.

In chapter 8 I will reflect on my relational approach and its components and suggest directions for further research.

ⁱ Because this is an odd numbered chapter, I use “he,” “him,” and “his” for the generic third person.

ⁱⁱ Catherine reported that she liked mathematics and was confident in her ability to do well. She was more conceptually than procedurally oriented towards mathematics learning (3.5 on a scale of 1 procedural through 5 conceptual, on the pre-*Mathematics Beliefs* Survey and 3.8 the highest in the class on the post-survey). Her presence in the class did not change over the course. She seemed comfortable; she was quietly (contributing only an average of .27 responses or questions per session) confident during the lectures, appearing to be processing and understanding the material; she worked on her own during problem-working sessions but was willing to interact with a neighbor if the neighbor initiated it (e.g., Mulder, Class 7). Since Catherine, unlike Lee, did not seek conceptual links during lecture discussions or problem working sessions nor participate in study groups or individual counseling, she may have been using some of her 5 hours per week of homework time doing that. Her mathematics testing anxiety was initially the lowest in the class (1.5 on a scale of 1 [none] through 5 [extreme]) but increased considerably to 2.1, nevertheless remaining relatively low.

ⁱⁱⁱ RN to BSN .RN = Registered Nurse; BSN= Bachelor of Science in Nursing.

^{iv} She had the characteristics of Davidson’s (1983) Mathematics Learning Style II learner or Krutetskii’s visual-pictorial (see chapter 2).

^v Evidenced in their Level 4 understanding of the algebraic variable on the *Algebra Test* but somewhat variable *Arithmetic for Statistics* responses.

^{vi} From Alain Boublil and Claude-Michel Schönberg’s musical adaptation of Victor Hugo’s novel *Les Misérables*.

^{vii} Karen did have a detached pattern of relating but it was defensive rather than independent and her mathematics depression was more prominent. All Category II, Type A students had issues of control but for Category II, Type B students it was much more prominent.

^{viii} For only four of fourteen questions did Floyd appear to understand what the question was asking for, know the formula, use the formula correctly and achieve a realistic answer; for two questions he used appropriate formulae but did not seem to understand the meaning of the question so substituted incorrect (though not unrelated) elements and achieved unrealistic answers; on six questions he did not seem to understand the meaning of the question; and on the other two he showed only partial understanding of the question and of the procedure required achieving a somewhat realistic answer in one of them.

^{ix} I accepted her anger and her depression and offered mathematics counseling as a secure base from which she could do the course. Towards the end of the course she had developed more self-reliance and chose when she did and didn’t need to come for help. She also changed in her stance towards the instructor learning to appreciate and rely on her. See chapter 6 for further discussion.

CHAPTER VIII

EVALUATION OF THE PILOT STUDY AND RECOMMENDATIONS
FOR FURTHER RESEARCH

When Jamie and I first met, mathematics was a storm that she was afraid would come back so she stayed inside. By the end of the counseling, it had changed to a partly sunny day and she could go out with her umbrella. Jamie's relationship with mathematics changed as she went to class, met with me in counseling, and struggled to understand and resolve the central conflict that had been sabotaging her conscious desire and her ability to do well in mathematics. Through this study I have developed a new way of providing mathematics support over a college semester, one that incorporates relational and cognitive counseling approaches. Here I turn from looking at the particulars of each participant's experience of counseling to looking at what those particulars might tell about the counseling approach itself as it emerged in this pilot application.

People can change. I found that out in this study. By crossing the lines drawn in traditional mathematics support in order to incorporate a relational counseling approach, I first changed how I looked at students and at myself, and they in turn changed how they looked at themselves, at the instructor and me, and at mathematics. We found that we could disembed ourselves from our entrenched theories about ourselves and each other and change our counterproductive patterns of relationship in mathematics learning when we recognized, explored, and challenged those patterns. To do so we each had to cross traditional lines to widen appropriate and useful objects of attention in academic support settings. We had to consciously attend to our relationships.

To explain further, I first evaluate this process of counseling and its elements, and evaluate the student categorizing system that emerged. Next I assess the limits of the

approach as it developed. Finally I discuss the limitations of the study and suggest future directions for research and development of this approach.

Understanding the Student: Who is She¹ and How Do I Know? How the Counselor Role Changed What I Knew

As an educator I used tests and surveys to try to classify each student's mathematics cognition and affect; I took what she told me about herself at surface value. As a counselor I gradually learned to listen not just to her "I" statements but to her behaviors, her metaphors, her "she should" and "they are" and "it is" statements, her transference (the role participants seemed to be casting me into), and to my countertransference (my feeling constrained to act out or react against the role imposed on me by the participant's transference). I looked for links between her history and her current mathematics performance. Jointly we looked at her exams and reviewed her grade, her thinking, her feelings and beliefs, her effort, and her contradictions. The chief difficulties I faced in understanding the student were (a) the now-dynamic nature of our relationship (The student changed, I changed, and our interaction changed.) and (b) the reality and power of the unconscious: The student spoke honestly about her realities, yet there was often good evidence that seemingly contradicted what she said; sometimes she reacted in surprising ways that seemed incongruent with the present reality. These difficulties provided the richest sources for understanding her (and me) as we worked together.

When I heard or found or sensed contradictions, at first I was angry and mentally accused the student of falsehood or cowardice (i.e., Brad's assertion that he earned a B in his last mathematics class when he had actually failed his last class). At times I felt hurt (i.e., Jamie only hinting and only at the end of the class that she was repeating

PSYC/STAT 104). I chose sides and went with one assertion, dismissing or refuting the other side.ⁱⁱ But when I drew on relational counseling insights through supervision and further readings, I recognized these contradictions and the conflict that they created to be equally genuine realities for the student. It was that very conflict that needed to be brought to consciousness and resolved. Formerly, in my educator-only role, I was not aware of wrestling with such nagging contradictions so I had not brought them to the student's attention. They remained as the "elephant in the living room"—known about by both of us at some level but unacknowledged. This lack of awareness or resolution of a student's conflicts may have precluded the possibility of improved mathematics mental health and success in mathematics courses.

When I encountered what seemed to be willful refusal to allow me to see areas of vulnerability or to help change behaviors or approaches (particularly with Autumn and Mitch), I felt frustrated. *I* could see what their problem was—why would they not discuss it with me, explore it, and resolve it? Through supervision some of my blind spots were identified. I had difficulty allowing a student to choose her own path, especially when that path seemed counterproductive to me. Instead I tried to get students to see their difficulties as I saw them and to change. In the cases of Autumn and Mitch, my behavior probably contributed to their becoming more entrenched in what I saw as their counterproductive approaches.

Working through a relational conflict perspective allowed me to understand that students had developed their current patterns of relationships in their attempts to protect and defend their vulnerable mathematics selves. When I brought together my educator and counselor roles I began to understand how a student's mathematics history might have influenced her current ways of functioning within her overall relationality. This

integration yielded the three broad categories of mathematics student (each with at least one subcategory) that I described in chapter 7 (see Figure 7.1).

Developing Categories to Understand Students.

Why Categorize Students?

In my endeavor to effectively support the whole person doing mathematics, I needed to understand the range of variations in students' responses to their mathematics learning histories. The significance of these variations and similarities helped me notice details as part of a whole rather than being distracted by them. These variations also helped me understand that quite different-looking symptoms could stem from similar sources and might call for a similar counseling approaches (i.e., Karen's under-confidence and empty depression and Mulder and Brad's over-confidence and grandiosity both were expressions of underdeveloped mathematics selves that stemmed from mathematics underpreparation and low self-esteem).

Emergent Categories

I developed categorical descriptions that were determined by interactions between a student's history and adequacy of mathematics preparation, and her mathematics self-esteem. My analysis of the case data gathered in this study led me to suggest that the condition of a student's underlying mathematics self may be classified into one of three categories: (a) Category I: A sound functioning mathematics self; (b) Category II: A relatively sound but undermined and vulnerable mathematics self; and (c) Category III: An underdeveloped mathematics self. Mathematics preparation and related self-esteem were the principal discriminator of these three categories (see chapter 7, Tables 7.1 and 7.2).

With my course participants, categories were further refined according to how a student had handled her compromised self-esteem, that is, by developing either malleable or inflexible (or unstable) mathematics relational patterns (see chapter 7, Table 7.3).

These categories are not exhaustive, however. Conceivably if these distinctions are applied to other groups of mathematics students, other subcategories could be identified within one or all of the three broad categories. I suspect that these three broad categories are sufficiently explanatory to encompass all students. It is possible however, that Category III might be helpfully divided into two categories according to whether the student had a low level of the algebraic variable with adequate arithmetic skills or inadequate levels of both. Further support for my categorization scheme is shown in the fact that students from my preliminary research on practice are relatively easily categorized with this scheme (e.g., Mary as Category II, Type A and Jane and Cara in Category III, Type A, see Knowles, 1998, 2001)

Comparison with Other Schemes

The only similar attempt to classify mathematics students is the tier sort Sheila Tobias proposes (personal communication, March 16, 2001; see chapter 4, pp. 131-134). Tobias' first, second, and third ("utilitarian") tiers more or less correspond to my Category I, Category II: Type A, and Category II: Type B, respectively. Her "underprepared" fourth tier and "unlikelies" fifth tier do not comfortably parallel any categories I found but my Category III: Types A and B have "underprepared" and "unlikely" characteristics. Tobias has researched her second tier in science classes (personal communication, April 5, 2003; Tobias, 1990 and her other tier categories come from experience and observation, again primarily based on science students although she applies them to mathematics students. The major contrast in our schemes is that mine is

based on underlying cognitive and relational differences in mathematics self development and learning history, while Tobias' focuses more on current cognitive preferences and behaviors. I find that mine provides more direction for counseling support and intervention.

Evaluating Cognitive Categorizing and Counseling Instruments

The principal means I used in this study to gauge students' mathematics cognitive functioning levels were: course assessments, the *Algebra Test*, the *Arithmetic for Statistics* assessment, and the *Statistics Reasoning Assessment* (SRA) (see Appendix C). All but the last provided valuable data both for categorizing students in order to develop a mathematics tutoring focus as well as for relational mathematics counseling. Taken together the first three data sources helped me sort students into the three categories of mathematics preparation: well prepared, adequately prepared, and underprepared (see also chapter 7, Table 7.2). A most important finding of this analysis for this group of students is that course the first exam grades did not provide in themselves an accurate indication of membership in a category or subtype of a category except perhaps for the students in Category I and to some extent Category III. The *Algebra Test* was a better indicator for the students who took it. It distinguished between students most appropriately described by Categories II and III. Students' arithmetic levels (as gauged by the *Arithmetic for Statistics* assessment and/or arithmetic samples gathered from exams, counseling and in class) discriminated well among all three categories.

The Algebra Test (Brown, Hart, & Kuchemann, 1985; Sokolowski, 1997)

I found the *Algebra Test* (see Appendix C) useful in mathematics counseling with students who scored at high concept levels of the variable but who had also developed negative or ambivalent beliefs about their own mathematical ability—Category II

students.ⁱⁱⁱ A high level provided some proof that their negative views were not accurate, and this was more objective than my reassurances or even the evidence of their coursework. It was worthwhile to use some mathematics counseling time to take this test because their difficulties did not seem to be fundamentally mathematical.

Once I established, for instance, that Jamie's level of the algebraic variable was high and not an issue for counseling, I determined to use this good result to refute her negative beliefs about her mathematics ability. The other three participants to whom I administered the *Algebra Test* during the course were all at Level 4 (see Appendix H, Table H1) and in each case this good result was used in counseling to allay concerns about each one's mathematical ability. Because Category II and III students' first exam results discriminated their category relatively poorly, the *Algebra Test* seemed to provide a more accurate way to clarify early her level of cognitive preparation especially when taken with Exam #1 grades and arithmetic preparation (see chapter 7, Table 7.2 and discussion).

Arithmetic for Statistics (AFS) Assessment

Each of the five students at level 4 or above on the *Algebra Test* took the *Arithmetic for Statistics* (AFS) assessment (see Appendix C) and performed adequately on it on at least seven of the eight categories tested.^{iv} The two students who were at level 2 on the *Algebra Test* (see chapter 6, Table 6.1). Karen and Mulder performed quite differently on the *Arithmetic for Statistics* assessment. Mulder performed adequately on all categories except large integer number sense and statistical sense. Karen however did not perform adequately on *any* category and her performance on operation sense, place value/decimal sense for numbers of magnitude greater than 5, and open ended arithmetical thinking/problem-solving was inadequate (see chapter 6, Table 6.2).

I developed the AFS assessment as the course was proceeding in order to more clearly isolate participants' arithmetical conceptual and procedural difficulties related to the mathematical requirements of the course (see Appendix C). Despite its limitations^v it revealed more precise data about participants' arithmetical issues than I could observe anecdotally in class or counseling. With modification, I believe it should be administered early in the counseling process, so that arithmetical issues may be addressed more systematically with the arithmetically weaker students. Adjusted to satisfy issues raised in endnote v, it should be a useful tool to be added to the *Algebra Test* and used at the beginning of the counseling process. This would help students with specific weaknesses or problem areas that impact their confidence and progress in mathematics.

Statistics Reasoning Assessment (Garfield, 1998)

The SRA was not useful in category placement or diagnosis for the strategic mathematics counseling of students taking PSYC/STAT 104 (see Appendix H, Table H2). Changes in scores from pretest to posttest did not parallel other changes students made over the course. This was not surprising because the course's design, direction, and implementation were not focused on confronting and changing individual students' misconceptions about statistics or probability, which the SRA was designed to measure. The primary focus of my counseling was to support students in their coursework so much of what is assessed by SRA did not match.

*Evaluating Affective and Relational Categorizing and
Counseling Instruments*

College Learning Metaphor Survey

Metaphor writing and analysis quickly provided rich, deep material that was directly relevant for both the participant and for me; it was key in establishing the central

conflict and relational focus (and in some cases the mathematics focus) for a participant's counseling. Given the brevity of available time the quick collection of data that revealed underlying issues was important. All but one participant found no difficulty in creating a metaphor and nearly all were open to jointly interpreting and exploring the meaning of personal metaphors.

The chief limitation in using metaphors lay in my tendency to assume that I understood when I should have remained open and probed more. It was easy to be diverted by other data and in some cases I initially failed to use those data in conjunction with the metaphor in order to see a clear common focus. In order to disembody the student from her own metaphor, both the student and I probed its meaning; explored its links to current practices, reflections, and automatic thoughts; explored ways to change; and finally, the student created a new metaphor to reflect on changes made.

The shared analysis of the meaning of students' metaphors and what I learned of their deeper meaning to the student often provided a unifier or common thread and even provided vital missing clues to the relational conflict, the mathematical focus or both. I discussed these insights with some participants, and we explored the implications together. However, in these cases I initially understood only part of the meaning; as counseling progressed more data emerged from the metaphor in the context of the student's approach to the counseling, to the course, and to the mathematics.^{vi}

With some participants I found the initial link between the metaphor and other presenting data was less accessible to me. Thus I found a conscious formulation of the central conflict and dynamic foci more difficult. For example, along with persistent negativity on the *JMK Mathematics Affect Scales* that contrasted with their course performance, the metaphors of Category III: Type A woman and the two Category II:

Type B students were an important piece in the diagnosis of mild to moderate mathematics depression, despite behaviors that initially indicated otherwise. The Category III: Type A men's metaphors at first seemed active and positive but in light of these students' initial poor performance, somewhat grandiose. However, further analysis revealed that these men's metaphors indicated a sense of being outside of the mathematics; their metaphorical characters used elusive and disconnected clues to try to understand the alien or mysterious mathematics, and I saw that the metaphors truly provided an accurate representation of how the students viewed and approached mathematics.^{vii}

The Category III: Type B student's metaphor held rich though indirect material and early indicated a lack of realism on her part about how she might need to change in order to succeed in the course.

Only one student refused to engage in exploring the meaning of a metaphor that seemed to me directly linked with his problems with the course (a Category II: Type B student). Even so, I was able to use the insights I gained to provide interventions such as giving him the *Algebra Test* to reassure him that he had the ability to opt out of his metaphor by passing PSYC/STAT 104.

JMK Mathematics Affect Scales

Both content and structure of this instrument made it extremely useful in the counseling situation (see Appendix B). The scalar design allowed for open-ended responses and its repeated use proved invaluable. The range from positive to negative allowed students to see their changes over time. Our shared discussion linked these changes to changes in their life circumstances, personal decisions, automatic thoughts, and unconscious patterns. Scale topics focused on students' immediate sense of their

mathematics self, world, and future (cf. Beck, 1977). The selected topics proved to be important but I found that they were difficult to address verbally at each session. My asking direct questions might have been perceived as accusatory or confrontational, and asking them at *each* session might have seemed to be nagging. The use of these scales avoided that conflictual situation.

People who are negative about themselves, their world, and their futures often rate themselves more negatively compared with their peers than may be warranted. To measure this I would add a new item to the JMK scales to investigate this perception: Compared with **others** in this class, I do mathematics better than/as well as/worse than most of them (see Appendix B for the original and revised versions).

There seemed to be a relationship between some students' metaphors and their responses to the *JMK Scales*. When a student's metaphor was negative, stable, and either passive (e.g., "cloudy" or "overcast") or indicating persecution (cf. Inspector Javert) there seemed to be an underlying mathematics depression as measured on the *JMK Scales*, yet when a student's metaphor was negative but unpredictable (e.g., storm), mathematics depression did not seem to be generally present—anxiety seemed to be more of an issue.

Beliefs Survey

I found each scale: *Procedural* vs. *Conceptual*; *Toxic* vs. *Healthy*; and *Learned Helpless* vs. *Mastery Oriented*, taken with other data, to be especially relevant for different participants. The first scale differentiated Category I students from the others and discriminated somewhat between Types A and B in Categories II and III. In most cases when a scale was highlighted with a participant in counseling, she became more aware of its relationship to her approach to the mathematics. She was usually able to clarify how it was manifested in mathematics testing and learning situations. From this

she often became more able to change in positive directions. If a post-course meeting to discuss post-scores, changes, and current beliefs had been possible with each participant, developing and discussing a long-term plan for each participant's mathematics future might have been feasible

A cluster of questions surveying learning versus performance motivation emerged in the post analysis as a discriminator between Types A and B in both Category II and III students, with Type A students being more learning-motivated than Type B students. A revised short *Beliefs* survey that highlights this factor is presented in Appendix C (*My Mathematics Orientation*).

Feelings Survey

Each of five students (except Lee) who reported very high mathematics testing anxiety^{viii} signed up for mathematics counseling during the pretesting session at the second class of the course when I offered counseling to all. Lee initially refused counseling but contacted me just before the first exam requesting support, citing her mathematics anxiety. These students also had the highest abstraction anxiety scores in the class (from 3.2 through 4.2 on a 1 through 5 scale). Jamie was the only one of these five for whom we eventually established the primary relational focus to be anxiety and the only one whose anxieties on this instrument all decreased substantially. I found this instrument to be useful in conjunction with other instruments in establishing a diagnosis although it did not seem precise when used alone (see Appendix C for the *My Mathematics Feelings* survey).

Mathematics *testing* anxiety of the class increased slightly overall but the class' average responses to individual items are of even more interest. "Signing up for a math course" or "Walking into a math class" now evoked considerably more anxiety than at

the beginning of the course (from 2.6 to 3.1 and from 2.1 to 2.7 respectively). In contrast, “Waiting to have a test returned” and “Receiving your final math grade in the mail” now created considerably less anxiety than at the beginning of the course (3 to 2.6 and 3.1 to 2.4 respectively). When taken with other evaluative data, these responses seem to indicate that in the context of *this* class students’ anxiety levels had decreased as their control and achievement had increased but that this improvement did not generalize to *future* or *other* mathematics classes. In fact, the prospect or memory of other courses now evoked more anxiety.

I paid little attention to the *number anxiety* results during the study since all number anxiety mean responses were at or below the mid point (3) of the scale and thus seemed to indicate low to moderate anxiety, especially when taken in contrast to the reported testing and abstraction anxieties that went as high as 4.1 and 4.2 respectively on the pretests. I realize now that the two participants whom I early recognized to have poor number and operation sense (Kelly and Karen) had the highest number anxiety scores in the class at 3 and 2.9 respectively. Lee, who was considerably more competent arithmetically than they, had a relatively high score of 2.8. On the other hand, on the *Arithmetic for Statistics* assessment, although Lee was generally adequate, she nevertheless had a marginal operation sense that likely contributed to heightened number anxiety. I would now flag scores in the middle of the number anxiety scale for immediate investigation of a student’s number and operation sense.

For some individuals the changes in their feelings on survey responses confirmed the direction and efficacy of the mathematics counseling. For example, Jamie’s testing and abstraction anxiety showed an overall significant decrease, with some aspects increasing while others decreased (see Appendix H, Table H3). In chapter 6, I discussed

the course of counseling with Jamie in detail. Her dynamic focus was specifically *social anxiety*^{ix} that was intensified in the mathematics learning environment, and it was this and her related practices that we worked together to change. All aspects of Jamie's testing anxiety that had a self-focused social public component ("Walking into a math class," "Raising your hand in a math class to ask a question,"^x and "Waiting to have a math test returned.") decreased over time. The aspects of Jamie's abstraction anxiety that had a self-focused social public aspect also decreased. By contrast, Jamie's anxiety about taking a final math exam in class increased. This seemed to have a mathematics self-competence, performance focus for her rather than a social self-focus.

The Role as Relational Counselor Transform My Tutoring Work

Counseling Use of Transference and Countertransference

The new need to attend to transference and countertransference immediately gave me conscious access to a fund of analyzable and usable data that I had previously largely ignored in mathematics tutoring practice. In typical tutoring situations, transference usually remains implicit as both student and tutor often continue to act out old patterns of interaction without the conscious reflection that my new approach encourages. Relationship patterns based on the student's internalized teacher presences of the past may pull or push the tutor into assuming the teacher role they demand. She may on the other hand react against assuming roles she believes are toxic for the student's mathematics mental health. Because these relationship patterns are not brought into the open the student may resent the refusal of the tutor to take on the expected role and the student's expectations are not realigned. But my new approach to the participants in this study allowed for this material to be brought into the open and dealt with consciously so that we could each adjust to more productive ways of relating.

In the academic setting, my use of insights gained from transference and countertransference was necessarily quite different from a psychoanalyst's use. Since the focus was on the mathematics and not on the resolution of personal psychological problems, interpretation and specific working through of transference was not appropriate. What *was* appropriate was noticing it and checking with the participant about shared insights. Most important was looking at patterns of interactions over time including the transference and countertransference so that a central relational conflict could be identified.

I found it challenging to attend to the student's transference and to my countertransference. In the past, I had found myself on occasion acting in ways that surprised and concerned me—for example, believing whiners and joining them, almost doing a student's work for her, agreeing to work with a student much more than appropriate, scolding, or panicking with them. It had not at all been my practice in the mathematics tutoring situation to consider what these behaviors might be telling me about the students' history, personality, approach, and practices, nor to consider my behaviors. During this study I needed to develop this reflection as a new practice. In the relational counseling situation, even in brief counseling, it is usual for counselor and client to discuss the transference and countertransference. Where the focus was mathematics learning, would that be appropriate or necessary? In the brief counseling situation in a college setting, the challenge was to establish for myself parameters for if, when, and how to use the transference and countertransference material in counseling with the student. I found that the following practices were appropriate in the mathematics counseling setting and allowed for effective use of the data from both transference and countertransference:

Incorporating data from instruments and from observation to consider students' conscious and unconscious expectations about their relationship with the present mathematics teacher and tutor. With every participant, I listened for, observed, and asked how she experienced the present class. I asked students questions about their past and the present to help them discover the ways they might be appropriately or inappropriately bringing their past into the present.^{xi} If it became clear that a student's experience of the class was discrepant from the present reality I drew her attention to it and invited her to consider how she might adjust to this new awareness.

Developing reflection and self-awareness regarding countertransferential reactions to the tutees. I filled out the mathematics counseling session summary sheet after each session to help me reflect immediately on the session. I listened to my tape recordings of counseling sessions and study groups and studied transcripts in order to observe myself in relation to participants. Supervision was central in some cases to recognizing my countertransferential reactions. Not surprisingly, my session notes were often ahead of my counseling practice.

Using self-revelation of countertransference. I found that when I did self-reveal in the counseling situation, both the participant and I became clearer about the relational patterns that might be keeping us both stuck. We were then more able to change our behaviors and to extricate ourselves from counterproductive patterns.^{xii} I found that I needed to present my experience of countertransference in a manner compatible with student's learning style or risk her not understanding and optimally benefiting.^{xiii}

Indirect use of transference and countertransference observations in situations where the student rejects or avoids a counseling approach (Category II: Type B students) or has a detached avoidant relating pattern (a Category III: Type A student in this study).

I could only talk about transference and countertransference *indirectly* with some students by noticing behaviors and perceptions and asking them to verify whether they were seeing the present relationships as different from the past, inviting them to notice the present relational reality, suggesting they evaluate the appropriateness of their beliefs and practices in relation to the present reality, and affirming their helpful choices and changes to appropriately deal with present reality.

Supervision by a person knowledgeable in counseling. I needed a knowing dispassionate ear to share my actions and judgments, particularly my subjectively experienced transference and countertransference. Preparing my cases for supervision forced me to reflect on each participant in a more global way than I had till then. Supervision itself provided me with affirming and challenging feedback on my progress thus far with each participant. It forced me to pay close attention to my own reactions and my tendency to impose my agenda on participants rather than facilitating their own choices and movement.^{xiv} It furnished me with possible new approaches for stuck situations (cf. paradoxical intention for Mulder). An even earlier supervision meeting may have helped me decide to do things differently from unwittingly acting out my countertransference.^{xv}

Counseling as Good-enough Tutor-Parenting

Winnicott's concept of good-enough freed me in a number of ways to be more available to my tutees and to help them be strategic in their choices. I am not neutral with respect to procedural (only) versus conceptual (including procedural) mathematics pedagogy, for example. In my experience, conceptual learning helps make students secure in their mathematics base. The tutoring role differs from the teaching role in that control over the curriculum lies with the teacher not with the tutor; the tutor must support

the student in mastering the curriculum whether the tutor “approves” of the curriculum or not. In the context of the 10- or 15-week mathematics course where students had the opportunity to struggle on problems in class with coaching support from the instructor, the conceptual aspects were only linked with the mathematical procedures when individuals asked the instructor during problem-working sessions. Opportunities in counseling to help tutees attain a more conceptual understanding of the material were limited by time and content, especially with students who were already deeply embedded in a procedural approach. I found that to support a student in doing well on a PSYC/STAT 104 exam, there were times when procedural advice superceded conceptual. I was able to see my mathematics tutoring as good-enough in providing for my students although it was less than (my) perfection. In line with good-enough parenting I also had to learn to better tolerate students’ mathematical goals when they differed from mine in contrast to my former approach of trying to badger or cajole them to take on my goals for them. On the other hand, I had to be careful not to allow this good-enough concept to lull me into lowering my expectations for what they could achieve.

Challenges and Limitations of this Approach: Integrating Counselor and Tutoring Roles into Mathematics Counseling

I found that to be a good-enough mathematics counselor is very difficult. My “successes” from my long enculturation and experience in traditional mathematics teaching had only relatively recently been called into question by the nagging failures that drove me into my doctoral program. A cognitive constructivist, conceptual, problem-solving approach to teaching and learning mathematics was the solution, I was convinced, but I found it difficult to be that teacher, to facilitate that learning. I had always been the one who worked out what the problem was and structured the solution

and told the student, who ran with it, or puzzled over it, or denied it, or ignored it. I grew to believe that the essence of constructivism was in the *student* seeing the problem and, with the teacher as guide, finding a solution for herself, but how to be a guide? I now know that *telling* spoiled it by making it mine and not hers (even if I was “right”). Now I have discovered that counseling is the same. I had learned through lay counseling ministry training and experience that a constructivist approach was essential for healing and growth. That was confirmed in my doctoral psychological counseling coursework. Now in this study I had to integrate my emerging but tentative constructivist teaching role with a constructivist counseling role to be a good-enough mathematics counselor.

The Challenge of Learning to be a Relational Counselor

Mulder taught me about counseling perhaps more than any other participant because he would not accept my *telling*; he resisted it and stood up to me and I learned to step aside and let him fight his own battle. Not that my input was not helpful—indeed it was! On his own, it is almost certain that Mulder would not have made the changes he did but in the end they were his *own* changes. If I had not stepped aside he may not have made the final crucial change. With other participants my propensity for prescribing my solutions for them was not as clear to me although my experience of transference and countertransference gave me clues. Dr. P. saw it and helped me to begin to see it in supervision. With some participants, though, it was only as I analyzed the transcripts, my session notes, other data, my own initial analysis of the student’s needs, and Dr. P.’s persistent supervision-style queries of that analysis that I finally heard myself telling and scolding and prescribing. And I finally realized how I could have done it differently, in a constructivist manner, because along with the telling I did some of that (work in a constructivist manner) and Dr. P. also pointed that out to me. All along I had the insights

and approaches of relational counseling to use; when I did use them students did well and found their own feet. In the end what I did with each participant (except for Kelly, Brad, and perhaps Autumn) was good-enough for them to gain insight into their restrictive mathematics relational patterns. This equipped them to make the changes necessary to succeed in PSYC/STAT 104.

The Challenge of Learning to be a Relational Tutor

I learned relational mathematics tutoring from the participants, especially Karen, Mulder, and Lee. I found that when I used constructivist, relational counseling approaches such as mirroring sound thinking (even in the midst of errors or low grades) to build up tentative and vulnerable mathematics selves, participants began to move into a competence they did not know they had and then to develop that competence. When instead I was drawn into participants' focus on the negatives (the errors or the low grade) and tried to fix it by telling the answer and teaching them more, I cut them off from that tentative mathematics self so that it could not grow.

Likewise when I heard their mathematical questions and responded to their pressing felt needs by telling, things did not go well; when I responded by eliciting from them what they already knew and we went from there (e.g., parallel problem-solving), they grew. In the end what I did with each participant (except for Kelly and Brad) was good-enough for them to gain access to their growing mathematics competence, develop insight into counterproductive mathematical beliefs and practices, and make the changes necessary to succeed in PSYC/STAT 104.

Relational counseling is based on the idea that the counselor and client are both adults, and the client chooses her path while the counselor supports her. The tutor-tutee relationship is usually an expert-novice relationship with regard to the mathematics

content and (theoretically at least) a novice-expert one with regard to the student's own affective *experience* of the mathematics. I had been learning how to negotiate the mathematics content in a constructivist, reciprocal way, but not the student's affective *experience* of the mathematics. In my prior tutoring practice the tutor-tutee relationship with regard to the student's own *experience* of the mathematics was more often a parent to child one. From this study, I found that *that* is the challenge for me in the practice of mathematics relational counseling—to learn how to be constructivist, non-directive, and supportive, while also learning from the student. This was needed not only when we dealt with the mathematics content, but also when we explored and gained insight into the affective areas of her mathematics cognition and her underlying relational patterns.

Just as a crucial assumption of this approach is the reality of student choice and responsibility for choices, this assumption applies equally to the counselor. The benefit for the student is in helping her become conscious of her choices and the extent of her power to choose differently. The danger in this approach is to appear to hold a person responsible for things she has little power to change. Thus Jamie could choose to sign up for individual mathematics counseling, but the shyness and prior negative experiences that dominated her interactional patterns, led to her choosing to hide and disappear rather than relate and approach. Because she was not consciously aware that she was making that choice, she seemed to remain powerless to choose differently. My choice to approach her was perhaps going against one of the maxims of counseling (Wait for the person to seek your help; that will mean she is ready and willing to receive it.) but because my choice was good-enough in this case, Jamie became aware of her choices and her power to chose differently. At other times my choices to be parent rather than peer with the

student were not good-enough (cf. Brad and Autumn). My awareness of my own power to choose my roles and the importance of my choices grew as the study proceeded.

BRIEF RELATIONAL MATHEMATICS COUNSELING: A SUMMARY

The traditional model for providing mathematics academic support typically excludes from consideration many aspects of the student's relationship to mathematical learning and compartmentalizes what *is* considered into content knowledge, and some aspects of affect. Tobias' concept of mathematics mental health provides a different perspective for viewing the struggling mathematics student. A brief relational counseling approach prioritizes students' mathematic mental health problems and provides a means of dealing with them in a holistic and productive way, without ignoring or minimizing important elements. My categorization scheme uses both the relational and cognitive diagnosis as a way of understanding and dealing with complexity.

It is important to highlight here crucial ways that this new approach differed from more traditional approaches to mathematics support. But first I must point out that this study alone, while it puts forward considerable evidence of student change, does not provide quantifiable comparisons between the effects of this approach and the effects of traditional mathematics support. For comparisons, the sample was small, there was no matched sample to receive control treatment (traditional mathematics support) and changes reported and the processes that led to these changes were in many ways not quantifiable nor easily verifiable. Nevertheless the differences I observed were striking.

Needy students do not necessarily access traditional mathematics academic support. Jamie would likely not have opted for academic support unless meeting with a tutor were a course requirement or there was a class-link tutor she could get to know first. Jamie's dilemma presented me a disturbing possibility that I had previously only vaguely

considered—there are likely unknown numbers of students who might benefit from this course intervention but would never come to a traditional Learning Assistance Center;

Class-linking provides unique observation opportunities for more thorough and effective counseling. Being able to observe student behaviors and practices in the classroom^{xvi} provides the tutor with data for timely and focused counseling interventions. If Jamie *had* come to a traditional Learning Assistance Center, the tutor would not have had the benefit of observing her in class, nor have been aware of how important that observation was. In my former role I would likely have worked with Jamie on her mathematics and found it to be relatively sound. We would not have found the real root of her mathematics learning issues nor explored ways she could see herself differently as a successful mathematics learner. Other students in the study who needed assistance also indicated that they would not have accessed my help if I, as the tutor, had not been in the classroom.^{xvii}

In contrast with current learning support experience, early and thorough diagnostic assessment of both mathematics cognition and mathematics relational issues is possible. Such timely diagnosis is key to growth in mathematics skill and improved mathematics mental health for students who are willing to explore both. Even for students who do not wish to explore their mathematics relational problems, their assessments can be used to design mathematics-only counseling interventions that assist them to make academic and indirectly, relational progress;

Without a relational counseling approach focused on the student's transference and the counselor's countertransference, the counselor might not be able to identify and deal with students' core mathematics mental health issues. If I had not identified Jamie's transference towards Ann and me as frightening, dangerous teachers, I might have acted

out the same kind of countertransference that Ann did (staying away in order not fulfill Jamie's fears). Instead I broke through and came close and was not dangerous;

For counseling to be efficacious, the importance of the classroom emotional climate established by the mathematics instructor cannot be underestimated. The positive emotional climate established by the instructor in this study created an environment for most students where damaged attachments to mathematics teachers could be repaired, where underdeveloped or vulnerable mathematics selves could grow, and where no further damage was done. By contrast, counseling students taking a course where they experience the instructor and the classroom as indifferent or abusive would likely have to take a different direction and would invariably have reduced efficacy in achievement and emotional healing for the student.

A counseling use of the mathematics addresses the various mathematics mental health problems caused by the ways the mathematics content has been and is being taught. Because a procedural approach to mathematics is closely related to conceptual linking difficulties (i.e., linking procedures with their conceptual base) and a tendency towards learned helplessness in the mathematics course environment, individualized mathematics-focused counseling approaches (e.g., mathematics course management and conceptual problem-solving counseling) may be called for. One use of exam analysis counseling is to help negative students break a negative focus by affirming or mirroring sound mathematical thinking, thus building up their underdeveloped selves. Conceptual linking counseling offers students a secure mathematics base they may not have previously experienced. Such intentional uses of mathematics tutoring as counseling contrast with traditional uses and demonstrate promise for improving the student's mathematics mental health.

LIMITATIONS OF STUDY

It is important to evaluate the conclusions of the study in terms of the sample, the measurement instruments and their uses, and the research methodology.

Sampling limitations. The number of participants in the study was small and they were taken from a small urban New England commuter university. The nature of the study necessitated a small sample but the fact that students were from widely varying backgrounds enhances its value. The small size of the sample restricted the use of quantitative results of the instruments to descriptive support for qualitative results within the sample, aiding the ongoing counseling process, and understanding of individual outcomes. The findings from the instruments may not be generalizable to students enrolled in other mathematics or statistics courses nor attending other types of college or university although uses of some instruments (e.g., the *Metaphor* and the *JMK Affect Scales*) seem applicable for counseling purposes in any setting.

Quantitative instrument limitations and uses. The only quantitative instrument reliably calibrated on large samples was the *Algebra Test* (see Appendix C). Apart from the first 20 questions of the *Feelings Survey* that were taken from the 98 item MARS which was normed thirty years ago (Suinn, 1972), all instruments with quantitative outcomes, except for the class exams and other class evaluations, were created or adapted and the results evaluated by the researcher. Individual results were compared with those of the (small) class group and individual changes are described in comparison with other researcher-observed changes and class achievement changes.

Researcher bias. The participants' words and actions were filtered through researcher bias. Relational data were collected via counseling session interaction where the researcher and the participant were working for change, and the interactions did

change both. Although ongoing analysis of the interactions by the researcher (with participant feedback) and clinical supervision (and later evaluation of researcher analysis by the clinical supervisor) were designed to monitor, interpret, and neutralize this bias, interpretations of the data by others might yield different conclusions about the relational outcomes of the study.

Possible omission of important student factors. My understandings of key aspects of mathematics functioning (cognition and affect)^{xviii}, were applied, integrated, and adapted in this study. I adapted Mitchell's relational conflict theory and his concept of three dimensions a person's relationality, and revised Tobias' five tier categorization of college mathematics students. In addition to Dweck, Seligman and Beck's work on learned helplessness and depression, the researcher's own findings were also used to ground the study. None of these, separately or together, has been used in a holistic study of college students' mathematics mental health or of interventions to improve it while the student was taking a college mathematic course. Because of this it is certainly possible that important aspects of students' mathematics mental health were not addressed. All findings in this study should be interpreted in this light.

RECOMMENDATIONS FOR FURTHER RESEARCH

Based on findings in this study and the limitations, I make the following recommendations for future research:

Counselor characteristics subtype B students and counselor-student match. My use of brief relational mathematics counseling helped students from all three categories identified. Students who benefited less were from subtype B of both Category II (Autumn and Mitch) and Category III (Kelly)—students whose ways of dealing with the vulnerability and under-confidence of their mathematics selves were relatively inflexible

or unstable. Counselor characteristics may have been a factor in this relative lack of success. Additional research needs to be done when offering this counseling to such students. It will be vital to investigate counselor characteristics, counselor-student match and interventions that may help them succeed.

Quasi-experimental studies. My adaptation of Mitchell's (1988) relational conflict theory to mathematics support in this study yielded an understanding of the three dimensions of participants' relationality and their central relational conflicts that enabled us to resolve that conflict well enough in the brief time available for them to be successful in the course (with the exceptions noted above). It is not possible to say what their outcomes would have been if they had not participated in the counseling intervention. Additional research using this approach with other college mathematics students, comparing their process and outcomes with those of matched samples of students who receive tutoring support only, and with matched samples of students who receive no support would further our understanding and test its generalizability, particularly the finding of increased achievement and improved mathematics mental health.

Gender differences. I found interesting gender differences in the way similar core problem were expressed. Men and women, especially those within Category II: Type A and Category III: Type A groups, whose core problems were the same, differed markedly in their emotional conditions, practices and ways of relating. With Category II: Type A students, the women expressed their insecure attachment to mathematics with anxiety; the man expressed his with a frantic and counterproductive attempt to learn it all. With Category III: Type A students, the woman expressed her underdeveloped mathematics self with underconfidence, empty depression and hostile detachment; the men expressed

theirs with overconfidence, unrealistic bravado (grandiosity) and resistance. Further research on gender differences within and across types may be called for to confirm (or disprove) that the presentation of similar core problems consistently differs predictably according to gender.

Counselor-student match and gender. I had considerable difficulty in overcoming my countertransference reactions to grandiose men of Category III: Type A. I wanted to deflate their inflated sense of prowess. My reactions seemed to come at least in part from our gender difference. One of these men, who spoke of his difficulties working with women, made only minimal progress in counseling. I wonder if a male counselor may have been more successful in supporting and developing his underdeveloped mathematics self. Research into the effects of counselor-student match by gender could shed light on this.

Identifying mathematics situational depression. This study suggests that some students may suffer from mathematics situational depression. The *College Learning Metaphor* (pre-and post) and repeated use (administered at every session) of the *JMK Mathematics Affect Scales* analyzed together seemed to aid diagnosis and help to monitor this condition. Further, it can alert the counselor to a need for a specific and timely intervention. Ongoing research using both the *College Learning Metaphor Survey* and the *JMK Mathematics Affect Scales* conducted with large numbers of students would investigate a possible relationship. Such a finding would investigate the simultaneous use of both instruments to rule out mathematics depression and aid accurate diagnosis

Mathematics relational counseling and other classroom conditions. This study was conducted in the context of a classroom where the instructor created a positive relational climate, where the mathematical demands were somewhat more procedural

than conceptual, and where the conceptual content was taught by lecture discussion and the mathematics procedures developed in problem-working sessions. Relational counseling in other contexts is likely to look different and have different outcomes for different categories of student. Classes may differ in how mathematics is taught: they may stress non-routine problem-solving; they may be designed to challenge student misconceptions; they may involve mathematics procedures only being demonstrated by the teacher on the board. Classes may differ in relational climate: the teacher may be disdainful of what she perceives to be students' low ability and poor understanding; she may be judgmental of certain student approaches; she may ignore or insult student who struggle. Further studies of the use of relational counseling to support students in different settings according to how mathematics is taught and according to relational climate would contribute to our understanding of the efficacy and limits of its use.

CONCLUSION

I learned during the summer of 2000 to open my learning specialist eyes wider and to see through the lens of relational conflict theory. This at once complicated and clarified my task. New complexities arose in having to look now not only at the student's mathematics tasks but also at her whole approach to the mathematics course, her mathematics self, her internalized presences, and her patterns of mathematics interpersonal interactions.

In this study I determined it was indeed possible for a mathematics learning specialist with some exposure to the field of psychological counseling to holistically help traditional and non-traditional aged college students taking an introductory level college mathematics course. Most came to understand their mathematics learning issues and found their own coherent explanatory frame for how the aspects of mathematics

cognition were personally configured within their relational history. Students became conscious of their areas of embeddedness as well as how they contributed to their own immobility, they made changes, and they improved their mathematics mental health. They attained “good-enough” success in the current course and some even seemed to develop the heart to tackle future mathematical challenges more effectively.

I found that the concept of relationality with its three dimensions did provide an adequate frame for me to understand and focus on each participant’s particular relational conflicts and I found that the approaches of relational and cognitive therapy were useful as elements of an approach designed to address those difficulties. The approach highlighted my role in the counselor-student dyad and I found that to the extent that I reflected on how I reacted and interacted with the student, the insights I gained led me to change in ways that promoted student growth. In sum, I found that the relational conflict perspective has given me a new, more nuanced, and authentic way of seeing students and helping them to see themselves and their interactions, in the academic support and course classroom setting.

ⁱ As this is an even numbered chapter, I use “she,” “her,” and “hers” for third person generic pronouns.

ⁱⁱ For example, I agreed with the Karen who said she probably could not do it and at first ignored the Karen who expressed and showed surprising competence; I agreed with the Mulder who said he could do it and dismissed the Mulder who expressed frustration and struggle.

ⁱⁱⁱ Each of these students (Autumn, Jamie, Lee, and Mitch) was ambivalent about her own mathematical ability. None saw him/herself in the category of “some people can do math” (question 9, Part II, *Mathematics Beliefs* survey) although all but Jamie initially believed her “ability in mathematics” could improve (question 8, Part II). The *Algebra Test* is not a test of ability; it shows a student approximately where she is on a developmental path. I was wary of feeding into any fixed trait beliefs about mathematical ability even the “I’m one of those people who *can* do mathematics” belief that saw others as *not* being able to. In fact, I took every opportunity to dispel such fixed trait beliefs. I deemed a developmental view that saw growth of self and others as always possible and expected improvement in relation to intelligent effort as much healthier. In the context of a 10 week college mathematics course, however, being at a higher algebra concept developmental level certainly gave a student an advantage over a student at a lower level.

^{iv} Two of these five did not exhibit an adequate operation sense (Pierre’s was inadequate and Lee’s was marginal), one did not exhibit adequate open ended arithmetical thinking /problem-solving (Autumn was inadequate), and one did not exhibit an adequate large (>1000) integer number sense (Jamie’s was marginal).

^v I found the AFS assessment to be too long, with a number of questions not relevant to this statistics course or discerning enough. The graph related questions not varied or discerning enough, there were not enough operation sense questions, and relevant categories such as order of operations were not addressed. In addition, within each category, the questions were not designed developmentally to reveal levels of understanding.

^{vi} For example, Jamie’s “fear of the storm coming back” metaphor was not principally about the mathematics itself but about a dangerous classroom environment with dangerous teachers. It took us some time to link this with Jamie’s own behaviors in the storm—staying inside in order to keep safe in this dangerous situation. Mulder himself linked his “Fox Mulder searching for aliens” metaphor with making mathematics hard for himself, so that became our initial focus—the *ways* Mulder did mathematics that made it hard for him. We at first missed the link for Mulder between the object of the search—aliens—and mathematics. An important piece to Mulder’s difficulties was that he was indeed seeing *mathematics* as alien, so he was using alien search techniques to master it rather than exploring and mastering it logically and conceptually.

^{vii} Mulder’s metaphor was Fox Mulder searching for aliens and Brad’s was Sherlock Holmes trying to crack a mystery.

^{viii} Each had an average score of 3.5 or above on a scale of 1 through 5.

^{ix} This social anxiety was related to and complicated by Jamie’s fear of too much success that invited attention, expectation, and pressure from her father for future performance.

^x Jamie answered 4 on her pretest on this but on her posttest she answered: 3 (not in front of class, individual work), 4 (our size class) and 5 (math lecture size class like at State University)

^{xi} I did this somewhat unevenly. For example, I indulged Lee because we were pals (staying with her transference and my countertransference); I did not become conscious enough of how her positive feelings towards me contrasted with her negative feelings toward the instructor so I did not help her to evaluate that against the reality and we did not ask ourselves how our reactions might have a detrimental effect on how she approached PSYC/STAT 104. We should have, for example, questioned the discrepancy between her effort in the class and on homework versus how much time she was spending with me.

^{xii} For example, with Mulder, I told him I felt like his scolding mother—that helped me in my process of moving out of that inappropriate role; I *could* have done it more clearly with Karen, e.g., “When I listen to what you say about yourself doing mathematics I feel depressed, but when I look at the mathematics you are doing and the ways you are changing I admire you and am hopeful that you can learn it and get a good grade in the class.” I believe that this would have had a positive effect on the development of her underdeveloped mathematics self.

^{xiii} For example, with Robin a visual learner with auditory processing difficulties, when I told her of my dual reactions: that in class she seemed to be acting as an intellectually incompetent female and I felt irritated and alarmed at the same time, whereas one-on-one in the mathematics counseling situation I found her to be intellectually competent and I admired and respected her ability to manage the content, she did not understand because I did not use visual pictures that she had introduced in her metaphor of the ditzy village girls versus Belle.

^{xiv} During supervision my strong frustration with Autumn became apparent and my judgment of her self-containment, her emotional distance, “I want to shake her” (Jillian, July 20, 2000). I found myself reporting a very similar reaction to Brad, but more for his refusal (from my perspective) to face the reality of his situation, and his bravado in the face of the realities. With both of these participants I had been very directive, partly the cause, Dr P gently suggested, for their digging in and my frustration. His reminding me that it was *their* motivations for change or stasis that needed to be revealed and respected and not *my* motivations for them imposed. He encouraged me to find what in me had been triggered by their behaviors and attitudes.

^{xv} In my session notes, I found that my insights and determinations were at times ahead of my actions. Earlier supervision might have alerted me to those discrepancies and led to a different approach with Brad or a timely intervention with Kelly.

^{xvi} What being a class-link contributes to the efficacy of the brief relational mathematics counseling model is: Counselor presence in the central context, leading to

1. Participation by the students in the counseling,
2. Inside perceptions of the instructor and the Course that could be used in counseling, and
3. In-class perceptions of student practices that could also be used in counseling

^{xvii} Seven of eight participants who finished the course and who responded to my post-study survey (December 2000) acknowledged that they would not have accessed mathematics learning assistance at the Learning Assistance Center if I had not been in the class with them. Of these seven, three, Karen, Mitch, and Pierre, said they will now access the Learning Support Center in the future when they are taking a mathematics course, one of these though, Karen, only “...as long as Jillian is there because you really helped me in my last math class.” Another, Lee, indicated a very conditional, “if I do [take another math course], and I find it difficult, I will use the Learning Assistance Center facilities if I think they will best help me.” The others, Autumn, Jamie, and Mulder, said they probably would not access the Learning Support Center in the future when they are taking a mathematics course, Jamie because she is “kind of shy” and a “helper” who doesn’t “really like to ask for help,” and Mulder because “Unless I am really struggling I do not go for help. I like to figure things out on my own.” Autumn gave no explanation.

^{xviii} Suggested by Schoenfeld (1992), Dweck (1975; 1982; 1986), Tobias (1993), Skemp (1987), Boaler (1997) and others.

Appendix A: Table A1

Proposed Brief Relational Mathematics Counseling

Relational Dimension	Student's Mathematics History		Mathematics Affect Now	Mathematics Cognition Now	Relational Diagnosis	Proposed Treatments
	How does this develop?	What can go wrong?	Expected Affective symptoms	Expected Cognitive symptoms	Expected Central Mathematics Relational Conflicts	
Mathematics Self	1. Mirroring 2. Idealization of teacher image 3. (a) Internalization and transformation of teacher image into values and ideals...superego, (b) manageable frustration => development of student's competence, confidence and basic mathematics self-esteem: healthy narcissism	1. Neglect ... chronic lack of mirroring => underdevelopment of self: low self-esteem, under-confidence 2. Failure to provide realistic curbing of grandiosity => underdevelopment of self: low self-esteem, ambivalent/over-confidence	1. empty depression; learned helplessness 2. grandiosity	1. sound mathematics preparation 2. adequate mathematics preparation 3. underpreparation	<i>Self: Conflict</i> between conscious ambition/desire to succeed in course and underlying belief in inability to succeed in course (low/undermined self-esteem)	<i>For Self:</i> Counselor mirrors student's mathematics self, provides self for idealization, provides manageable frustrations to push student to development & realization of competence
Mathematics Internalized Presences	1. Installation of bad internalized teacher presence in the unconscious (Note: degree of badness) 2. Identification of the ego with the bad internalized presence (Note: extent of identification) => development of defenses to protect the ego from these bad internalized presences	1. Experience of endangerment by bad-enough teacher => moral conversion to self as bad internalized presence or repression of bad internalized teacher object 2. Experience of mathematics as punitive internal saboteur: superego => sense of moral failure	1. guilt, shame => depression 2. fear of judgment => anxiety	1. internalized presences supportive or at least not detrimental to mathematics self and internalized mathematics values (superego) 2. internalized presences undermining mathematics self 3. punishing mathematics superego ("internal saboteur") making mathematics self feel guilt/shame	<i>Internalized Presences: Conflict</i> between conscious desire to and perhaps belief in self for success in course and Internalized presences insisting that one is bad/cannot succeed	<i>For Bad Internalized Presences:</i> Counselor provides self (and points to instructor) as good replacements for bad presences and refutes claims of bad internalized presences
Mathematics Attachments	<i>Mathematics teachers:</i> 1. Teacher provides good-enough caregiving: responsive & available => teacher as secure base 2. Student develops secure attachment => able to explore and return to secure base when needed <i>Mathematics:</i> 1. Teacher has good-enough grasp of fundamental arithmetic/ transitions to algebra/algebra 2. Teacher promotes mathematics rather than self as authority for correctness 3. Teacher believes in student's prowess and provides developmentally appropriate mathematical tasks; student has necessary tools => student develops secure attachment to mathematics	<i>Mathematics teachers:</i> Teacher unavailable and/or unresponsive => student develops insecure attachment to teacher: anxious, ambivalent, or disorganized attachment <i>Mathematics:</i> Teacher does not know and/or teach mathematics well enough=> student develops anxious, ambivalent, or disorganized attachment to arithmetic and/or algebra	1. grief/loss => depression 2. separation anxiety from teacher and/or mathematics	1. sound mathematics attachment 2. traumatized mathematics attachment 3. failure of mathematics attachment	<i>Attachments: Conflict</i> between conscious desire to succeed in course and detached attachment pattern that prevent student from getting the help he/she needs or dependent relational pattern that prevents student from taking responsibility with support or ambivalent unstable attachment pattern	<i>For Compromised Attachments:</i> Counselor provides self (and points to instructor if applicable) and promotes mathematics as secure teacher and secure mathematics bases on which student can rely

APPENDIX B

Individual Mathematics Counseling
Assessment and Treatment Tools

1. Mathematics Counseling Session Reflection
2. *Student Mathematics History Interview Protocol*
3. *College Learning Metaphor Survey*
4. Negativity/Positivity Survey
 - a. *JMK Mathematics Affect Scales*
 - b. *JMK Mathematics Affect Scales*, revised
5. *Survey Profile Summary Sheet*
6. *One-On-One Mathematics Counseling Evaluation*

1. Mathematics Counseling

Session # _____

Name _____

Course/Semester _____

Counselor _____

Date _____

Notes

Transference/Countertransference

Self

Object

Space-in-between

Summary:Thoughts for the next Session:

MATHEMATICS HISTORY INTERVIEW PROTOCOL

NAME _____ AGE _____

DATE _____ MAJOR _____

1. Tell me how you usually **feel** when you are doing mathematics.

In the class you are in now, how does it **feel** to be:

- in class? _____
- doing homework? _____
- doing an exam? _____

2. Describe your **best** experience doing mathematics? _____

Why? _____

3. Describe your **worst** experience doing mathematics? _____

Why? _____

4. A. Have you always felt this way about doing mathematics? [If not, when and why and how did how you feel doing mathematics **change**?]

5. Is doing mathematics the same as or different from doing other subjects for you? How? _____

6. Do you do mathematics outside of classes—when do you do it in your daily life?

7. Tell me how well **you** do in mathematics courses, in daily life.

8. What is mathematics anyway? How would you describe it to a friend?

9. If doing mathematics is different for you from doing **other activities**, why do you think that is so?

10. How important do you think math/stats will be for you in your **future**? How does that make you feel?

11. Are any parts of math comfortable for you to do? Tell me a little about it...

12. What are your **least favorite** types of mathematics to do? Tell me a little about it...

13/14/15 In **Elementary/Middle/High** school, what was mathematics like for you?

What type of mathematics did **you** do? _____

What tools did you use? _____

Do you remember the teacher? _____

Was there anything about you/your family that you felt made a difference in how the teacher treated you/her expectations of you? _____

How did you get through school math when it got hard? [when you felt unable to do it well] _____

Did you receive any extra help? How was that for you? _____

How do you think math **should** have been taught/the learning environment should have been for you to have done better in it?

How do you think **YOU** could have done things differently in mathematics for you to have done better in it?

16. List the mathematics courses you took in high school, the year you took each, and the grade you earned in each:

	Mathematics Course	Year (e.g., 1995)	Grade earned
9 th Grade			
10 th Grade			
11 th Grade			
12 th Grade			

17. Did your parents work with you with math at home?_____ How was that? Their attitudes to math? to you doing math? Any brothers? Or sisters?

3. College Learning Metaphor Survey:

The *College Learning Metaphor* was administered at the beginning of the first session to all participants. When possible, it was also administered in the final session to assess any changes (but see also *One-on-One Counseling Evaluation* below).

COLLEGE LEARNING METAPHOR SURVEY

Name _____

Date _____

1. Make a list of metaphors that show how you FEEL about MATHEMATICS/YOURSELF DOING MATHEMATICS. For example, if it were a **color** what **color** would it be? If it were **weather** or an **animal** or a **fictional character** or ... what would it be?
2. Now choose **one** of the metaphors from 1. that most closely describes your relationship with MATHEMATICS and write more about why this metaphor describes your relationship with MATHEMATICS.
3. As you reread your metaphors, what do they tell you about your attitudes as you do MATHEMATICS? your expectations of yourself doing MATHEMATICS? your predictions about your success in MATHEMATICS?

Adapted from: Gibson, H. (1994). "Math is like a used car": Metaphors reveal attitudes towards mathematics. In D. Buerk (Ed.), *Empowering students by promoting active learning in mathematics* (pp. 7-12). Reston, West Virginia: National Council of Teachers of Mathematics.
© Jillian Knowles, revised December 1999.

4 a.

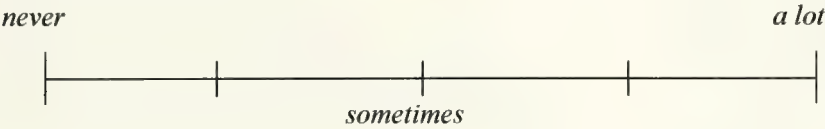
JMK Mathematics Affect Scales

Name _____ Date _____

On this questionnaire is a group of scales. Please read each scale carefully. Then indicate the part of each scale which best describes the way you have been feeling while doing mathematics during the PAST WEEK, INCLUDING TODAY. If an interval on the scale better describes your range of feelings rather than point, indicate that range with a line. If the words on the scale do not accurately describe your feelings, supply your own.

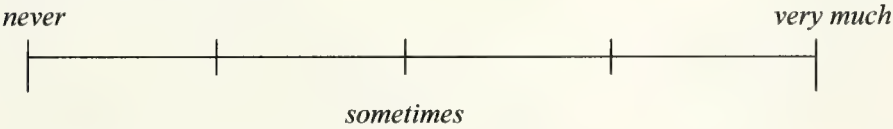
1. When I think about **doing** mathematics,

I tend to put work off:



2. If I think about how I **experience** my problems with mathematics,

I tend to feel discouraged:



3. When I think about my mathematics **future**,

I feel:



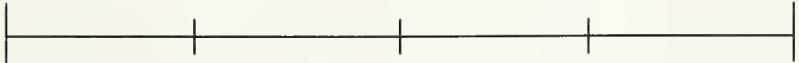
4. When I think about the **mathematics course** I am taking now,

I:

like it

I:

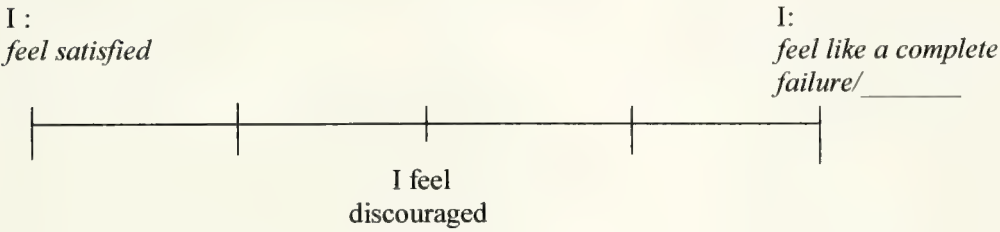
would withdraw if I
could



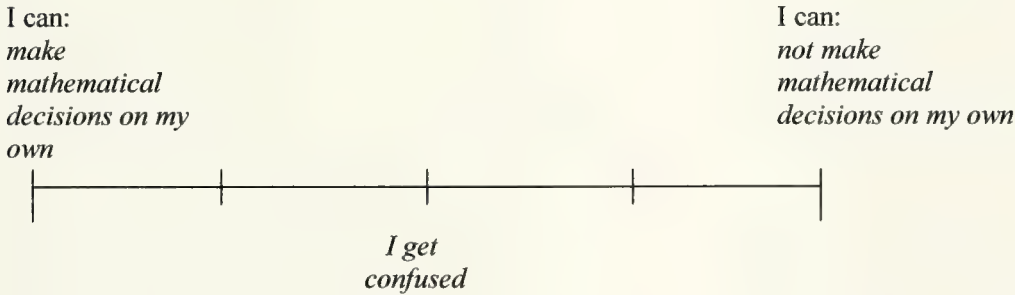
5. When I think about **how** I do mathematics,



6. When I think of my **mathematical achievements**,



7. **While** I am **doing** mathematics,

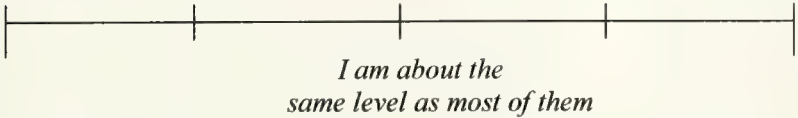


4. b. I revised the *JMK Mathematics Affect Scales* following the study by adding an eighth scale to gauge responder's sense of himself in relation to the rest of the class. See chapter 6 for discussion.

8. When I **compare myself** with **others** in my mathematics class,

I am:
*better at
mathematics
than most of them.*

I am:
*much worse
at mathematics
than most of them.*



5. As a way of integrating students data and using it for ongoing insight and intervention in counseling, I placed an individual's scores with the class extreme scores for each scale and discussed the concepts and implication with participants during counseling sessions. See chapter 5 and 6 for discussion.

Survey Profile Summary Sheet

Name _____ Class _____ Pre/Post _____ Date _____

MATHEMATICS FEELINGS

Math Testing Anxiety

I

Not at all

1

2

3

4

5

very much

Number Anxiety

II

Not at all

1

2

3

4

5

very much

Abstraction Anxiety

III

Not at all

1

2

3

4

5

very much

MATHEMATICS BELIEFS SURVEY

I

Procedural Math

1

2

3

4

5

Conceptual Math

II

Toxic /Negative

1

2

3

4

5

Healthy/Positive

III

Learned Helpless

1

2

3

4

5

Mastery Orientated

OVERALL SUMMARY

Negative

1

2

3

4

5

Positive

5. *One-On-One Mathematics Counseling Evaluation*. Participants were asked to respond to whether and in what ways they had changed in their approach to mathematics learning during the course and to write about factors to which they attributed any changes. As part of this they were invited to write a different metaphor if a new one was applicable. See chapters 6 and 8 for a discussion of responses.

I administered this to individual counseling participants during class posttesting, July 31,

2000 _____

One-On-One Mathematics Counseling Evaluation

Name (optional) _____ Date _____

Please answer the following questions as honestly as possible from your point of view. Please be open with any criticisms, questions or suggestions you have. Use the back if necessary.

1 (a) What was your initial motivation for signing up to meet with me for one-on-one mathematics counseling?

(b) Did that motivation change? If so, how and why?

2. Did the way you see yourself as a mathematics learner change in any way as you were doing PSYC/STAT 104 this summer? If so, in what ways did you change? Did your math metaphor change? To what? What, do you think, were the main factors in that change? (e.g., the way the class was taught?, the professor? the testing style?, meeting with me?, the math content? a personal change? ...a combination?)

3. Do you think your meetings with me affected how you were approaching PSYC/STAT 104?
If so, in what ways?

4. Do you think your meetings with me affected your success in PSYC/STAT 104? If so, in what ways?

5. With regard to Question 4, how do you define “success in PSYC/STAT 104” ?
6. How, if at all, do you think your overall experience in PSYC/STAT 104 this summer will affect how you will approach your next mathematics–related course or challenge?

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Appendix C

Class Assessment and Observation Tools

1. Beliefs Surveys:
 - a. *Modified Mathematics Beliefs Survey*, June 5, 2000
 - b. *Modified Mathematics Beliefs Survey*, Revised Version, August 2002
 - c. *My Orientation to Mathematics Survey*, Short Revised Version, May 2003
2. Mathematics History, Feelings and Evaluations Surveys:
 - a. *Pretest Mathematics Background Survey* and *My Mathematics Feelings* survey
 - b. *Pretest Mathematics Background Survey* Revised Version, August 2002
 - c. *Posttest Course Reflection and Evaluation* Survey that preceded the posttest *My Mathematics Feelings* survey
 - d. Class-Link Evaluation.
3. Arithmetic Assessment:
 - a. *Arithmetic for Statistics* (AFS) Assessment
 - b. *Arithmetic for Statistics* (AFS) Assessment Profile form
4. Statistical Reasoning Assessment (SRA)
5. The Algebra Test and sample scoring sheet
6. Observation Tools:
 - a. Music Staff Class Interaction Observation Chart
 - b. Class Layout Observation Form
 - c. Problem Working Session Interaction Chart (Class 13)
 - d. *Class Summary* analysis sheet

1. Belief Surveys:

When Erna Yackel, with counseling psychologist Ann Knudsen (and later Carolyn Carter) developed and ran a mathematics anxiety reduction course, they aimed at challenging and changing students' procedural, helpless, and mythical beliefs about mathematics and themselves and reducing anxiety levels while the students learned conceptual mathematics using a problem-solving, constructivist approach (E. Yackel, personal communication, January 21, 2000; Carter & Yackel, 1989).

Yackel created a two-part mathematics beliefs survey as a before and after instrument for the course. The first part assesses beliefs about mathematics along a continuum from beliefs about mathematics as conceptual (Skemp's (1987) "relational" mathematics) through mathematics as procedural (Skemp's "instrumental" mathematics). In the second part Yackel had included questions that she felt from her experience as a mathematics educator to be important for a healthy approach to mathematics, questions she "found interesting" (personal communication, January 21, 2000). Because Carter and Yackel used Kogelman and Warren's (1978) anxiety reduction approach in their workshops, I reviewed Kogelman and Warren's list of myths and used in my survey ones related to the topics I surveyed:

- 1 Men are better at math than women.
- 2 Math requires logic, not intuition.
- 3 You must always know how you got the answer.
- 4 Math is not creative.
- 5 There is always a best way to do a math problem.
- 6 It's always important to get the answer exactly right.

- 7 It's bad to count on your fingers.
- 8 Mathematicians do problems quickly, in their heads.
- 9 Math requires a good memory.
- 10 Math is done by working intensely until the problem is solved.
- 11 Some people have a math mind and some don't.
- 12 There is a magic key to doing math.

These beliefs can be grouped into three broad categories: Some of these myths relate to an erroneous or procedural *view of mathematics* and *self* (e.g., 2, 3, 5, 6, 8, and 9); some relate to learning style bias and constricted *pedagogy* (e.g., 3, 4, 7); others are embedded in American *cultural tradition* (1, 11, 12). Yackel's survey used versions of myths 1, 4, 5, 6, 9, 11 isolated by Kogelman and Warren. I added question 19 (Part II) that Kogelman and Warren isolated (cf. their #7) and Yackel had not included.

I also added some perceived usefulness questions (Part II questions 22 and 23) to touch on Sherman and Fennema's (and others') usefulness factor found to be related to mathematics learning motivation and achievement although Yackel had already included two usefulness items. Yackel's survey touched on male domain and mathematics-related affect factors identified on Fennema-Sherman Attitude scales and I added a parent/teacher item (Part II #21) that I believed may be linked to learned helplessness. Yackel and I did not include any success items (Fennema & Sherman, 1976; Mulhern & Rae, 1998).

In order to elucidate student's beliefs around their control of the situation I modified the second part, adding questions related to learned helplessness (Licht & Dweck, 1984) such as:

7. I think my ability to do mathematics can improve. SD D U A SA

(SD means “strongly disagree,” SA means “strongly agree,”), that asked whether the respondent has a fixed trait mathematics theory about herself or not.

A number of questions relating to learned helplessness were already in the first part since, for example, a belief in mathematics as procedural sees the mathematics as outside one's control, leading to a helpless response if one does not “recognize” the problem or if one “forgets” the procedure. I thus created a Learned Helplessness through Mastery Orientation (LM) Scale within the larger scales. There were fourteen questions that pertained to student beliefs and behaviors on this continuum. (Since these questions are embedded in the larger survey I labeled the Learned Helpless/Mastery Oriented questions as LM and signed them LM– to indicate a Learned Helpless and LM + to indicate Mastery Orientated belief or behavior respectively. The label LM –and the LM+ signs did not appear in the student administered version of the Modified Mathematics Belief Scale.)

During post analysis as I looked for factors that discriminated among the categories of students I identified (see chapter 7), I found that questions in this *Beliefs Survey* (Part I, Items 4, 7, 9, and 10) that related to achievement motivation contributed to that identification. Since these questions are embedded in the larger survey I labeled the performance/learning achievement motivation questions as P/L and signed them P/L – to indicate a performance achievement motivation and P/L + to indicate learning

respectively. The labels P/L –and the P/L + did not appear in the student administered version of the Modified Mathematics Belief Scale.

Thus my *Modified Mathematics Beliefs Systems Survey* yielded three measures of belief and attitude:

1. mathematics as procedural through conceptual,
2. mathematics learning approaches and attitudes as toxic through healthy, and
3. learned helpless through mastery orientation, and

to provide a starting point for discussion, challenge, and reeducation in the mathematics counseling setting, and a fourth: performance through learning achievement motivation, to aid post analysis .

I used this information with each participant by discussing their positions on their individualized *Surveys Profile Summary* (see Appendix B), by investigating individual item responses, and by explaining the concepts involved and their ramifications to mathematics learning.

I gave the *Mathematics Beliefs Systems Survey* as a posttest to ascertain if any changes had been made over the summer. I had opportunity to discuss these changes with only one participant, Jamie. See chapters 3, 6, and 7 for further discussion.

1. a. *Modified Mathematics Beliefs Systems Survey* administered as a pre- and posttest to the class on June 5, 2000 and on July 31, 2000 respectively.

Modified Mathematical Belief Systems Survey

Name/Number _____ Date _____

All individual responses to this survey will be kept strictly confidential. Your responses will be used to study relationships among student beliefs about mathematics, past teaching methods used, effects of mathematics learning assistance and certain other variables such as mathematics background.

For each item, circle the response that indicates how you feel about the item as indicated below. PLEASE add your own comments or questions at any point in the Survey.

Strongly				Strongly
Disagree	Disagree	Undecided	Agree	Agree
SD	D	U	A	SA

Part I

- | | | | | | | |
|--|----------|----|---|---|----|----|
| 1. Doing mathematics consists mainly of using rules. | SD | D | U | A | SA | |
| 2. Learning mathematics mainly involves memorizing procedures and formulas. | SD | D | U | A | SA | |
| 3. Mathematics involves relating many different ideas. | SD | D | U | A | SA | |
| 4. Getting the right answer is the most important part of mathematics. | P/L- | SD | D | U | A | SA |
| 5. In mathematics it is impossible to do a problem unless you've first been taught how to do one like it. | LM- | SD | D | U | A | SA |
| 6. One reason mathematics is so much work is that you need to learn a different method for each new class of problem. | LM- | SD | D | U | A | SA |
| 7. Getting good grades in mathematics is more of a motivation than is the satisfaction of learning the mathematics content. | P/L- | SD | D | U | A | SA |
| 8. When I learn something new in mathematics I often continue exploring and developing it on my own. | LM+ | SD | D | U | A | SA |
| 9. I usually try to understand the reasoning behind all the rules I use in mathematics. | P/L+ LM+ | SD | D | U | A | SA |
| 10. Being able to successfully use a rule or formula in mathematics is more important to me than understanding why and how it works. | P/L- | SD | D | U | A | SA |
| 11. A common difficulty with taking quizzes and exams in mathematics is that if you forget relevant formulas and rules you are lost. | LM- | SD | D | U | A | SA |

- | | | | | | |
|---|-----|---|---|---|----|
| 12. It is difficult to talk about mathematical ideas because all you can really do is explain how to do specific problems. | SD | D | U | A | SA |
| 13. Solving mathematics problems frequently involves exploration. | SD | D | U | A | SA |
| 14. Most mathematics problems are best solved by deciding on the type of problem and then using a previously learned solution method for that type. | SD | D | U | A | SA |
| | LM- | | | | |
| 15. I forget most of the mathematics I learn in a course soon after the course is over. | SD | D | U | A | SA |
| | LM- | | | | |
| 16. Mathematics consists of many unrelated subjects. | SD | D | U | A | SA |
| 17. Mathematics is a rigid uncreative subject. | SD | D | U | A | SA |
| 18. In mathematics there is always a rule to follow. | SD | D | U | A | SA |
| 19. I get frustrated if I don't understand what I am studying in mathematics. | SD | D | U | A | SA |
| 20. The most important part of mathematics is computation. | SD | D | U | A | SA |

Part II

- | | | | | | | |
|---|-----|----|---|---|----|----|
| 1. I usually enjoy mathematics. | SD | D | U | A | SA | |
| 2. Mathematics is boring. | SD | D | U | A | SA | |
| 3. When I work on a difficult mathematics problem and I can't see how to do it in the first few minutes, I assume I won't be able to do it and I give up. | SD | D | U | A | SA | |
| | LM- | | | | | |
| 4. When I read newspaper and magazine articles I skip over numbers, graphs, and numerical material. | SD | D | U | A | SA | |
| 5. I only take mathematics courses because they are required. | LM- | SD | D | U | A | SA |
| 6. I think mathematics is fun and is a challenge to learn. | LM+ | SD | D | U | A | SA |
| 7. I think my ability to do mathematics can improve. | LM+ | SD | D | U | A | SA |
| 8. Mathematics/statistics, in my experience, has no connection to the real world. | SD | D | U | A | SA | |
| 9. Mathematics is a subject that some people can do and others can't. | LM- | SD | D | U | A | SA |
| 10. My overall feeling towards math is positive. | SD | D | U | A | SA | |
| 11. Mathematics is used on a daily basis in many jobs. | SD | D | U | A | SA | |
| 12. Mathematics is easy for me. | SD | D | U | A | SA | |

13. I like to work on hard mathematics problems.	LM+	SD	D	U	A	SA
14. Most mathematics courses go too fast for me.		SD	D	U	A	SA
15. Mathematics is a subject men do better in than women.		SD	D	U	A	SA
16. I would like to learn more about mathematics/statistics.		SD	D	U	A	SA
17. I was better at Geometry than at Algebra.		SD	D	U	A	SA
18. I have to understand something visually before I can “get” it auditorily/verbally		SD	D	U	A	SA
19. I think having to use fingers or other calculating manipulatives is childish and shows you are not very good at mathematics.		SD	D	U	A	SA
20. I have avoided/delayed taking a mathematics class because of my worry about my ability to succeed in it.	LM-	SD	D	U	A	SA
21. I have had a math teacher/guidance counselor/parent who has made me feel I did/do not have the ability to take higher level math classes.		SD	D	U	A	SA
22. I’ll need mathematics/statistics in my future schooling.		SD	D	U	A	SA
23. I’ll need mathematics/statistics in my future work.		SD	D	U	A	SA

Other Comments and Questions:

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1. b. *Modified Mathematics Beliefs Survey*, Revised Version, August 2002, recommended for use after analysis of dissertation data.

Modified Mathematical Belief Systems Survey

Name/Number _____ Date _____

All individual responses to this survey will be kept strictly confidential. Your responses will be used to study relationships among student beliefs about mathematics, past teaching methods used, effects of mathematics learning assistance and certain other variables such as mathematics background.

For each item, circle the response that indicates how you feel about the item as indicated below. PLEASE add your own comments or questions at any point in the Survey.

Strongly				Strongly
Disagree	Disagree	Undecided	Agree	Agree
SD	D	U	A	SA

Part I

- | | | | | | |
|--|----|---|---|----------|----|
| 1. Doing mathematics consists mainly of using rules. | SD | D | U | A | SA |
| 2. Learning mathematics mainly involves memorizing procedures and formulas. | SD | D | U | A | SA |
| 3. Mathematics involves relating many different ideas. | SD | D | U | A | SA |
| 4. Getting the right answer is the most important part of mathematics. | SD | D | U | A | SA |
| | | | | P/L- | |
| 5. In mathematics it is impossible to do a problem unless you've first been taught how to do one like it. | SD | D | U | A | SA |
| | | | | LM- | |
| 6. One reason mathematics is so much work is that you need to learn a different method for each new class of problem. | SD | D | U | A | SA |
| | | | | LM- | |
| 7. Getting good grades in mathematics is more of a motivation than is the satisfaction of learning the mathematics content. | SD | D | U | A | SA |
| | | | | P/L- | |
| 8. When I learn something new in mathematics I often continue exploring and developing it on my own. | SD | D | U | A | SA |
| | | | | LM+ | |
| 9. I usually try to understand the reasoning behind all the rules I use in mathematics. | SD | D | U | A | SA |
| | | | | P/L+ LM+ | |
| 10. Being able to successfully use a rule or formula in mathematics is more important to me than understanding why and how it works. | SD | D | U | A | SA |
| | | | | P/L- | |
| 11. A common difficulty with taking quizzes and exams in mathematics is that if you forget relevant formulas and rules you are lost. | SD | D | U | A | SA |
| | | | | LM- | |
| 12. It is difficult to talk about mathematical ideas because all you | SD | D | U | A | SA |

can really do is explain how to do specific problems.

- | | | | | | |
|---|-----|---|---|---|----|
| 13. Solving mathematics problems frequently involves exploration. | SD | D | U | A | SA |
| 14. Most mathematics problems are best solved by deciding on the type of problem and then using a previously learned solution method for that type. | SD | D | U | A | SA |
| | LM- | | | | |
| 15. I forget most of the mathematics I learn in a course soon after the course is over. | SD | D | U | A | SA |
| | LM- | | | | |
| 16. Mathematics consists of many unrelated subjects. | SD | D | U | A | SA |
| 17. Mathematics is a rigid uncreative subject. | SD | D | U | A | SA |
| 18. In mathematics there is always a rule to follow. | SD | D | U | A | SA |
| 19. I get frustrated if I don't understand what I am studying in mathematics. Item broken into two parts and moved to Part II | SD | D | U | A | SA |
| 20. The most important part of mathematics is computation. | SD | D | U | A | SA |

Part II

- | | | | | | |
|---|--------|---|---|---|----|
| 1. I usually enjoy mathematics. | SD | D | U | A | SA |
| 2. Mathematics is boring. | SD | D | U | A | SA |
| 3. When I work on a difficult mathematics problem and I can't see how to do it in the first few minutes, I assume I won't be able to do it and I give up. | SD | D | U | A | SA |
| | LM- | | | | |
| 4. When I read newspaper and magazine articles I skip over numbers, graphs, and numerical material. | SD | D | U | A | SA |
| 5. I only take mathematics courses because they are required. | LM- SD | D | U | A | SA |
| 6. I think mathematics is fun and is a challenge to learn. | LM+ SD | D | U | A | SA |
| 7. I think my ability to do mathematics can improve. | LM+ SD | D | U | A | SA |
| 8. Mathematics/statistics, in my experience, has no connection to the real world. | SD | D | U | A | SA |
| 9. Mathematics is a subject that some people can do understand and others can't. | SD | D | U | A | SA |
| | LM- | | | | |
| 10. My overall feeling towards math is positive. | SD | D | U | A | SA |

(new) I rate my ability in mathematics as:

poor; below average; average; above average; excellent (circle one)

- | | | | | | |
|---|----|---|---|---|----|
| 11. Mathematics is used on a daily basis in many jobs. | SD | D | U | A | SA |
|---|----|---|---|---|----|

12. Mathematics is easy for me.		SD	D	U	A	SA
(new) Pt I, 19 (a) I am able to learn mathematical procedures		SD	D	U	A	SA
		(no score on scale)				
(new) Pt I, 19 (b) I do not expect to be able to understand what		SD	D	U	A	SA
I am doing in mathematics or why						
13. I like to work on hard on mathematics problems until I master them.	LM+	SD	D	U	A	SA
14. Most mathematics courses go too fast for me.		SD	D	U	A	SA
15. Mathematics is a subject men do better in than women.		SD	D	U	A	SA
16. I would like to learn more about mathematics/statistics.		SD	D	U	A	SA
17. I was better at Geometry than at Algebra.		SD	D	U	A	SA
18. I have to understand something visually before I can "get" it — auditorily/verbally		SD	D	U	A	SA
19. I think having to use fingers or other calculating manipulatives is childish and shows you are not very good at mathematics.		SD	D	U	A	SA
20. I have avoided/delayed taking a mathematics class because of my worry about my ability to succeed in it.	LM-	SD	D	U	A	SA
21. I have had a math teacher/guidance counselor/parent who has made me feel I did/do not have the ability to take higher level math classes.		SD	D	U	A	SA
22. I'll need mathematics/statistics in my future schooling.		SD	D	U	A	SA
23. I'll need mathematics/statistics in my future work.		SD	D	U	A	SA

Other Comments and Questions:

1. c. *My Orientation to Mathematics Survey*, Short Revised Version, May 2003

This is a shorter Learned Helpless-Mastery Oriented focused Revised Version of *Modified Beliefs Survey*. This shortened form includes items from the *Beliefs Survey* that investigate students' beliefs about (a) the nature of mathematics (conceptual versus procedural), (b) ability and effort beliefs and attributions items (that include some U.S. cultural beliefs), (c) student mathematics practices items, and (d) achievement motivation items, but no usefulness beliefs, or mathematics attractiveness attitudes items. I have added a category of items (e) to investigate student social practices related to social learned helplessness in accessing support (see chapter 6 and 7).

MY ORIENTATION TO MATHEMATICS LEARNING

Name _____ Course _____ Date _____

SD = Strongly Disagree; D = Disagree; N = Neutral; Agree; SA = Strongly Agree

- | | | | | | |
|---|----|---|---|---|----|
| 1. In mathematics it is impossible to do a problem unless you've first been taught how to do one like it. | SD | D | N | A | SA |
| 2. One reason mathematics is so much work is that you need to learn a different method for each new class of problem. | SD | D | N | A | SA |
| 3. When I learn something new in mathematics I often continue exploring and developing it on my own. | SD | D | N | A | SA |
| 4. I usually try to understand the reasoning behind all the rules I use in mathematics. | SD | D | N | A | SA |
| 5. A common difficulty with taking quizzes and exams in mathematics is that if you forget relevant formulas and rules you are lost. | SD | D | N | A | SA |
| 6. Most mathematics problems are best solved by deciding on the type of problem and then using a previously learned solution method for that type. | SD | D | N | A | SA |
| 7. I forget most of the mathematics I learn in a course soon after the course is over. | SD | D | N | A | SA |
| 8. When I work on a difficult mathematics problem and I can't see how to do it in the first few minutes, I assume I won't be able to do it and I give up. | SD | D | N | A | SA |
| 9. I only take mathematics courses because they are required. | SD | D | N | A | SA |
| 10. I think my ability to do mathematics can improve. | SD | D | N | A | SA |
| 11. I rate my ability to do mathematics as: (circle one) poor below average above excellent
 average average | | | | | |
| 12. Mathematics is a subject that some people can do and others can't. | SD | D | N | A | SA |

- | | | | | | |
|--|----|---|---|---|----|
| 13. I can do mathematical procedures. | SD | D | N | A | SA |
| 14. I don't expect to be able to understand what I am doing in mathematics or why. | SD | D | N | A | SA |
| 15. In the past, working hard has not changed how I did in mathematics. | SD | D | N | A | SA |
| 16. I work on hard mathematics problems until I master them. | SD | D | N | A | SA |
| 17. If I get a good grade in mathematics it is only because I work hard, not because I am smart. | SD | D | N | A | SA |
| 18. I delay taking mathematics classes because of my worry about my ability to succeed in them. | SD | D | N | A | SA |
| 19. If I got a bad grade in mathematics it is only because I didn't work hard. | SD | D | N | A | SA |
| 20. Teachers should not pick out particular students to answer questions in class. | SD | D | N | A | SA |
| 21. I would never volunteer to answer a question a teacher asked in class even if I knew the answer. | SD | D | N | A | SA |
| 22. If I didn't understand what the professor was saying about a math problem I would | | | | | |
| a) ask her in class | SD | D | N | A | SA |
| b) go to her office hours to ask her | SD | D | N | A | SA |
| c) ask a student sitting near me | SD | D | N | A | SA |
| d) go to the Learning Center to ask a tutor | SD | D | N | A | SA |
| e) do nothing and hope it would be covered in the next class | SD | D | N | A | SA |
| 23. Getting the right answer is the most important part of mathematics. | SD | D | N | A | SA |
| 24. Getting good grades in mathematics is more of a motivation than is the satisfaction of learning the mathematics content. | SD | D | N | A | SA |
| 25. Being able to successfully use a rule or formula in mathematics is more important to me than understanding why and how it works. | SD | D | U | A | SA |

1.	SD	D	N	A	SA	
	5	4	3	2	1	_____
2.	SD	D	N	A	SA	
	5	4	3	2	1	_____
3.	SD	D	N	A	SA	
	1	2	3	4	5	_____
4.	SD	D	N	A	SA	
	1	2	3	4	5	_____
5.	SD	D	N	A	SA	
	5	4	3	2	1	_____
6.	SD	D	N	A	SA	
	5	4	3	2	1	_____
7.	SD	D	N	A	SA	
	5	4	3	2	1	_____
8.	SD	D	N	A	SA	
	5	4	3	2	1	_____
9.	SD	D	N	A	SA	
	5	4	3	2	1	_____
10.	SD	D	N	A	SA	
	1	2	3	4	5	_____
11.	p	b/a	av	a/av	e	<u>0</u>
12.	SD	D	N	A	SA	
	5	4	3	2	1	_____
13.	SD	D	N	A	SA	<u>0</u>
14.	SD	D	N	A	SA	
	5	4	3	2	1	_____
15.	SD	D	N	A	SA	
	5	4	3	2	1	_____
16.	SD	D	N	A	SA	
	1	2	3	4	5	_____
17.	SD	D	N	A	SA	
	5	4	3	2	1	_____
18.	SD	D	N	A	SA	
	5	4	3	2	1	_____
19.	SD	D	N	A	SA	
	1	2	3	4	5	_____
20.	SD	D	N	A	SA	<u>0</u>
21.	SD	D	N	A	SA	<u>0</u>
22 e)	SD	D	N	A	SA	
	5	4	3	2	1	_____
23.	SD	D	N	A	SA	
	5	4	3	2	1	_____
24.	SD	D	N	A	SA	
	5	4	3	2	1	_____
25.	SD	D	N	A	SA	
	5	4	3	2	1	_____

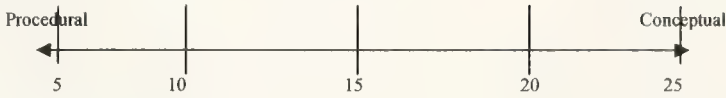
TOTAL: _____



Score My Mathematics Orientation
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Fall 2002.
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Subscales From MY ORIENTATION TO MATHEMATICS LEARNING

Scale 1: **Mathematics** as:



Total from Questions 1, 2, 5, 6, 7: _____

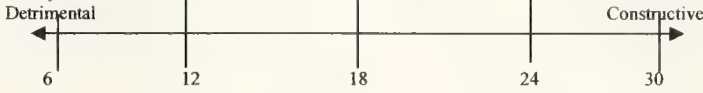
Scale 2: My mathematics **practices** as
Learned Helpless



Total from Questions 3, 4, 8, 9, 16, 21: _____

Score 21. SD D N A SA
5 4 3 2 1

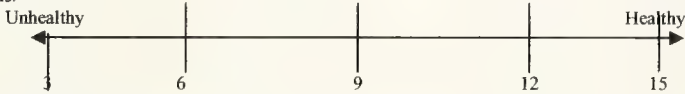
Scale 3: My beliefs about my mathematics **self** as



Total from Questions 10, 11, 12, 13, 14, 18: _____

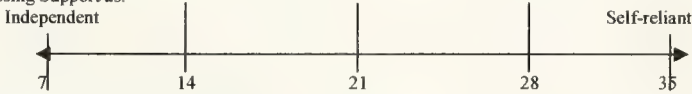
Score 11. p b/a av a/av e
2 2 4 4 3
13. SD D N A SA
1 2 3 4 5

Scale 4: Attributions as:



Total from Questions 15, 17, 19: _____

Scale 5: Social/ Accessing Support as:



Total from Questions 20, 21, 22 a, 22b, 22c, 22d, 22e: _____

Score 20. SD D N A SA
5 4 3 2 1
21. SD D N A SA
5 4 3 2 1
22a. SD D N A SA
1 2 3 4 5
22b. SD D N A SA
1 2 3 4 5
22c. SD D N A SA
1 2 3 4 5
22d. SD D N A SA
1 2 3 4 5
22e. SD D N A SA
5 4 3 2 1

Scale 5: Achievement Motivation as:



Total from Questions 4, 23, 24, 25: _____

2. *Mathematics Feelings Surveys.*

The *Mathematics Anxiety Rating Scale* (MARS) has been normed and is perhaps the most used in the field (Richardson & Suinn, 1972). It is long however (98 items), only yields one measure, but seems to address anxiety in a number of different settings that it would be helpful to differentiate. Rounds and Hendel did a factor analysis of 94 of the items of MARS and identified 30 items that they found measured two relatively homogeneous factors (15 items each) they called “mathematics testing anxiety” and “numerical anxiety respectively” (Rounds & Hendel, 1980). Ron Ferguson created a three-factor instrument from this using the twenty items that loaded most heavily on these factors (10 each) and adding ten items to measure a factor he labeled “abstraction anxiety” to make an instrument more applicable to a college setting. Factor analysis showed that his items did measure a factor different from the two that Rounds and Hendel identified (Ferguson, 1986). I have slightly changed some of Ferguson’s items and adopted his instrument, calling it *Measuring Mathematics Feelings* rather than Ferguson’s suggestive “Phobus” (a moon of Mars and the root of the word phobia). Ferguson has placed his items in the public domain and I have purchased MARS (adult form) from Dr. Suinn so that I could use the 10 mathematics testing anxiety and the 10 numerical anxiety MARS items that Ferguson used from Rounds and Hendel’s factor analysis. Ferguson’s instrument is not normed but, as my primary use of it is in the counseling situation, its ability to quickly assess three pertinent factors of a student’s anxiety, two of which relate to the type of mathematics, thus providing a point of discussion, made it more useful for this study than the full MARS. The principle reason for assessment in this study was not to compare an individual or group with equivalent

people in the wider population, but to compare an individual with herself as she made changes.

I used this information with each counseling participant by discussing their positions on their individualized *Survey Profile Summary* (see Appendix B), by discussing individual item responses, and by explaining in more detail the concepts involved.

I gave the *Measuring Mathematics Feelings* as a post test to ascertain if any changes had been made over the summer. I had opportunity to discuss these changes with only one participant, Jamie. Discussing her changes on the instrument highlighted another aspect of anxiety that was particularly pertinent to her—the interaction of social anxiety with the mathematics learning or performance situation. It was this element of Jamie’s mathematics anxiety that had been reduced over the summer.

I have therefore coded each item of *Measuring Mathematics Feelings* as:

1. Position in relation to others:
 - a. P for primarily public,
 - b. S for solitary,
 - c. S/P for solitary with a public component,
 - d. P/S for public with a solitary component, depending on the relational setting implied or explicitly referred to in the question, and
2. Setting of activity
 - a. Cl to indicate primarily classroom setting for the activity and
 - b. Cl/H for an activity that occurs both at home and in the classroom.

I have done this to aid analysis with the student responder and for post analysis. My coding may change in discussion with a student who feels the question situation as more or less public or more or less solitary. For further discussion see *Jamie and Me* chapter 6.

Note: Dr. Richard Suinn has given me permission to include here as samples (to be used by readers only with his permission) ten of the twenty items that I took from his *Mathematics Anxiety Rating Scale* (MARS) that form parts I and II of the *My Mathematics Feelings* survey. I have deleted the other ten items but retained the above categorization of them.

2. a. *Measuring Mathematics Feelings* Pretest Survey, administered in class, June 5, 2000. This survey also includes the *Pretest Mathematics Background* Survey questions.

Statistics in Psychology PSYC 402, Summer 2000

Please fill in whatever of the following you feel comfortable sharing. All the data will be kept confidential. Participation or non-participation in this study will not affect your grade in this class in any way.

Name/Number _____ Date _____

Major _____ Is this class required for your major? _____ If yes, why do you think it is required? _____

Last math class taken before this one _____ Year _____ Grade _____

What statistics have you studied before? _____

What, in your opinion is the relationship between mathematics and statistics? _____

What grade do you hope for in this class? _____ What grade do you expect? _____

Measuring Mathematics Feelings

Each question below describes a mathematics-related activity or situation. Please indicate on the scale of 1 through 5 how much you are scared by that mathematics-related activity or situation nowadays.

	Not at all				Very much
	1	2	3	4	5
1. S. Signing up for a math course.*					
2.P/CIWalking into a math class.*					
3. P/CI Raising your hand in a math class to ask a question.*	1	2	3	4	5
4. S/P/CI.Taking an examination (final) in a math class.*	1	2	3	4	5
5.S.**	1	2	3	4	5
6.S.**	1	2	3	4	5
7.S/P/CI Waiting to have a math test returned.*	1	2	3	4	5
8.S.**	1	2	3	4	5
9.S. Receiving your final math grade in the mail.*	1	2	3	4	5
10.S/P/CI**	1	2	3	4	5

* Sample items from the *Mathematics Anxiety Rating Scale*. The *Mathematics Anxiety Rating Scale* (MARS) is copyrighted by Richard M. Suinn, Ph.D. Any use of the MARS items requires the permission of Dr. Suinn: suinn@lamar.colostate.edu. I retained these items because class and/or individual response changes on them over the course were notable (see chapter 8 for further discussion)

** The items from MARS used are omitted here as per agreement with Dr. Richard Suinn.

PART II

	Not at all				Very much
	1	2	3	4	5
1.P. Determining the amount of change you should get back from a purchase involving several items.*					

2.P. Listening to a salesperson show you how you would save money by buying his higher priced product because it reduces long-term expenses.*

1 2 3 4 5

3.P.**

1 2 3 4 5

4.S. Reading your W-2 form (or other statement showing your annual earnings and taxes).*

1 2 3 4 5

5.P.

1 2 3 4 5

6.P. Hearing friends make bets on a game as they quote the odds.*

1 2 3 4 5

7. P/S.**

1 2 3 4 5

8. S. **

1 2 3 4 5

9. S. **

1 2 3 4 5

10.S. **

1 2 3 4 5

* Sample items from the *Mathematics Anxiety Rating Scale*. The *Mathematics Anxiety Rating Scale* (MARS) is copyrighted by Richard M. Suinn, Ph.D. Any use of the MARS items requires the permission of Dr. Suinn: suinn@lamar.colostate.edu. I chose to retain these items because they elicited the highest anxiety responses or changed most over the course.

** The items from MARS used are omitted here as per agreement with Dr. Richard Suinn.

PART III

		Not at all			Very much
1.S/Cl. Having to work a math problem that has x's and y's instead of 2s and 3s.	1	2	3	4	5

2.P/Cl. Being told that everyone is familiar with the Pythagorean Theorem.	1	2	3	4	5
---	---	---	---	---	---

3.S/P/Cl. Realizing that my psychology professor has just written some algebraic formulas on the chalkboard.	1	2	3	4	5
--	---	---	---	---	---

4. S/Cl. Being asked to solve the equation $x^2 - 5x + 6 = 0$	1	2	3	4	5
---	---	---	---	---	---

5. P/Cl. Being asked to discuss the proof of a theorem about triangles.	1	2	3	4	5
---	---	---	---	---	---

6. S/Cl. Trying to read a sentence full of symbols such as: $SS_{xx} = \sum x^2 - \frac{(\sum x)^2}{N}$	1	2	3	4	5
---	---	---	---	---	---

7. P. Listening to a friend explain something she just learned in calculus.	1	2	3	4	5
---	---	---	---	---	---

8. S, Cl/H. Opening up a math book and not seeing any numbers, only letters, on an entire page.	1	2	3	4	5
---	---	---	---	---	---

9.S. Reading a description from a college catalog of the topics to be covered in a math course.	1	2	3	4	5
---	---	---	---	---	---

10.P, Cl/H. Having someone lend me a calculator to work a problem and not knowing which button to push to get the answer.	1	2	3	4	5
---	---	---	---	---	---

The 98 item *Mathematics Anxiety Rating Scale* (MARS) was developed by Richardson and Suinn in 1972 (Richardson & Suinn, 1972). Ron Ferguson created *Phobus* (a moon of Mars) by first choosing 20 items from MARS, ten found by Rounds and Hendel to be related to Mathematics Test Anxiety (Part I) and the other ten to be related to Number Anxiety (Part II) (Rounds & Hendel, 1980). Ferguson then added ten more items to measure what he calls Abstraction Anxiety (Part III) (Ferguson, 1986, 1998). I have slightly changed items 2 and 3 of Part II and 6 and 7 of Part III.

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2. b. Following the study I found that there were a number of background details I had failed to ascertain from participants. To remedy this I sent an e-mail survey to which most participants responded. The following is my revision of the *Pretest Mathematics Background* Survey that I would recommend to avoid these difficulties I encountered. Revision additions are **bolded**. Revision deletions are shown as strike throughs.

Statistics in Psychology PSYC 402, Summer 2000

Please fill in whatever of the following you feel comfortable sharing. All the data will be kept confidential. Participation or non-participation in this study will not affect your grade in this class in any way.

Name/Number _____ Date _____

Major _____ Is this class required for your major? _____ If yes, why do you think it is required? _____

Last **high school** math class taken _____ Year (e.g., 1997) _____ Grade (e.g., A) _____

Last **college** math class taken _____ Year(e.g., 1997) _____ Grade (e.g., A) _____

Have you ever repeated a mathematics course? _____ If so what course and when? _____

Are you repeating PSYC/STAT 104? _____

Have you taken the Brookwood State University mathematics placement test?

_____ **If yes, what mathematics course was recommended?** _____

Did you take that course? _____ **If yes, when and what grade (e.g., B)** _____

What statistics have you studied before? _____

~~What, in your opinion is the relationship between mathematics and statistics?~~ _____

What grade do you hope for in this class? _____ What grade do you expect? _____

Describe your worst experience in a mathematics class? _____

_____ **How old were you?** _____

Describe your best experience in a mathematics class _____

_____ **How old were you?** _____

2 . c. The *Posttest Course Reflection and Evaluation* survey questions preceded

Measuring Mathematics Feelings Posttest Survey, administered in class, July 31, 2000.

The *Measuring Mathematics Feelings* part was identical with that on the Pretest Survey so it is not included here.

Statistics in Psychology PSYC 402, Summer 2000

Please fill in whatever of the following you feel comfortable sharing. All the data will be kept confidential. Participation or non-participation in this study will not affect your grade in this class in any way.

Name/Number _____ Date _____

Please describe how taking this summer course, PSYC 402, was for you? _____

What did you learn about yourself as a mathematics learner doing this course? _____

How much time did you spend on studying/homework per week on the course? _____

What about the statistics covered in this class is still puzzling to you? _____

Will you try to find out more about it? _____ How? _____

What is the most meaningful concept/idea you learned about statistics in this class? _____

Why? _____

What grade did you hope for in this class? _____ What grade are you getting? _____

How satisfied are you with this grade? With what you learned? With your own approach? _____

2. d. *Class-Link Evaluation*. This is an evaluation form designed by Learning Assistance Center personnel for students to evaluate the class-link tutor and the instructor's use of her. I administered this form to the class during posttesting, July 31, 2000. See chapters 8 for discussion.

Class Link Evaluation

Tutor's Name: _____

Course: _____

Instructor's Name: _____

Year: _____

[] Fall

[] Spring

1. How often did you see a tutor for this course?
2. Describe how the tutor worked with you. For example, did s/he demonstrate? Ask questions? Read aloud?
3. In what specific ways was the tutor helpful?
4. Were there areas where you feel s/he could have been more helpful? If so, how?
5. Did the tutor ever confuse you? If so, how?

Please answer the following questions by circling the most descriptive response.
The scale ranges from strongly *disagree* (1) to strongly *agree* (5).

Class links are an asset to a class.

1 2 3 4 5

I would prefer to deal with the instructor rather than the class link.

1 2 3 4 5

I sometimes feel the instructor used the class link in order to avoid student contact.

1 2 3 4 5

I do not like having another student involved in my work.

1 2 3 4 5

The class link was knowledgeable about the course content.

1 2 3 4 5

I was usually able to get in contact with my class link when a need arose.

1 2 3 4 5

The class link was reliable in keeping appointments.

1 2 3 4 5

The class link should have been better informed about the requirements and materials of the course.

1 2 3 4 5

The class link was easy to talk with.

1 2 3 4 5

My work improved through my association with the class link.

1 2 3 4 5

The instructor and the class link communicated sufficiently.

1 2 3 4 5

The class link made me feel comfortable in the learning process.

1 2 3 4 5

I sometimes felt that the class link was too critical.

1 2 3 4 5

It is generally helpful to have a class link with whom to discuss ideas.

1 2 3 4 5

I feel that the instructor relied too much on the class link.

1 2 3 4 5

Thank you for taking the time to complete this evaluation. Please add any other comments you wish to make about the tutor(s) and/or the Learning Center on the back of this form.

3. I developed the *Arithmetic for Statistics* instrument during the course as a diagnostic for participants in response to the type of arithmetical reasoning errors I saw and lack of the type of arithmetical reasoning that if used might have led students to correct their errors. I used operation sense and number sense questions suggested by Marolda and Davidson (Marolda & Davidson, 1994), items used by Liping Ma in her assessment of elementary teachers' profound understanding of fundamental arithmetic (Ma, 1999a), a proportional reasoning question, number line scale questions investigating small (decimal) and large numbers, some normal curve area under the curve and horizontal scale questions, a coordinate graph question and a pie graph question. Some questions were open-ended; others closed; some asked for a written explanation. I was able to use it with some of the participants during the course but others completed it during the post-testing session after the MINITAB project presentations in the second to last class meeting (July 31, 2000) and others mailed theirs to me. Robin and Brad did not complete theirs. For further discussion see chapters 6, 7, and 8.

This diagnostic was administered during individual counseling sessions with some students and given to all the students who had not already taken it during the July 31, 2000 post- test session after the MINITAB project presentations.

**Arithmetic for Statistics
Assessment**

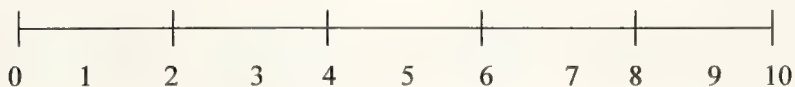
Name _____

Date _____

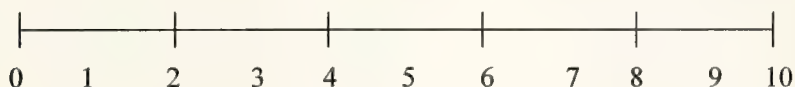
1. When you multiply 61.2 and 3.5 the product is 21.4; 264.2; 2,142 or 214.2?
2. When you divide 12 by 0.12 you get a number smaller than 12? A number smaller than 1? A number larger than 12?
3. When you multiply, you always get a number bigger than the one you started with?
Yes/No Explain.
4. When you divide, you always get a number smaller than the one you started with?
Yes/No Explain.
5. If you earn 10% interest per year on your investment of
 - \$1million, how much would you earn?
 - \$1billion, how much would you earn?
 - \$1thousand, how much would you earn?
 - \$1hundred, how much would you earn?
 - \$10.00, how much would you earn?
 Now work out your earnings if the interest rate is 8%
6. Does $\frac{3}{4}$ lie between $\frac{7}{12}$ and $\frac{2}{3}$? Explain?
7. $1\frac{3}{4} \div \frac{1}{2} =$ Explain.
8. Given that 1 is the largest probability you can get, what could you say about a probability of 0.099? 0.99? 0.119?
9. Is .099 closer to 1 or to 0? Explain.
10. a. Which is a better sale, $\frac{2}{5}$ off or 40% off or .04 off? Why?

b. In a group of 48 students, 1 out of 8 is of African origin, 2 out of 8 is Latino, and 4 out of 8 is of European origin, and the rest are of Asian origin. How many students are there from each racial category in the whole group?

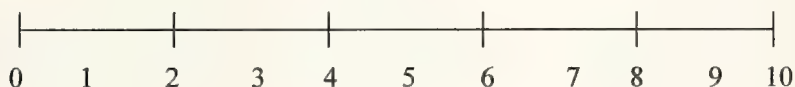
11. On this line place the point 9.9.



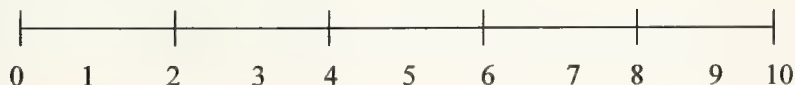
12. On this line, place the points 9 and 0.9 and 0.09 and 0.009



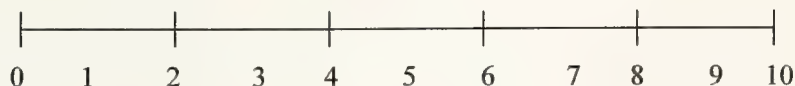
13. On this line place the point 0.99



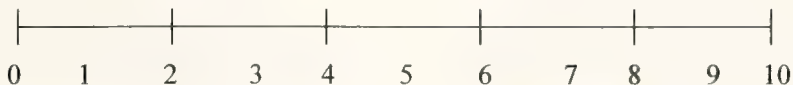
14. On this line place the point 4.19



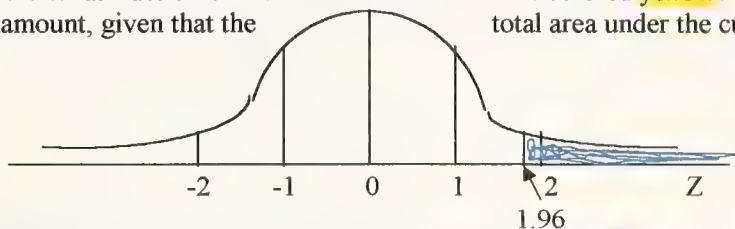
15. On this line place the point 3.99



16. On this line place the point 6.49

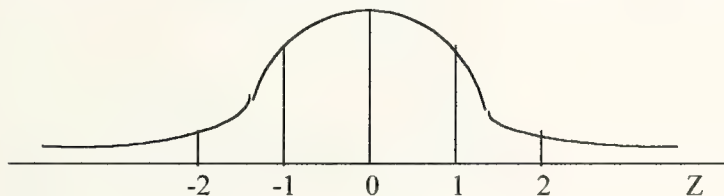


17. What fraction of the area under the curve is colored yellow? What percent? What amount, given that the total area under the curve is 1 unit?

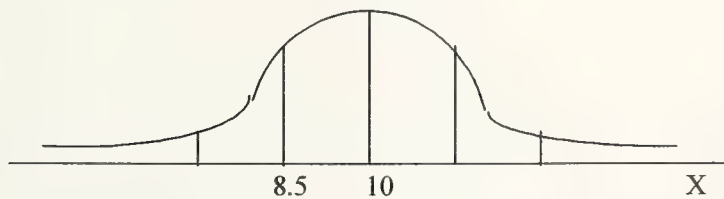
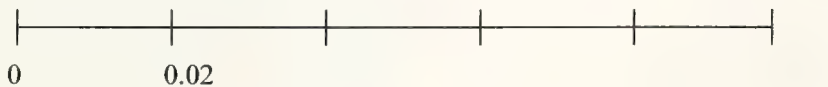


What fraction of the area under the curve is colored blue? What percent? What amount, given that the total area under the curve is 1 unit?

18. $Z = -1.645$. Where should it be on this Standard Normal Graph?



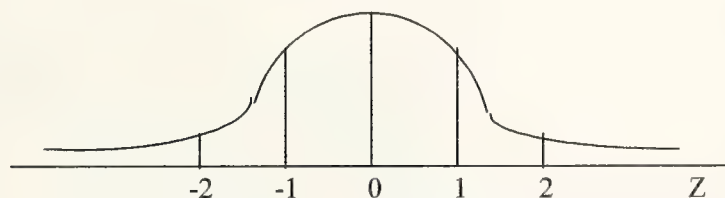
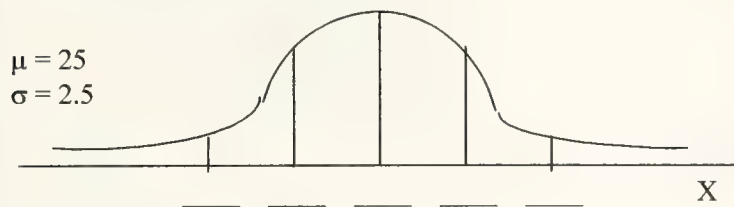
19. Fill in the missing number labels for the points on the line:



20. Fill in the missing number labels for the points on the line



21. a. For the following normal distribution of continuous data, fill in the missing number labels for the points on the line.



b. How is this standard normal distribution graph related to the one above with $\mu = 25$ and $\sigma = 2.5$?

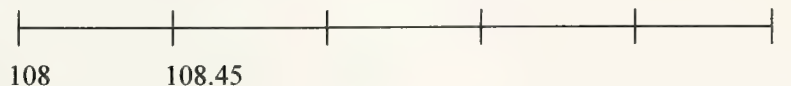
22. Fill in the missing number labels for the points on the line:



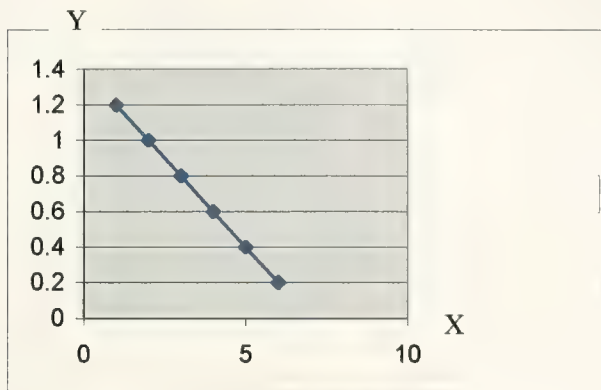
23. Place the points 1.85 and -1.85 on this number line.



24. Fill in the missing number labels for the points on the line:

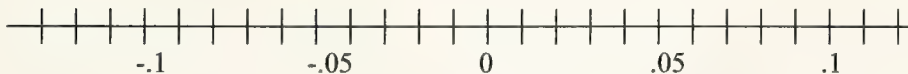
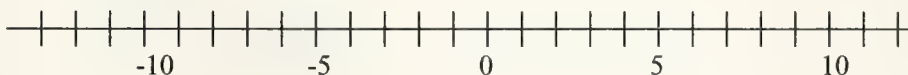
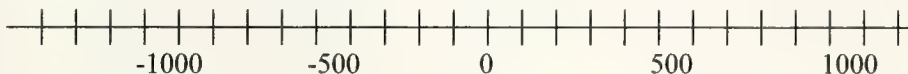


25. From the function graph below find the value of Y for which the X value is 5. Think of a situation in which one variable is related to another in the way shown on the graph below. Fill in the table with data of all the points shown on the function line.



X	Y

26. On **each** of the following three number lines think of three different numbers appropriate for that scale and plot them.



Create a scale on this number line to plot these numbers and then plot them: 25, 150



Create a scale on this number line to plot these numbers and then plot them: 0.04, 0.45, 3.05



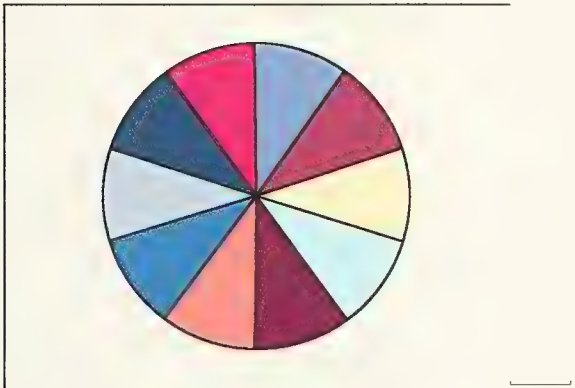
Create a scale on this number line to plot these numbers and then plot them: 1800, 85





27. Block out 0.35 of this pie graph. How much is left?

28. The pie graph below represents the population of 1,500 students at a small liberal arts college. 35% are freshmen; 25% are sophomores; 25% are juniors and the rest are seniors. How many are in each class? Show them on the pie graph. 60% of each of the freshman and sophomore classes are women. 44% of the junior class and 40% of the senior class are men. Create a chart to show the make-up of the college by gender. Show it on the other pie or other graph.



3. b. _____'s Arithmetic for Statistics Understanding Profile

Course: _____

Date: _____

Number Correct	Statistical sense	Small (<1000) Integer Number sense	Large Integer Number sense	Proper Fractional Sense	Place Value/Decimal/ Percent Sense	Operation Sense	Open Ended Arithmetical thinking/ problem solving	Overall level
	17a(3) 17b(3) 18 19b(3) 21a(5) 21b 25(6)	5c. 5d. 5e. 5f(3). 26b(3) 26d(2)	5a. 5b. 5f(2). 26a(3) 26f(2)	6. 7. 10a. 17a(3) 17b(3)	$ a < 5$ 8(3) 9. 10a 12a 12b 12c 12d 13 14 15 18 19a(4) 20(4) 22(4) 23(2) 26c(3) 26e(3) $ a > 5$ 1. 11 16 19b(3) 21a(5) 24(4)	1. 2. 3 4. 7.	3b 4b 6b 7b 8(3) 9b 25(1) 26a(3) 26b(3) 26c(3) 26d(2) 26e(3) 26f(2) 27(2) 28(6)	
Totals	22 or 16	11	9	9 or 3	$ a < 5$: 33 $ a > 5$: 15	5	33or 27w/o28	
%								
Not Attempted								
Comments								

4. *Statistical Reasoning Assessment*. I was aware at the beginning of the study of a number of factors that led me to conjecture that the chief aim of PSYC/STAT 104 would not be to change students' misconceptions about probability and statistics or to develop their statistical reasoning. Instead, I supposed the aim would be to use a traditional lecture and test approach to have students become familiar with standard means of sorting and describing data (descriptive statistics) and with recognizing when and knowing how to use standard parametric and nonparametric statistical analysis to test hypotheses about populations (inferential statistics), that is, to introduce potential social scientists to procedures they would later use to do their own research (see chapter 5). This is not to say that these two aims are necessarily incompatible but it has been demonstrated that even with deliberate and concerted effort and active student involvement with data the former aim is very difficult to accomplish, and without such effort extremely unlikely (Garfield, 1992; Shaughnessy, 1992). In mathematics counseling, however, I hoped to have opportunities to address mathematical and statistical misconceptions, so I felt that a pre- and post- statistical reasoning assessment might reveal changes related to that. Joan Garfield's 20-item multiple choice *Statistical Reasoning Assessment* is well constructed and investigates such faulty heuristics as representativeness (e.g., items 9, 11, 14), the gambler's fallacy (e.g., item 10), base-rate fallacy (e.g., item 12), and correlation as causality (e.g., item 16).

I gave this assessment as a pre-test at the beginning and a posttest at the end of the course. The *Statistical Reasoning Assessment* was used with permission its author Joan Garfield (1998) for purposes of research. See also chapter 8 for discussion of usefulness of this instrument in this study.

Statistical Reasoning Assessment

Name: _____
Date: _____

© Joan Garfield, (1998)

University of Minnesota.

Used with permission, by Jillian Knowles, 12/15/16, Summer 2000

The following pages consist of multiple-choice questions about probability and statistics. Read the question carefully before selecting an answer.

1. A small object was weighed on the same scale separately by nine students in a science class. The weights (in grams) recorded by each student are shown below.

6.2 6.0 6.0 15.3 6.1 6.3 6.2 6.15 6.2

The students want to determine as accurately as they can the actual weight of this object. Of the following methods, which would you recommend they use?

- _____ a. Use the most common number, which is 6.2.
- _____ b. Use the 6.15 since it is the most accurate weighing.
- _____ c. Add up the 9 numbers and divide by 9.
- _____ d. Throw out the 15.3, add up the other 8 numbers and divide by 8.
2. The following message is printed on a bottle of prescription medication:

WARNING: For applications to skin areas there is a 15% chance of developing a rash. If a rash develops, consult your physician.

Which of the following is the best interpretation of this warning?

- _____ a. Don't use the medication on your skin — there's a good chance of developing a rash.
- _____ b. For application to the skin, apply only 15% of the recommended dose.
- _____ c. If a rash develops, it will probably involve only 15% of the skin.
- _____ d. About 15 of 100 people who use this medication develop a rash.
- _____ e. There is hardly a chance of getting a rash using this medication.

3. The Springfield Meteorological Center wanted to determine the accuracy of their weather forecasts. They searched their records for those days when the forecaster had reported a 70% chance of rain. They compared these forecasts to records of whether or not it actually rained on those particular days.

The forecast of 70% chance of rain can be considered very accurate if it rained on:

- _____ a. 95% - 100% of those days.
- _____ b. 85% - 94% of those days.
- _____ c. 75% - 84% of those days.
- _____ d. 65% - 74% of those days.
- _____ e. 55% - 64% of those days.
4. A teacher wants to change the seating arrangement in her class in the hope that it will increase the number of comments her students make. She first decides to see how many comments students make with the current seating arrangement. A record of the number of comments made by her 8 students during one class period is shown below.

	Student Initials							
	A.A.	R.F.	A.G.	J.G.	C.K.	N.K.	J.L.	A.W.
Number of comments	0	5	2	22	3	2	1	2

She wants to summarize this data by computing the typical number of comments made that day. Of the following methods, which would you recommend she use?

- _____ a. Use the most common number, which is 2.
- _____ b. Add up the 8 numbers and divide by 8.
- _____ c. Throw out the 22, add up the other 7 numbers and divide by 7.
- _____ d. Throw out the 0, add up the other 7 numbers and divide by 7.

5. A new medication is being tested to determine its effectiveness in the treatment of eczema, an inflammatory condition of the skin. Thirty patients with eczema were selected to participate in the study. The patients were randomly divided into two groups. Twenty patients in an experimental group received the medication, while ten patients in a control group received no medication. The results after two months are shown below.

Experimental group (Medication)

Improved	8
No Improvement	12

Control group (No Medication)

Improved	2
No Improvement	8

Based on the data, I think the medication was:

_____ 1. somewhat effective

_____ 2. basically ineffective

If you chose option 1, select the one explanation below that best describes your reasoning.

- _____ a. 40% of the people (8/20) in the experimental group improved.
- _____ b. 8 people improved in the experimental group while only 2 improved in the control group.
- _____ c. In the experimental group, the number of people who improved is only 4 less than the number who didn't improve (12-8), while in the control group the difference is 6 (8-2).
- _____ d. 40% of the patients in the experimental group improved (8/20), while only 20% improved in the control group (2/10).

If you chose option 2, select the one explanation below that best describes your reasoning.

- _____ a. In the control group, 2 people improved even without the medication.
- _____ b. In the experimental group, more people didn't get better than did (12 vs 8).
- _____ c. The difference between the numbers who improved and didn't improve is about the same in each group (4 vs 6).
- _____ d. In the experimental group, only 40% of the patients improved (8/20).

6. Listed below are several possible reasons one might question the results of the experiment described above. Place a check by every reason you agree with.

- ☐ a. It's not legitimate to compare the two groups because there are different numbers of patients in each group.
- ☐ b. The sample of 30 is too small to permit drawing conclusions.
- ☐ c. The patients should not have been randomly put into groups, because the most severe cases may have just by chance ended up in one of the groups.
- ☐ d. I'm not given enough information about how doctors decided whether or not patients improved. Doctors may have been biased in their judgments.
- ☐ e. I don't agree with any of these statements.

7. A marketing research company was asked to determine how much money teenagers (ages 13 - 19) spend on recorded music (cassette tapes, CDs and records). The company randomly selected 80 malls located around the country. A field researcher stood in a central location in the mall and asked passers-by who appeared to be the appropriate age to fill out a questionnaire. A total of 2,050 questionnaires were completed by teenagers. On the basis of this survey, the research company reported that the average teenager in this country spends \$155 each year on recorded music.

Listed below are several statements concerning this survey. Place a check by every statement that you agree with.

- ☐ a. The average is based on teenagers' estimates of what they spend and therefore could be quite different from what teenagers actually spend.
- ☐ b. They should have done the survey at more than 80 malls if they wanted an average based on teenagers throughout the country.
- ☐ c. The sample of 2,050 teenagers is too small to permit drawing conclusions about the entire country.
- ☐ d. They should have asked teenagers coming out of music stores.
- ☐ e. The average could be a poor estimate of the spending of all teenagers given that teenagers were not randomly chosen to fill out the questionnaire.
- ☐ f. The average could be a poor estimate of the spending of all teenagers given that only teenagers in malls were sampled.
- ☐ g. Calculating an average in this case is inappropriate since there is a lot of variation in how much teenagers spend.
- ☐ h. I don't agree with any of these statements.

8. Two containers, labeled A and B, are filled with red and blue marbles in the following quantities:

Container	Red	Blue
A	6	4
B	60	40

Each container is shaken vigorously. After choosing one of the containers, you will reach in and, without looking, draw out a marble. If the marble is blue, you win \$50. Which container gives you the best chance of drawing a blue marble?

- _____ a. Container A (with 6 red and 4 blue)
- _____ b. Container B (with 60 red and 40 blue)
- _____ c. Equal chances from each container

9. Which of the following sequences is most likely to result from flipping a fair coin 5 times?

- _____ a. H H H T T
- _____ b. T H H T H
- _____ c. T H T T T
- _____ d. H T H T H
- _____ e. All four sequences are equally likely

10. Select one or more explanations for the answer you gave for the item above.

- _____ a. Since the coin is fair, you ought to get roughly equal numbers of heads and tails.
- _____ b. Since coin flipping is random, the coin ought to alternate frequently between landing heads and tails.
- _____ c. Any of the sequences could occur.
- _____ d. If you repeatedly flipped a coin five times, each of these sequences would occur about as often as any other sequence.
- _____ e. If you get a couple of heads in a row, the probability of a tails on the next flip increases.
- _____ f. Every sequence of five flips has exactly the same probability of occurring.

11. Listed below are the same sequences of Hs and Ts that were listed in Item 8. Which of the sequences is least likely to result from flipping a fair coin 5 times?

_____ a. H H H T T

_____ b. T H H T H

_____ c. T H T T T

_____ d. H T H T H

_____ e. All four sequences are equally unlikely

12. The Caldwells want to buy a new car, and they have narrowed their choices to a Buick or a Oldsmobile. They first consulted an issue of Consumer Reports, which compared rates of repairs for various cars. Records of repairs done on 400 cars of each type showed somewhat fewer mechanical problems with the Buick than with the Oldsmobile.

The Caldwells then talked to three friends, two Oldsmobile owners, and one former Buick owner. Both Oldsmobile owners reported having a few mechanical problems, but nothing major. The Buick owner, however, exploded when asked how he liked his car:

First, the fuel injection went out — \$250 bucks. Next, I started having trouble with the rear end and had to replace it. I finally decided to sell it after the transmission went. I'd never buy another Buick.

The Caldwells want to buy the car that is less likely to require major repair work. Given what they currently know, which car would you recommend that they buy?

- _____ a. I would recommend that they buy the Oldsmobile, primarily because of all the trouble their friend had with his Buick. Since they haven't heard similar horror stories about the Oldsmobile, they should go with it.
- _____ b. I would recommend that they buy the Buick in spite of their friend's bad experience. That is just one case, while the information reported in Consumer Reports is based on many cases. And according to that data, the Buick is somewhat less likely to require repairs.
- _____ c. I would tell them that it didn't matter which car they bought. Even though one of the models might be more likely than the other to require repairs, they could still, just by chance, get stuck with a particular car that would need a lot of repairs. They may as well toss a coin to decide.

13. Five faces of a fair die are painted black, and one face is painted white. The die is rolled six times. Which of the following results is more likely?

_____ a. Black side up on five of the rolls; white side up on the other roll

_____ b. Black side up on all six rolls

_____ c. a and b are equally likely

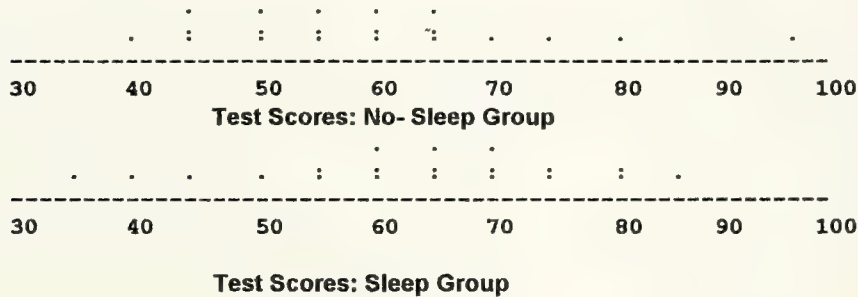
14. Half of all newborns are girls and half are boys. Hospital A records an average of 50 births a day. Hospital B records an average of 10 births a day. On a particular day, which hospital is more likely to record 80% or more female births?

_____ a. Hospital A (with 50 births a day)

_____ b. Hospital B (with 10 births a day)

_____ c. The two hospitals are equally likely to record such an event.

15. Forty college students participated in a study of the effect of sleep on test scores. Twenty of the students volunteered to stay up all night studying the night before the test (no-sleep group). The other 20 students (the control group) went to bed by 11:00 p.m. on the evening before the test. The test scores for each group are shown in the graphs below. Each dot on the graph represents a particular student's score. For example, the two dots above the 80 in the bottom graph indicate that two students in the sleep group scored 80 on the test.



Examine the two graphs carefully. Then choose from the 6 possible conclusions listed below the one you most agree with.

- _____ a. The no-sleep group did better because none of these students scored below 40 and the highest score was achieved by a student in this group.
- _____ b. The no-sleep group did better because its average appears to be a little higher than the average of the sleep group.
- _____ c. There is no difference between the two groups because there is considerable overlap in the scores of the two groups.
- _____ d. There is no difference between the two groups because the difference between their averages is small compared to the amount of variation in the scores.
- _____ e. The sleep group did better because more students in this group scored 80 or above.
- _____ f. The sleep group did better because its average appears to be a little higher than the average of the no-sleep group.

16. For one month, 500 elementary students kept a daily record of the hours they spent watching television. The average number of hours per week spent watching television was 28. The researchers conducting the study also obtained report cards for each of the students. They found that the students who did well in school spent less time watching television than those students who did poorly.

Listed below are several possible statements concerning the results of this research. Place a check by every statement that you agree with.

- ☐ a. The sample of 500 is too small to permit drawing conclusions.
- ☐ b. If a student decreased the amount of time spent watching television, his or her performance in school would improve.
- ☐ c. Even though students who did well watched less television, this doesn't necessarily mean that watching television hurts school performance.
- ☐ d. One month is not a long enough period of time to estimate how many hours the students really spend watching television.
- ☐ e. The research demonstrates that watching television causes poorer performance in school.
- ☐ f. I don't agree with any of these statements.

17. The school committee of a small town wanted to determine the average number of children per household in their town. They divided the total number of children in the town by 50, the total number of households. Which of the following statements must be true if the average children per household is 2.2?

- ☐ a. Half the households in the town have more than 2 children.
- ☐ b. More households in the town have 3 children than have 2 children.
- ☐ c. There are a total of 110 children in the town.
- ☐ d. There are 2.2 children in the town for every adult.
- ☐ e. The most common number of children in a household is 2.
- ☐ f. None of the above.

18. When two dice are simulataneously thrown it is possible that one of the following two results occurs:

Result 1: A 5 and a 6 are obtained.

Result 2: A 5 is obtained twice.

Select the response that you agree with the most:

- a. The chances of obtaining each of these results is equal
- b. There is more chance of obtaining result 1.
- c. There is more chance of obtaining result 2.
- d. It is imposible to give an answer. (Please explain why)

19. When three dice are simultaneously thrown, which of the following results is **MOST LIKELY** to be obtained?

- a. *Result 1:* "A 5, a 3 and a 6"
- b. *Result 2:* "A 5 three times"
- c. *Result 3:* A 5 twice and a 3"
- d. All three results are equally likely

20. When three dice are simultaneously thrown, which of these three results is **LEAST LIKELY** to be obtained?

- a. *Result 1:* "A 5, a 3 and a 6"
- b. *Result 2:* "A 5 three times"
- c. *Result 3:* A 5 twice and a 3"
- d. All three results are equally unlikely

5. The *Algebra Test*, adapted from the *Chelsea Diagnostic Algebra Test*. This test is one of ten designed as a diagnostic instrument to be used “both for ascertaining a child’s [aged 12 through 15+ years] level of understanding and to identify the incidence of errors” by the mathematics research team of the British Social Science Research Council Program ‘Concepts in Secondary Mathematics and Science’ (CSMS)(Brown, Hart, & Kuchemann, 1985). The research was carried out “broadly within a Piagetian framework.” In particular, the algebra test specifies four levels of understanding of the algebraic from level 1 at which a letter can be evaluated by recalling an arithmetical relationship and letter objects to be collected are all of one type, through level 4 at which the letter is understood at least as specific unknowns or generalized numbers (and in some cases as variables) and two operations can be coordinated. Sokolowski designed a fifth level at which the letter is understood as having “a range of numbers (a dynamic view) that is, as a true variable and coordinated operations can be reordered and reconfigured” (Sokolowski, 1997, pp. 97-98). Sokolowski’s level 5 items have not been subjected to the rigorous clustering and leveling analysis applied by CSMS to the level 1 through 4 questions, however. Sokolowski also made minor language and setting changes in the test to make it comprehensible to students in the New England area of the U.S.

In this study it was expected that some participants’ difficulties with the mathematics could be linked directly to weak mathematical backgrounds, gaps, and primitive understanding of the algebraic variable. Others’ difficulties were expected to be *in spite of* sound mathematical and algebraic concepts. I believed that the *Algebra Test* would be a valuable tool for helping pinpoint a symptomatic (mathematical) focus for the

former, and an explanation (removable by education) *other than intrinsic inability* for their troubles. For the latter it could be used as evidence to refute their negative opinions of their mathematical functioning that were contributing to their helplessness and poor achievement. I used the *Algebra Test* with permission. See chapter 5 and 6 for further discussion. The *Algebra Test* was used with permission for the purposes of this research. See chapter 6, 7, and 8 for further discussion.

Name

Date

Algebra Test

Practice Item 1

1. What number does $a + 4$ stand for if $a = 2$ _____

if $a = 5$ _____

Practice Item 2

2. Fill in the blanks:

Work down the page

$$x \rightarrow 3x$$

$$2 \rightarrow 6$$

$$5 \rightarrow \underline{\hspace{2cm}}$$

$$x \rightarrow x + 3$$

$$5 \rightarrow 8$$

$$4 \rightarrow \underline{\hspace{2cm}}$$

$$n \rightarrow \underline{\hspace{2cm}}$$

$$x \rightarrow 7x$$

$$2 \rightarrow \underline{\hspace{2cm}}$$

$$x \rightarrow x + 8$$

$$3 \rightarrow \underline{\hspace{2cm}}$$

1. Fill in the blanks:

$$x \rightarrow x + 2$$

$$x \rightarrow 4x$$

$$6 \rightarrow \underline{\hspace{2cm}}$$

$$3 \rightarrow \underline{\hspace{2cm}}$$

$$r \rightarrow \underline{\hspace{2cm}}$$

2. Write the smallest and the largest of these:

smallest

largest

$$n + 1, \quad n + 4, \quad n - 3, \quad n, \quad n - 7$$

3. Which is larger, $2n$ or $n + 2$?

Explain.

4. 4 added to n can be written as $n + 4$. n multiplied by 4 can be written as $4n$.

Add 4 to each of these:

Multiply each of these by 4:

$$8$$

$$n + 5$$

$$3n$$

$$8$$

$$n + 5$$

$$3n$$

5. If $a + b = 43$

If $n - 246 = 762$

If $e + f = 8$

$$a + b + 2 = \underline{\hspace{2cm}}$$

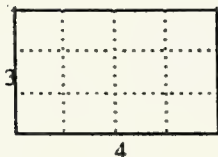
$$n - 247 = \underline{\hspace{2cm}}$$

$$e + f + g = \underline{\hspace{2cm}}$$

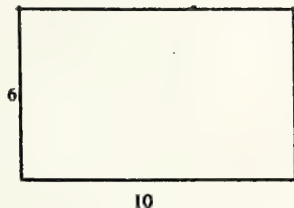
6. What can you say about a if $a + 5 = 8$

What can you say about b if $b + 2$ is equal to $2b$

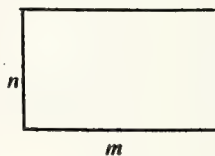
7. What are the areas of these shapes?



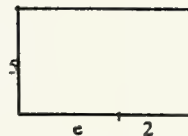
$A =$ _____



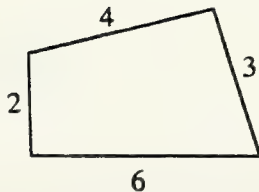
$A =$ _____



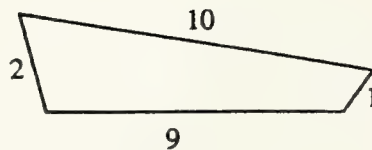
$A =$ _____



$A =$ _____

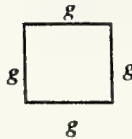


8. The perimeter of this shape is equal to $6 + 3 + 4 + 2$, which equals 15.



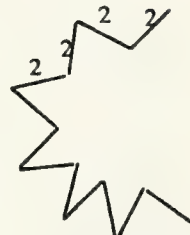
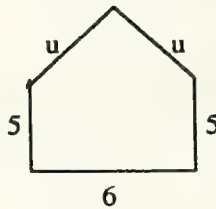
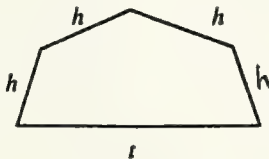
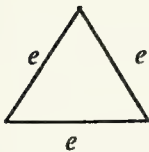
What is the perimeter of this shape? $P =$ _____

9. This square has sides of length g .



So, for its perimeter, we can write $P = 4g$.

What can we write for the perimeter of each of these shapes?



Part of this figure is not drawn. There are n sides altogether, all of length 2.

$P =$ _____

$P =$ _____

$P =$ _____

$P =$ _____

10. Small apples cost 8 cents each and small pears cost 6 cents each.

If a stands for the number of apples bought
and p stands for the number of pears bought,
what does $8a + 6p$ stand for? _____

What is the total number of fruits bought? _____

11. What can you say about u if $u = v + 3$

and $v = 1$ _____

What can you say about m if $m = 3n + 1$

and $n = 4$ _____

12. If John has J compact discs and Peter has P compact discs, what could you write for the number of compact discs they have altogether? _____

13. $a + 3a$ can be written more simply as $4a$.

Write these more simply, where possible:

$$2a + 5a = \underline{\hspace{2cm}}$$

$$2a + 5b = \underline{\hspace{2cm}}$$

$$(a + b) + a = \underline{\hspace{2cm}}$$

$$2a + 5b + a = \underline{\hspace{2cm}}$$

$$(a - b) + b = \underline{\hspace{2cm}}$$

$$3a - (b + a) = \underline{\hspace{2cm}}$$

$$a + 4 + a - 4 = \underline{\hspace{2cm}}$$

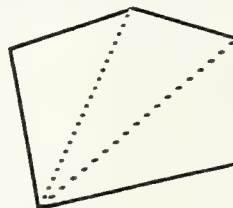
$$3a - b + a = \underline{\hspace{2cm}}$$

$$(a + b) + (a - b) = \underline{\hspace{2cm}}$$

14. What can you say about r if $r = s + t$

and $r + s + t = 30$? _____

15. In a shape like this you can determine the number of diagonals from one vertex



by taking away 3 from the number of sides.

So, a shape with 5 sides has 2 diagonals;

a shape with 57 sides has _____ diagonals;

a shape with k sides has _____ diagonals.

16. What can you say about c if $c + d = 10$

and c is less than d _____

17. Mary's basic wage is \$200 per week.

She is also paid another \$7 for each hour of overtime that she works.

If h stands for the number of hours of overtime that she works, and
if W stands for her total weekly wages (in \$),
write an equation connecting W and h . _____

What would Mary's total weekly wages be if she
worked 4 hours of overtime? _____

18. When are the following true – always, never, or sometimes?

Underline the correct answer:

$A + B + C = C + A + B$ Always Never Sometimes, when _____

$L + M + N = L + P + N$ Always Never Sometimes, when _____

19. $a = b + 3$. What happens to a if b is increased by 2? _____

$f = 3g + 1$ What happens to f if g is increased by 2? _____

20. Bagels cost b cents each and muffins cost m cents each.

If I buy 4 bagels and 3 muffins,

what does $4b + 3m$ stand for? _____

21. If this equation $(x + 1)^3 + x = 349$ is true when $x = 6$,
then

what value of x will make this equation, $(5x + 1)^3 + 5x = 349$, true?

$x =$ _____

22. Fine point black pens cost \$3 each and medium point red pens cost \$2 each.

I went to Staples in Salem, New Hampshire, and bought some of each type of pen, spending a total of \$25.

If b is the number of black pens

and if r is the number of red pens bought,

what can you write about b and r ? _____

23. You can feed any number into this machine:



Can you find another machine that

has the same overall effect?



Note: The *Chelsea Diagnostic Algebra Test* (Brown et al., 1985) was used for this research with the written permission of its publishers.

Algebra

Levels of Understanding

Name _____

Course/Semester _____

Date _____

Last Math Course _____

Level 1	Level 2	Level 3	Level 4	Level 5
5 (a)	7 (c)	4 (c)	3	19a
6 (a)	9 (b)	5 (c)	4 (e)	19b
7 (b)	9 (c)	9 (d)	7 (d)	23
8	11 (a)	13 (b)	13 (e)	
9 (a)	11 (b)	13 (h)	17 (a)	
13 (a)	13 (d)	14	18 (b)	
	15 (a)	15 (b)	20	
		16	21	
			22	
4/6	5/7	5/8	6/9	

Totals 1: _____ 2: _____ 3: _____ 4: _____

Math Course Taking History

High School

Freshman _____

Sophomore _____

Junior _____

Senior _____

College

Freshman _____

Sophomore _____

Junior _____

Senior _____

6. Observation Tools:

a. Music Staff Class Interaction Analysis Chart

Start Time:

End time:

I: S ^a	
I: Q	
S: Q	
S: S	

Start Time:

End time:

I: S	
I: Q	
S: Q	
S: S	

Start Time:
time:

End

I: S	
I: Q	
S: Q	
S: S	

^aI: S indicates Instructor's Statement; I: Q indicates Instructor's Questions; S: S indicates Student's Statement; S: Q indicates Student's Questions

6. b. Class Layout Observation Form

Front Right

Front Left

A diagram of a classroom layout. The main area is a large rectangle divided into four quadrants by a vertical line and a horizontal line. Surrounding this central area are boxes representing student seating. On the left side, there are six boxes arranged vertically. On the right side, there are six boxes arranged vertically. At the bottom, there are two boxes side-by-side. At the top, there is a long horizontal box. In the top right corner, there are two parallel diagonal lines.

6. c. Problem Working Session Interaction Chart: With seating for Class 13

Time	
P ^a	
A	
R	
B	
Jillian	
Ka	
C	
M	
J	
L	
M	

Note. A = Autumn; B = Brad; J = Jamie; Ka = Karen; Ke = Kelly; L = Lee; Mi = Mitch; Mu = Mulder; P = Pierre; R = Robin.

^aSeated with Pierre front right going counterclockwise to Mitch seated front left.

6. d. Class # _____ Summary

Course/Semester _____

Date _____

Professor _____

Location _____

Classroom configuration and individual's locations:

Summary of class

Interactions

Teaching/Learning

Participant/observer issues

Thoughts for next class:

Appendix D

Research Information and Informed Consent Forms

Learning Assistance for a College Undergraduate Mathematics Class

Doctoral Dissertation Research

Jillian Knowles

Summer, 2000

My aim in this research is to investigate the role of a number of different types of learning assistance interventions in helping students who are taking a required undergraduate mathematics course to not only pass the course, but also to improve their grasp of and approach to mathematics. In order to do this, in all aspects of the research, I will be investigating each participating student's own ideas and feelings on his or her issues around mathematics learning at college.

I, as the participant researcher, will be:

- attending and observing all the Summer 2000 PSYC/STAT 104 classes. This will include my giving the class two pre and post surveys on beliefs and feelings around mathematics. [Complete confidentiality is assured.]
- I will be organizing and observing (including audio-taping these sessions) a weekly study group that will meet before class at 4:30pm on Wednesdays in University Center, Room 254 [Complete confidentiality is assured.]
- I will be offering Drop-In and by-appointment mathematics tutoring in the Learning Assistance Center, Greenville campus, Room 203. [Complete confidentiality is assured], and, finally,
- I will be offering one-on-one mathematics counseling (audio-taped, transcribed and analyzed) to volunteers who want to work on their emotional and mathematical background issues in order to improve their approach to and achievement in mathematics in life and in college. There is a possibility of follow-up of individuals per mutual agreement with me. [Complete confidentiality is assured.]

Please Note:

1. The personal identity of each participant in this study will be kept confidential. Each participant will be assigned an assumed name (You can choose!). All analysis and reporting will use these assumed names and the setting will be disguised.

2. **Your participation or non-participation in this study will in no way affect your grade in this course.** If you do **not** wish to participate you will indicate that by **not** filling in the class surveys and by **not** signing the permission sheet at the study-group and Drop-In. Alternately, on the class surveys, you may be willing to complete them using a number rather than your name, remembering the same number for the post tests. In that way, there will be complete data for the class but your individual responses will not be directly linked to you.
3. You are encouraged to take advantage of any or all of the above learning assistance offerings. Being involved in one does NOT mean you cannot take advantage of others.

Analysis will involve some quantitative and much qualitative work. Quantitative analysis of the pre and post surveys using, amongst other tests, Student's t test difference of means for dependent groups will be clarified using qualitative data. Qualitative analysis will involve developing grounded theory. This means that I will have to be continually noting and setting aside my own assumptions about what are your key issues around your mathematics learning and listening to and hearing you. I will work at producing draft theories for you to look at and critique, until a grounded theory is developed. This study will then be reported in my doctoral dissertation for Lesley College, Cambridge MA.

Jillian Knowles,

Local Identification and Contact Information

A Call for VOLUNTEERS

I'm looking for people who want to learn how to do their mathematics more effectively. I need several volunteers who will agree to meet with me regularly (for 1 hour per week or once every other week for 1 hour per session) for the duration of the Statistics in Psychology PSYC/STAT 104, Summer 2000 course, to engage in one-on-one mathematics counseling—working on both your mathematics and also your emotional issues around mathematics.

If you have issues around mathematics learning that you feel may make it harder to succeed in this course, maybe this could help. I have worked with college students, teaching, tutoring, and helping them with their mathematics for many years. In my doctoral studies I have been looking for better, more effective ways to do this. In this dissertation research project, I wish to explore these new ways with students who want to improve how they do mathematics. It will be completely confidential and should lead to improved ways of doing mathematics.

If you would like to work with me, please respond “Yes” on the attached index card which I will collect with your surveys. If you want more time to think about it, come to see me at the Learning Assistance Center, Room 203, Greenville campus, call me at the Learning Assistance Center at ____ or at home ____ or e-mail me at ____ or at _____. I DO need to know by Wednesday, June 7, because the course time is so short, so you could let me know at the study group or in class on June 7.

Jillian Knowles
Local Contact Information
Doctoral student
Lesley College,
Cambridge, MA

DISSERTATION RESEARCH INFORMED CONSENT FORM

I, _____, a) affirm that I have read and Jillian Knowles has explained the objectives of her research, the procedures to be followed and the potential risks and benefits. yes/no

b) understand that my participation or nonparticipation in this research project will not affect my grade in Dr Paglia's Statistics in Psychology PSYC 402 Summer 2000 class yes/no

c) understand that I am free NOT to respond to any part of the research yes/no

d) understand that I can withdraw from the research at any time yes/no

e) affirm that I have volunteered to be involved in this research of my own free will, without coercion by Jillian Knowles or any other person yes/no

f) agree that the information I give may be discussed only with Jillian Knowles' dissertation committee members at Lesley College, Cambridge, MA, using my name/under an assumed name, and used to write her dissertation for her doctoral degree. Otherwise all materials and information about me she gathers will be kept completely confidential—in particular, they will NOT be shared with any persons or institutions within the University of New Hampshire at Manchester

yes/no

g) assert that if Jillian Knowles chooses at some time to include any information I give in a published article/book, she may do so with/without my written/verbal consent yes/no

h) Jillian Knowles will not publish materials about me without having allowed me to review the relevant part of article/book first yes/no

i) Jillian Knowles will keep audio-tapes and transcripts of this data in a secure place and will only allow direct access to it by her dissertation committee. Access by others will only be allowed with my verbal/written permission yes/no

Signed by me _____ this ____ day of _____, 20____

Name _____

Address _____

Phone _____ e-mail _____

Individual Mathematics Counseling

Sign-Up Card

A personalized copy of this 4inch by 6inch response card was given to each student in the PSYC/STAT 104 class during the second class of the course. All students responded and returned their cards at that time.

Student Name	6/5/00
I would like to meet with Jillian Knowles for	
<input type="checkbox"/>	1 hour per week
<input type="checkbox"/>	1 hour every other week
[Please check one]	
beginning this week (if possible) until the end of the summer 2000, PSYC/STAT 104 Statistics course, to do one-on-one mathematics counseling.	
Yes/No	[Please circle one]
Signed by (optional).....	

Appendix E

Coding and Analysis

In this study looked at students' sense of mathematics self, their mathematics internalized presences, and their mathematics attachments to better understand their state of mathematics functioning or mathematics mental health that would lead to strategic approaches to helping them negotiate their college mathematics course, I analyzed our interactions, their behaviors and utterances in class, study group, and in counseling sessions, their responses to the instruments and their mathematical products in terms of these three dimensions. I wished to determine if the three dimensions provided a reasonable framework for understanding their mathematics functioning but also if there were important elements that could not be understood this way. I wanted to see if students' affective and cognitive symptoms of dysfunction could be better understood via this framework.

The central task for the relational mathematics counselor in this study was continually culling relevant data from the voluminous observations and then processing the data in order to help the student grow in his mathematical functioning and relationships. That processing as Arlow (1995) and others in the psychoanalytic tradition point out has "an aesthetic [aspect] that depends on empathy, intuition, and introspection" (p.144) and a cognitive aspect that "depends on rationally assembled, methodologically disciplined conclusions from the data of observation" (p. 44). Since in psychoanalysis as in mathematics counseling, life and class events change the context and meaning of observations, the many variables are impossible to control; hence the need to limit the dimensions of the issues under investigation in any empirical investigation (Arlow, 1995). The stance that psychoanalysis and empirical investigation are antithetical is giving way to more and more nuanced standardized methodologies such as using guided central relationship measures to guide the therapy more systematically and allow for more empirical evaluation of

techniques, their underlying rationales, and outcomes (Luborsky & Luborsky, 1995). For my purposes here the use of a modified guided central mathematics relationship measure to guide the counseling and, via analysis, to trace its path retrospectively, seems appropriate.

My task in tracing the path of mathematics counseling and analyzing its efficacy is in some senses easier than the task of the psychoanalyst. Since the central symptomatic focus for each tutee is mastering the mathematics course, his mathematical behaviors in the classroom (see Table E2 and Table E3) and in the counseling sessions and his mathematical products for the course: homework, projects and especially exams(see Table E4), provide central data for charting his progress. I was also able to follow targeted mathematics affective symptoms and their changes through pre and post feeling and belief/attitude surveys. It was the relational changes that I hypothesize underlie his mathematical cognitive and affective changes that I need a guided central relationship measure to gauge (see Table E1). In this also I have an advantage over the psychoanalyst, who only sees the client in the counseling setting, since I see the student not only in the counseling setting but also in the central forum of his present mathematics life—the classroom—so what he reports in the counseling session of his experiences in class I and the instructor also observe (see Table E1).

On the other hand the major disadvantage in trying to formulate a student's central relational pattern or conflict lies in the fact that the central focus in the mathematics counseling is on the student doing mathematics rather than on his relational, albeit mathematics relational, conflicts. This means that relatively little time is spent in a mathematics counseling session in *talking* about his past and present mathematics relationships. Therefore there is substantially less direct student-initiated relational data from the sessions, especially relational data with respect to the counselor. This may be related to the predominance, from the student's perspective, of the tutor role over the counselor role in

this setting and his concomitant expectations of and desires for what mathematics counseling would entail—that is, mathematics tutoring.

The basic organizational unit I used to do identify a student's central mathematics relational pattern was the *relational episode*. This involved first locating and identifying narrativesⁱ (called relationship episodes) and then reviewing the relationship episodes and extracting the central relationship theme from them (Luborsky & Luborsky, 1995). Three components that Luborsky (1976) finds prominent in these relational episodes are: what the patient wanted from other people; how the other people reacted; and how the patient reacted to their reaction. Other researchers include disguised allusions, acts of self, expectations of others, consequent acts of others towards self and consequent acts of self towards self (cf. Gill and Hoffmann, 1982; Schacht et al., 1984)

My adaptations for mathematics achievement setting are: Central relationship pattern with respect to self:

1. what student wants/expects from self;
2. student's achievements;
3. how other people reacted to student's achievements;
4. how the student has reacted to others' reactions to his achievements; and
5. mathematics as part of self

Central Relationship Pattern with respect to internalized presences:

1. what student wanted/expected from other people;
2. how other people reacted to student;
3. how the student reacted to their reactions; and
4. mathematics as internalized other

Central (interpersonal) Attachment Pattern:

1. what student wants/expects from other people;
2. how other people react to student;
3. how the student reacts to their reactions; and
4. attachment to mathematics

A crucial concern in understanding a student's central relationship pattern from a relational conflict perspective is the understanding that the student is dealing with patterns of conflict, parts of which he is conscious and parts of which he is unconscious. This means that his verbal statements and behaviors will likely include ones that appear to and some that do contradict his basic wishes. In order to identify a student's central relationship pattern given this difficulty I adopted the following four principles developed by Luborsky and Luborsky (1995):

1. The central conflictual relationship theme may have an opposite conflicting less conscious theme,
2. A wish frequently expressed may have a less frequent (but perhaps more intense) version of that wish in reduced awareness,
3. Instances of denial are likely to point to content that is in reduced awareness,
4. If a student refers to a history of difficulties with awareness this might infer present similar difficulties that he does not acknowledge. (p. 345)

Table E1

Analysis scheme for Counseling Session Data: Student's Mathematical Relationality

	Mathematics Self	Mathematics Internalized Presences	Mathematics Interpersonal Environment
Relational Assessment Categories	Mathematics Identity Central Relationship Patterns 1. With self 2. With mathematics	Object relations Central Relationship Patterns 1. With internalized others 2. With mathematics	Attachments Central Relationship Patterns 1. With others now 2. With mathematics now
Central relationship measure categories	what a student wants/expects from self; a student's achievements; how other people reacted to student's achievements; how the student reacted to others' reactions to his achievements; and mathematics as part of self <i>Metaphor Survey^a</i>	what student wanted/expected from other people; how other people reacted to student; how the student reacted to their reactions; and mathematics as internalized other <i>Metaphor Survey</i>	what a student's wants/expects from other people; how other people react to student; how the student reacts to their reactions; and attachment to mathematics <i>Metaphor Survey</i>
Mathematics Affect	Testing or mathematics anxiety as extinction anxiety: <i>History (re teacher's mirroring and invitation to idealize), Feelings Survey, Metaphor Survey, Test Taking behaviors</i> . Mathematics depression, learned helplessness as empty depression; mathematics grandiosity <i>History (mirroring and invitation to idealize), Beliefs Survey, JMK Affect Scales; class and counseling behaviors</i>	Testing or mathematics anxiety as social anxiety, adjustment disorder, <i>PSTD</i> , phobia, ... <i>Feelings Survey, Metaphor Survey, History (re critical incidents); classroom behaviors versus classroom "reality."</i> Mathematics depression related to a severe mathematics super ego/ internal saboteur	Testing or mathematics anxiety as separation (from teacher or mathematics) anxiety <i>History (re separation, loss, change)</i> <i>Feelings Survey Metaphor Survey; teacher and counselor related behaviors.</i> Mathematics depression related to separation or loss
Mathematics Cognition	Elementary Mathematics: <i>PSYC/STAT 104 Exams, Arithmetic for Statistics Assessment</i> ; High School Mathematics: <i>HS courses/grades, The Algebra Test</i> ; Intro Statistics: <i>PSYC/STAT 104 Exams</i>	Elementary Mathematics: <i>PSYC/STAT 104 Exams, Arithmetic for Statistics Assessment</i> ; High School Mathematics: <i>HS courses/grades, The Algebra Test</i> ; Intro Statistics: <i>PSYC/STAT 104 Exams</i>	Elementary Mathematics: <i>PSYC/STAT 104 Exams, Arithmetic for Statistics Assessment</i> ; High School Mathematics: <i>HS courses/grades, The Algebra Test</i> ; Intro Statistics: <i>PSYC/STAT 104 Exams</i>

Counselor's

countertransference

^a The items in italics are instruments, protocols, mathematics products, demographic and behavioral data that were used in conjunction with audiotaped counseling session data to develop a profile of a student's mathematics functioning and his central mathematics relational pattern.

Table E2

Analysis of Lecture Session Student Exchanges with Instructor

Student Questions	Student Answers	Student Comments
Timing ^a	Timing	Timing
Relevance	Accuracy	Relevance
Topic: 1. current content: mathematics; application; personal 2. course strategy 3. grading	Topic: 1. current content: mathematics; application; personal 2. course strategy 3. grading	Topic: 1. current content: mathematics; application; personal 2. course strategy 3. grading
Level of certainty: 1. affective, 2. cognitive	Level of certainty: 1. affective, 2. cognitive	Level of certainty: 1. affective, 2. cognitive
Frequency	Frequency	Frequency
Development	Development	Development
Implications re student's 1. mathematics self 2. internalized presences 3. attachments: to teacher; to mathematics Implications re student's auditory processing	Implications re student's 1. mathematics self 2. internalized presences 3. attachments: to teacher; to mathematics Implications re student's auditory processing	Implications re student's 1. mathematics self 2. internalized presences 3. attachments: to teacher; to mathematics Implications re student's auditory processing
Central relational conflict or theme		

^aTiming is judged in terms of the extent to which the student's verbalization is linked in a timely manner with the instructor's utterance. For example, on a number of occasions Robin answered Ann's question with the correct answer to a *previous* question; her timing was off.

Table E3

Analysis of Student's Problem Working Session Behaviors					
P A R A M E T E R S	Topic/Task:	Seated beside: 1. Left 2. Right	Tools: 1. text 2. provided by instructor 3. student aids: calculator, notes,...	Interaction with Instructor	Interaction with researcher
	Cognitive preparation: 1. Background 2. Homework	Peer relational behaviors: 1. social learner ^a 2. voluntary loner ^a 3. involuntary loner ^a	Mathematics learning style Behaviors: 1. Analytic (Mathematics Learning Style I) ^b 2. Global (Mathematics Learning Style II) ^b 3. Harmonic I ^c 4. Harmonic II ^c	Student- teacher relational behaviors: 1. Secure attachment ^d 2. Insecure avoidant ^d 3. Insecure dependent ^d 4. Insecure disorganized ^d	Student- researcher relational behaviors: 1. Secure attachment ^d 2. Insecure avoidant ^d 3. Insecure dependent ^d 4. Insecure disorganized ^d
Changes					
R E L A T I O N A L I T Y	Implications Re: 1. Self 2. Internalized presences 3. Attachments	Implications Re: 1. Self 2. Internalized presences 3. Attachments	Implications Re: 1. Self 2. Internalized presences 3. Attachments	Implications Re: 1. Self 2. Internalized presences 3. Attachments	Implications Re: 1. Self 2. Internalized presences 3. Attachments
	Central relational conflict or theme				

^a I designated as social learners in this group Lee, Mulder, and Robin because they always chose to work with people beside them if they were willing; I designated as involuntary loners Pierre and Jamie because they seemed to be working alone not by choice but because of personal issues; I designated as voluntary loners Autumn, Catherine, and Karen because they showed no interest in working with others (except Ann or me). Catherine was willing to help someone if he asked (e.g., Mulder) but never asked to check with anyone. ^b See Davidson, 1983; Witkin et al, 1967 and chapter 2 discussion. ^c Harmonic I balance of Mathematics Learning style I & II more I; Harmonic II balance of Mathematics Learning style I & II more II. See Krutetskii, 1976, and chapter 2. ^d See Bowlby, 1973 and chapter 2.

Table E4

Protocol for Analysis of Exam Question Solutions						
The question	Pre-Exam: Class treatment, student reaction and Counseling preparation	Student's out of class preparation	Errors	Trouble-shooting efforts:	Instructor Grading	Post-exam Counseling
Defining the problem: concepts			1. understanding the question 2. misconceptions 3. confusions with ... 4. other	1. affective 2. cognitive		
Planning the solution: procedures			1. formula sheet 2. data-symbol linking 3. choice of formula 4. strategy/ layout	1. affective 2. cognitive		
Carrying out the solution: algebra			1. multiple uses of letter symbols 2. algebra	1. affective 2. cognitive		
Carrying out the solution: arithmetic			1. arithmetic 2. order of operations	1. affective 2. cognitive		
Conclusion Checking and reporting			1. reasonableness of solution 2. units 3. interpretation of solution	1. affective 2. cognitive		
Individual Patterns						
Class Patterns						

¹ These narratives are extracted not only from direct student reports but also from discussion of classroom interactions, metaphor and survey responses and mathematics focused interactions with the counselor.

Appendix F

Researcher and Student Seating

Ann in her final interview (Interview 3) noted that she had not taught a class before in which there seemed to be so much change in seating arrangement. She wondered if the different physical arrangement of the classroom from the usual rows of individual chairs was a factor. In previous classes she had taught students had mostly maintained the seating positions they had taken in the first class, changing only to sit in seats adjacent to the original. We both also wondered about the effect of my choices of seating on the choices by the students.

Researcher seating. I had struggled with my seating choices throughout the course (see Figure F1). In my role of researcher, I wanted to be as much an observer and as little a participant as possible. As the class progressed, and I realized that I was

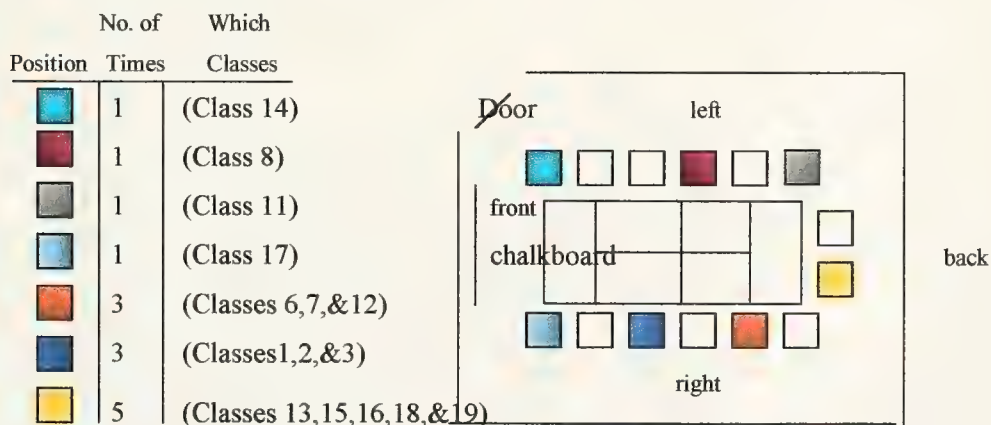
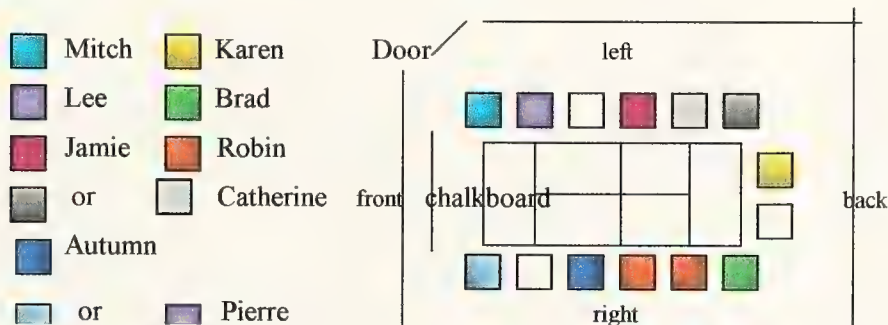


Figure F1. Jillian's seating positions for the PSYC/STATS 104 course, second floor, Riverside Center, Brookwood State University, Summer 2000

choosing to sit only on the right side, albeit in various positions, I decided that I needed a perspective of the class and students from the left. Eventually, towards the end of the

course, I decided that a better perspective of the whole class might come from the end of the table beside Karen (see Figures 1 and 2).

Student seating. Karen (12 times) and Catherine (10 times) (left back corner, see Figure I2) were the most consistent in their choice of seating of the class, although Autumn (9 times) mostly right middle and Jamie (9 times) left middle were almost as predictable. Brad (right back corner), Robin (right side mostly next to Brad in the right back corner), Mitch (left middle to left front), and Lee (mostly left side beside Mitch) each had her or his discernable pattern (7 to 8 times each). Pierre usually sat close to the front evenly on left and right sides—presumably to maximize the use of his tape-recorder for each lecture. Mulder showed the most inconsistency, perhaps because he was usually a few minutes late to class (he had to transport his mother) so he had to find an empty space—he was more often on the right or at the back (See Figure F2 for a most representative seating arrangement).



Ellen only came to the first class and sat on the left at the front. Kelly dropped the class the day of the second test (the 9th class). She sat towards the front either on the left or the right as she tended to be late and there was usually a seat or two unoccupied towards the front. Floyd came to only four classes in this room and sat at the back each time. This data contrasts somewhat with Ann's perception of constant change in student seating. Most students were relatively consistent in their seating choices or patterns.

Appendix G

PSYC/STAT 104 Instructor Syllabus and Selected Handouts

PSYC/STAT- Statistics in Psychology

104

Summer 2000

M/W 6:00 – 8:20

May 31st – August 2nd

Professor: Ann Porter Ph.D.

Office Information:

Phone:

e-mail:

Summer Office Hours: by appointment

Due to several advisory responsibilities at [redacted] and [redacted] my summer schedule is extremely inconsistent. **PLEASE DO NOT** take this to mean I am inaccessible, just that my schedule fluctuates from week-to-week. Please, feel free to contact me anytime to schedule an appointment.

Required Text Book:

Pagano, R.R. (1998). Understanding Statistics in the Behavioral Sciences – Fifth Edition. Pacific Grove, CA. Brooks/Cole Publishing Company.

Course Overview:

Psyc/Stat104 – Introduction to Statistics in Psychology will provide a comprehensive overview of the basic statistical concepts utilized in psychological research. Many, if not all, of these concepts are utilized in other disciplines as well. In order to comprehend statistics, it will be necessary to initially learn the material at a conceptual level. Calculations and computer modules will be required to advance your understanding of the statistical concepts.

Computers are an essential part of the psychology program at [redacted] and are extensively used in the field psychology. These computer assignments are intended to illustrate the ease that computer statistical analysis provides with large data sets. Although there will be some initial frustration, as you become familiar with the computer program itself, you will witness the convenience that computers provide to statisticians. Familiarity with Mini-tab, a computer program available for statistical analysis, is a university-wide requirement for this course. Mini-tab is available on the [redacted] mainframe computer. This version of Mini-tab is somewhat archaic, but will provide the necessary exposure to statistical analysis on the computer.

Course Goals:

1. To provide a basic overview of the statistical concepts utilized in empirical research.
2. To facilitate a comfortable relationship with statistical concepts.
3. To increase the conceptual understanding of the various statistical analyses utilized in research.
4. The increase understanding and critical thing about the statistics that the media presents.
5. To increase familiarity with statistical calculations and computer analysis.

Course Requirements:

1. You are expected to attend class on a regular basis. You are expected to read the text and compute statistical calculations in preparation for class lectures and the tests.
2. **Tests: There will be a total of 5 tests.** Each of the first 4 tests will be worth **20%** of your final grade. The 5th test is a conceptual comprehensive exam and is worth **10%** of your final grade. You will have the full class time to complete the exams. All 5 of the combined exam grades will determine **90%** of your final grade.
3. **Computer Assignments:** You will be required to complete one computer module independently for **2%** of your final grade. Additionally, you will be required to complete an additional computer module with a few of your classmates. In addition to completing the module as a group, you will be required to present this information to the class. The group presentation and paper are worth **8%** of your final grade. Handwritten papers will not be accepted! Computer Assignments must be typed!!! If papers are turned in after the due date, you will loose one letter grade for each date that the paper is late.

The group presentation and paper should include:

- All members of the group contributing to the oral presentation.
- A review of the method that the module illustrates.
- A visual display of the entire statistical analysis.
- An overview of the "Interpretation" portion of the assignment.
- A brief written commentary (specific form will be distributed to the class) of the efforts of the group (ex. meetings, attendance at meetings, designation of tasks, etc.) completed **separately** by each of the group members. *If any member does not contribute to the group assignment, it will be reflected on that individual's grade for this assignment.*

4. **Homework:** "Questions & Problems" are located at the end of each chapter in the text. You are not required to turn in the homework to me, but be sure to do these assignments, as they are essential to your understanding of the course material. These "Questions and Problems" provide an excellent review for the tests.

UNH Grading Scale:

Final grades will be based on the following scale:

	A = 93% - 100%	A- = 90% - 92%
B+ = 87% - 89%	B = 83% - 86%	B- = 80% - 82%
C+ = 77% - 79%	C = 73% - 76%	C- = 70% - 72%
D+ = 67% - 69%	D = 63% - 66%	D- = 60% - 62%
	F = 0% - 59%	

Course Policies:

A calculator with a square root key is required for this course.

Rescheduling/Missed Exams:

With good reason & advanced notification, you may take an exam earlier than the scheduled date. If you miss class on the date of the exam, you will be required to take a comprehensive exam (conceptual & calculations) at the end of the summer session in place of the missed exam – **no exceptions**. If you have not missed an exam, you may take the comprehensive exam to replace your lowest exam grade. If you choose this option and the comprehensive exam grade is lower than your lowest exam grade, the grade will not be averaged with your final grade. Absence on an exam date may be subject to the approval of the Dean of the College

Policy on Cheating / Plagiarism:

DO NOT CHEAT OR PLAGIARIZE!!!

Any student caught cheating or plagiarizing will be penalized in accordance to the policies stated in the 1999-2000 UNH Student Rights, Rules, and Responsibilities. **(NO EXCEPTIONS!)**

Students with Disabilities:

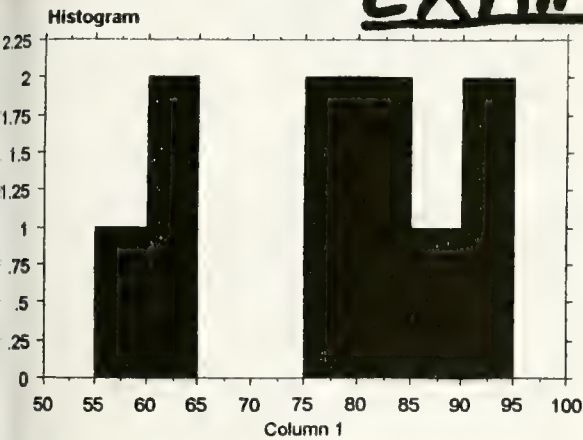
If you have a disability that requires special accommodations, you must obtain written documentation from Carol Swiech.

Course Schedule *(All Dates Are Subject to Change!)*

★★★★★ Computer Orientation - June 14th★★★★★

Assignment	Scheduled Date
Chapters 1 – 5	Exam #1 – Monday - June 12 th
Computer Orientation	<i>Tentatively Scheduled</i> Wednesday - June 14 th
Chapters 6 – 9	Exam #2 – Wednesday - June 28 th
Work independently or in groups on computer projects!!	Monday - July 3 rd
Chapters 10 – 14	Exam #3 – Monday – July 17 th
Chapters 15, 16, & 18	Exam #4 – Wednesday – July 26 th
Minitab Projects & chapter 19 review	Presentations - Monday - July 31 st
Chapter 19	Conceptual Comprehensive Exam #5 – Wednesday – August 2 nd

EXAM #3



mean = 77
St.dev. = 12.08
range = 33

Review Exam #4

Ch. 15

1-4, 6, 8, 9, 10, 11, 12, 15, 16 & 17(formula used in class)

Ch. 16

1, 2, 8(Only class notes for #3)

Ch. 18

1-3, 5, 6, 8-10, 12, 13, 15, 22

Symbols:

α	F_{obt}	F_{crit}	S_B^2	S_W^2
χ^2_{obt}	χ^2_{crit}	f_o	f_e	

Procedure for Testing the Null Hypothesis

1. **State the Null Hypothesis (symbols &/or words)**
2. **State the Alternative Hypothesis (symbols &/or words)**
- directional (1 tailed) / non-directional (2 tailed)
3. **Choose an alpha level / decision rule**
4. **Determine the most appropriate statistical analysis**
5. **Compute calculations**
6. **Make a decision (reject or fail to reject the null hypothesis)**
7. **Draw conclusions in the context of the problem**

Don't
write in
formula
sheet

1/10

6:00

Class

Study
Group 6

For
Exam #3
& 4

For the following, determine null/alternative, alpha, & the most appropriate statistical analysis:

1. An ecologist suspects that kingbirds found in Switzerland have more feathers than the rest of the kingbirds in the ~~nation~~^{world}. An exhaustive worldwide study was conducted last year to assess the number of feathers on all of the kingbirds in the ~~nation~~^{world}.
2. An investigator conducts an experiment to determine the importance of frequency of psychotherapy on depression for men and women. Men and woman suffering depression are randomly assigned to one of three frequencies of treatment conditions (3X per week, 1X per week, 1X per month). The depression scores are assessed after 6 months.
3. Prior to the superbowl, a survey was conducted to determine whether there was a relationship between gender and team preferences (Tampa Bay Buccaneers or New England Patriots).
4. A health educator wants to evaluate the effect of a dental film on the frequency with which children brushed their teeth. Eight children were randomly selected for the experiment. First, a baseline of the number of times children brush their teeth in a month was established. Next the children are shown the dental film. Again, the numbers of teeth brushings are recorded for a month.
5. A student at Midwest college is interested in whether women or men take more time in the shower. 8 women & 8 men are randomly selected to determine weekly shower time.
6. A traffic safety officer noticed that he was giving more speeding tickets to older people, so he conducted an experiment to determine whether there is a relation between people's ages & driving speeds.
7. A professor of women's studies is interested in determining if stress affects the menstrual cycle. Ten women are randomly sampled & divided into two groups. One of the groups is subjected to high stress for 2 months, while the other group lives in a stress-free environment for 2 months. The professor measures the menstrual cycle for all of the women.
8. A researcher believes that women in her town are taller today than in previous years. The researcher compares her data to that of a local consensus collected 20 years ago.
9. An investigator conducts an experiment to determine the importance of frequency of psychotherapy on depression. Subjects suffering depression are randomly assigned to one of three frequencies of treatment conditions (3X per week, 1X per week, 1X per month). The depression scores are assessed after 6 months.
10. During the past 5 years there has been a consistent inflationary trend in milk prices. You have yearly average in milk prices for the past year. You are an elementary school administrator and need to predict the cost of milk in 2005.

APPENDIX H

Descriptive and Comparative Data for PSYC/STAT 104 Class of Summer 2000

Table H1

Students' Expectations & Hopes in Relation to Effort, Grades and Scores, Summer 2000

	Pre: Grade Hoped For?	Pre: Grade Expected in course (June 5)	Grade in Exam #1	Post: Grade Hoped For?	Post: Grade So Far? (July 31)	Time on H'wk per week	Prior High School Highest Math	Algebraic Variable Level ^a	Final Grade in PSYC/STAT 104
Autumn 4^b (II B)^c	A	A	B ⁺ (E=R) ^c	A ⁻	A or A ⁻	2hrs/wk	AlgII,Disc/Stats (Finite Math in college)	4/5 [50]	A (E=R) ^d
Brad 4 (III A)	A	A	C ⁻ (E>R)		[D/C ⁻]		Algebra ? (F in PSYC/STATS in college)		WP (E>R)
Catherine (I)	A	A	A (E=R)	A	A	5hrs/wk	? (Calc1 in college)	5 [50]	A (E=R)
Ellen									
Floyd (III A)	A	B	F (E>R)		[F]		?		AF (E>R)
Jamie 5 (II A)	B	C	A (E<R)	C	B	5hrs/wk	Precalculus (D ⁺ in Psyc Stats in college)	4 [41]	B ⁺ (E<R)
Karen 5 (III B)	B	C	D ⁻ (E>=R)	C ⁺	B	6-7hrs/wk	Algebra II (F in PSYC/STATS in college)	2 [26]	B (E<R)
Kelly 3 (III B)	B ⁻	B ⁻	F/D ⁻ (E>R)		[F/D ⁻]		Algebra II		AF (E>R)
Lee 6 (II A)	A	A	C (E>R)	B	B	20min/wk	Precalc/calc (Finite math, college)	4 [45]	A ⁻ (E=R)
Mitch 4 (II B)	A	A ⁻	C ⁺ (E>=R)	A	B	3-5hrs/wk	Alg I, Geom (repeat), (F in PSYC/STATS in college)	4 [43]	B (E>=R)
Mulder 5 (III A)	B	B	D (E>R)	B	?	3hrs/wk	Algebra II	2 [25]	B (E=R)
Pierre 8 (II A)	A	B	D ⁺ (E>R)	B	B	17hrs/wk	College prep	4 [44]	B ⁻ (E=R)
Robin 3 (I)	A	A	B ⁺ (E=R)	A	B	10hrs/wk	"College" Alg. (in HS)		A ⁻ (E=R)

Notes: ^aLevels of understanding of the algebraic variable on the *Algebra Test* from 0 the least, through 5 the most sophisticated (see Appendix C). The number in the [] is the number of items correct out of 53. ^bNames of individual counseling participants are bolded and the number beside their names is the number of their counseling sessions. ^cCategory Type number (see chapter 7). ^dE=grade expectation, R=grade reality; = less than one grade discrepancy; < or > more than one grade discrepancy

Table H2

Student Tier(Tobias) and Category (Knowles) in Relation to Class Rank after Exam #1 and Pre- and Post-Statistical Reasoning Assessment (SRA) scores.

Student in order of score (1s) on pre SRA	Tobias' Tier level/Knowles Type	Exam #1 class Rank	PRE-Statistics Reasoning Assessment [SRA] (6/12/2000)		POST-Statistics Reasoning Assessment [SRA] (7/31/2000)	
			Number of 1s (correct reasoning)	Number of 0s (misconceptions)	Number of 1s (correct reasoning)	Number of 0s (misconceptions)
Catherine	1 st Tier/Category I	1 st	13	4	13 (+0)	4 (+0)
Robin	1 st Tier/ Category I	3 rd	12	6		
Kelly	Unlikely/ Category III, type B	11 th	11	6		
Floyd	Unlikely/ Category III, type A/B?	12 th	10	6		
Mulder	____/ Category III, type A	9 th	9	3	7 (-2)	6 (+3)
Autumn	Utilitarian/ Category II, type B	4 th	9	4	9 (+0)	4 (+0)
Brad	____/ Category III, type A	7 th	9	4		
Mitch	Utilitarian/ Category II, type B	5 th	9	7	9 (+0)	4 (-3)
Pierre	ESOL 2 nd Tier/ Category II, type A	8 th	7	8	7 (+0)	9 (+1)
Jamie	2 nd Tier/ Category II, type A	2 nd	7	10	7 (+0)	9 (-1)
Lee	2 nd Tier/ Category II, type A	6 th	6	9	7 (+1)	8 (-1)
Karen	____/ Category III, type A	10 th	3	11	4 (+1)	6 (-5)
Class Average:			8.75 (n=12)	6.5 (n=12)	7.875 (n=8)	6.25 (n=8)

Notes: ESOL: Pierre was an English Speaker of Other Languages

Table H3

Students' Pre and Post Positions on Feelings and Beliefs Surveys with Net Number of Changes

	MATHEMATICS FEELINGS			MATHEMATICS BELIEFS				N E T	F I N A L
	Testing Anxiety	Number Anxiety	Abstraction Anxiety	Procedural to Conceptual	Toxic to Healthy	Learned Helpless to Mastery Oriented	Performance to Learning Achievement Motivation		
CATEGORY I Type A									
Catherine	ok↑**ok	ok↑ok	ok↑ok	ok↑ok	ok↑ok	ok↑ok	ok↑ok	0/ 0/ +1	7OK
Robin	ok↑nok	ok=ok	ok↑ok	ok↑ok	ok↑ok	ok↑ok	ok↑ok	-1/ +1/ +2	6OK
CATEGORY II Type A									
Jamie	nok↓*nok	ok↓*ok	nok↓*ok	nok↓nok	nok↑nok	nok↑nok	nok↑~ok	+1/ +4/ +5	2OK
Lee	nok↑nok	nok↓ok	nok↑nok	ok↑ok	ok↓ok	ok=ok	ok=ok	+1/ -1/ -1	5OK
Pierre	ok↓*ok	ok=ok	ok↑ok	nok↑~ok	ok↑**ok	ok↑ok	nok↓nok	+1/ +3/ +2	6OK
Type B									
Autumn	ok↑nok	ok=ok	ok↑nok	nok=nok	ok↑ok	nok↑nok	nok↑ok	-2/ 0/ +1	2OK
Mitch	nok↓nok	ok↓ok	ok↓**ok	nok↑nok	nok↑*ok	nok↓nok	nok↑nok	+1/ -4/ +5	3OK
CATEGORY III Type A									
Brad	nok	ok	nok	ok	ok	ok	ok		
Frank	nok	ok	ok	ok	ok	ok	ok		
Karen	nok↑*nok	nok↓nok	nok↓*ok	nok↑*ok	ok↓ok	nok=nok	ok↓ok	+2/ +1/ 0	4OK
Mulder	nok↓nok	ok↓ok	ok↓ok	nok↑ok	ok↑*ok	nok↑*ok	ok↑ok	+2/ +6/ +7	6OK
Type B Kelly	nok	nok	nok	nok	ok	nok	nok		

Notes: On scale of 1 through 5: nok: 3.5 to 5 (anxiety) or 1 to 2.5 (beliefs); nok: 3 to 3.4 (anxiety) or 2.6 to 3 (beliefs); ok: 2.6 to 3 (anxiety) or 3-3.4 (beliefs); ok: 1 to 2.5 (anxiety) or 3.5-5 (beliefs); ↑/↓: increase/decrease; ↑*/↓*: significant increase/decrease ($p < .05$); ↑**/ ↓**: significant increase/decrease ($p < .01$) using Student t test of difference between means, dependent samples.

Appendix I

Summer 2000, PSYC/STAT 104 Class Calendar of Events

WEEK 1

Interview 1 with Ann Porter May 31, 2000

Class 1 Wednesday, May 31, 2000, Introductions, the syllabus and schedule, and chapter 1: Statistics and the Scientific Method

WEEK 2

Class 2 Monday, June 5, 2000, chapter 2: Basic Mathematical and Measurement Concepts and chapter 3: Frequency Distributions. I administered pretest feelings and beliefs surveys and invited volunteers to participate in individual mathematics counseling.

STUDY GROUP 1. Wednesday, June 7, 2000 4:30 p.m. 5:45 p.m.

Riverside Center

Brad, Lee, Jamie, Pierre (later)

Class 3. Wednesday, June 7, 2000, chapter 4: Measures of Central Tendency and chapter 5: The Normal Curve and Standard Scores

Individual Sessions. Kelly June 8, 2000

WEEK 3

Drop-In. June 12, 2000 Kelly, Karen

Individual Session. Karen June 12, 2000

Class 4 Monday, June 12, 2000 Exam 1 on chapters 1 through 5. I administered the *Statistics Reasoning Assessment* as a pretest.

Individual Session. Autumn June 12, 2000

Brad June 13, 2000 cancelled

Mitch June 14, 2000

STUDY GROUP 2 Wednesday, June 14, 2000 4:30 p.m. 5:45 p.m.

Greenville campus in the Learning Assistance Center, Mitch, Lee, Kelly

Class 5 Wednesday, June 14, 2000, Minitab Computer Orientation Computer Lab
Greenville campus
Minitab Module 1

Individual Session. Robin June 14, 2000
Kelly June 16, 2000 11:30 a.m.

WEEK 4

Class 6 Monday, June 19, 2000, chapter 6: Correlation

Individual Session. Jamie June 20, 2000 5:30 p.m.
Brad June 20, 2000 6:30 p.m.
Mulder June 21, 2000 9:30 a.m.
Lee June 21, 2000 3:20 p.m.

STUDY GROUP 3 Wednesday, June 21, 2000

Lee

Class 7 Wednesday, June 21, 2000, chapter 7: Linear Regression

Individual Session. Kelly June 21, 2000 8:20 p.m.
Pierre June 22, 2000 6:00 p.m.-9:00 p.m.
Floyd June 23, 2000 9:00 a.m. cancelled

WEEK 5

Individual Session. Karen June 26, 2000 4:00 p.m.
Mitch June 26, 2000 5:00 p.m.

Class 8 Monday, June 26, 2000 chapter 8: Random Sampling and Probability and
chapter 9: Binomial Distribution

Individual Session. Pierre June 27, 2000 6:30 p.m.

STUDY GROUP 4 Wednesday June 28, 2000 4:30 p.m.

Mitch, Lee, Robin, Jamie, Karen (from back of the room), Brad
(watching), Autumn (with her own questions, Pierre (late), Carol (just
checking)

Class 9 Wednesday June 28, 2000, Exam 2

Individual Session. Autumn June 28, 2000 7:30 p.m.
Mulder June 29, 2000 8:00 a.m.

WEEK 6

Class 10 Monday July 3, 2000, no class meeting

Individual Session. Jamie July 3, 2000 7:00 p.m. – 8:30 p.m.

Lee July 5, 2000 3:30 p.m.

STUDY GROUP 5 Wednesday July 5, 2000

Lee

Class 11 Wednesday, July 5, 2000, chapter 10: Introduction to Hypothesis testing

Using the Sign Test; entirely lecture ...didn't get to Mann Whitney

Individual Session. Robin July 5, 2000 7:30 p.m.

Mulder July 6, 2000 12:00 noon

WEEK 7

Interview 2 Ann Porter July 10, 2000 3:00 p.m.

Individual Session. Karen July 10, 2000 4:00 p.m.

Class 12 Monday July 10, 2000, chapter 11: Mann-Whitney U Test and chapter

12: Sampling Distribution of the sample means, the Normal Deviate (z) Test

Individual Session. Brad July 10, 2000 8:20 p.m.

Jamie July 11, 2000 10:00 a.m.

Drop-In Learning Center (with Jillian)

Lee July 12, 2000 1:00 p.m. - 3:00 p.m.

Individual Session. Mitch July 12, 2000 3:30 p.m.

STUDY GROUP 6 Wednesday July 12, 2000

Mitch, Lee

Class 13 Wednesday July 12, 2000, chapter 13: Student's t Test for Single

Samples, chapter 14: Student's t Test for Correlated and Independent Groups

Individual Session. Mulder July 12, 2000 8:20 p.m. cancelled

Pierre July 13, 2000 11:00 a.m.

Brad July 13, 2000 6:00 p.m.

Pierre July 14, 2000

Robin July 15?, 2000

WEEK 8

Individual Session Mulder 9:00 a.m. July 17, 2000

Drop-In: Learning Center (with Jillian)

Karen 1:00 p.m. - 4:00 p.m.

Jamie didn't come

Individual Session Karen 4:00 -5:00 p.m. July 17, 2000

Drop-In: Riverside (with Ann 4:00 p.m. – 6:00

p.m., with Ann and Jillian 5:00 p.m. – 6:00 p.m.)

Lee 4:30 p.m. – 6:00 p.m.

Jamie

Autumn (doing her own thing)

Catherine (doing her own work)

Mitch 5:20 p.m.

Karen 5:00 p.m. (doing her own thing)

Class 14 Monday July17, 2000, Exam 3

Individual Session Autumn July17, 2000 7:40 p.m.

Lee July19, 2000 3:30 p.m.

STUDY GROUP 7 Wednesday July19, 2000

Lee

Class 15 Wednesday, July19, 2000, chapter 15: Introduction to the Analysis of Variance chapter 16 Multiple Comparisons, did one-way, talked about setting up two-way

Individual Session Robin July19, 2000 8:20 p.m. cancelled

WEEK 9

Individual Session Karen July24, 2000 4:00 p.m.

Class 16 Monday July24, 2000, chapter 18: Chi-Square and other Nonparametric Tests, namely, one-way and two-way χ^2 and Wilcoxon Matched-Pairs Test

Individual Session Mulder July25, 2000 9:00 a.m.

Robin July25, 2000 ??

Pierre July26, 2000 8:00 a.m.

Jamie July26, 2000 10:00 a.m.

Mitch July26, 2000 3:3

STUDY GROUP 8 Wednesday July 26, 2000

Mitch, Lee, Autumn, Jamie, Mulder, Pierre [Robin, Brad,
Catherine came later]

Class 17 Wednesday July 26, 2000 Exam 4

Individual Session Autumn July 26, 2000 7:40 p.m.

WEEK 10

Drop-In Mulder 1:00 p.m.

Individual Session Karen 4:00 p.m. cancelled

Class 18 Monday July 31, 2000, Minitab Project Presentations, chapter 19:

Review of Inferential Statistics; I administered research posttests, Brad absent

Drop-In Karen cancelled

Individual Session Pierre, August 2, 2000 8:00 p.m.

Lee, August 2, 2000 3:30 p.m.

STUDY GROUP 9 Wednesday August 2, 2000

Lee, Mitch, Autumn, Pierre, Robin, Jamie, Catherine

Class 19 Wednesday, August 2, 2000 Exam 5, Brad absent

Individual Session Pierre August 3, 2000 8:00 a.m.

Interview3 Ann Porter, August 3, 2000, 1:30 p.m.

Drop-In: Learning Center (with Jillian)

Mulder August 3, 2000 (for Finite Math)

WEEK 11

Individual Session Jamie August 6, 2000 7:30 p.m.

Pierre August 7, 2000 8:00 a.m.

Lee August 7, 2000 9:00 a.m.

Appendix J

Sample Mathematics Counselor Tutoring Handouts

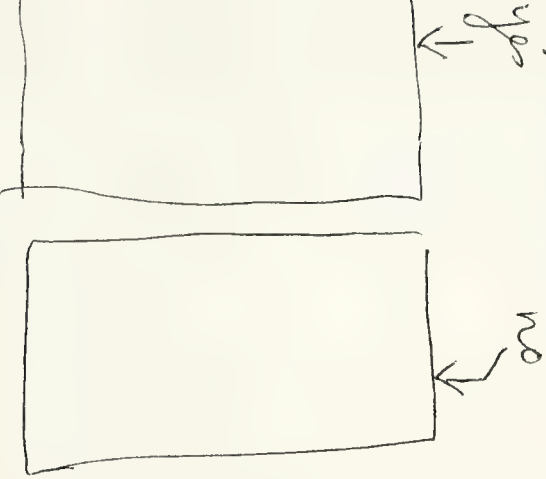
7/17/00
Exam #3

Experimental Design

Single Sample

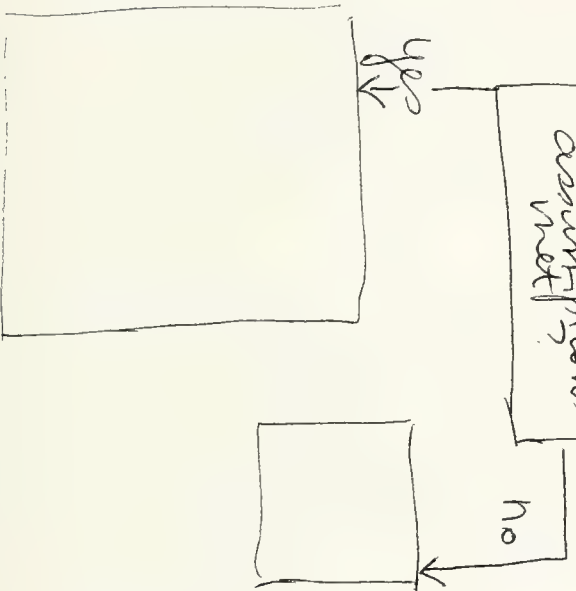
Z & t
assumptions
met?

yes
Is σ
known?



Correlated
groups

t -test
assumptions
met?

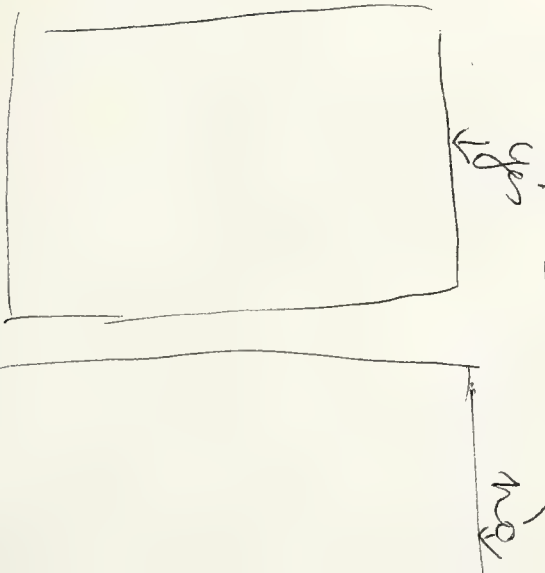


Correlated or
Independent
groups?

two samples or
2 conditions

Independent
groups

t -test
assumptions
met?



8/2/00

before
Exam #5

1. A university is considering implementing one of the following three grading systems: (1) All grades are pass-fail, (2) all grades are on the 4.0 system, and (3) 90% of the grades are on the 4.0 system and 10% are pass-fail. A survey is taken to determine whether there is a relationship between undergraduate major and grading system preference. A random sample of 200 students with engineering majors, 200 students with arts and science majors, and 100 students with fine arts majors is selected. Each student is asked which of the three grading systems he or she prefers.

2. A physician employed by a large corporation believes that due to an increase in sedentary life in the past decade, middle-age men have become fatter. In 1970, the corporation measured the percentage of fat in their employees. For the middle-age men, the scores were normally distributed with a mean of 22%. To test her hypothesis, the physician measures the fat percentage in a random sample of 12 middle-age men currently employed by the corporation. The fat percentages found were as follows: 24, 40, 29, 32, 33, 25, 15, 22, 18, 25, 16, 27. On the basis of these data, can we conclude that middle-age men employed by the corporation have become fatter?

3. Assume you are a nutritionist who has been asked to determine whether there is a difference in sugar content among the three leading brands of breakfast cereal (brands A, B, and C). To assess the amount of sugar in the cereals, you randomly sample six packages of each brand and chemically determine their sugar content. The following percentages of sugar were found:

4. A professor has been teaching statistics for many years. His records show that the overall mean for final exam scores is 82 with a standard deviation of 10. The professor believes that this year's class is superior to his previous ones. The mean for final exam scores for this year's class of 65 students is 87. What do you conclude?

5. A neuroscientist suspects that low levels of the brain neurotransmitter serotonin may be causally related to aggressive behavior. As a first step in investigating this hunch, she decides to do a correlative study involving nine rhesus monkeys. The monkeys are observed daily for 6 months, and the number of aggressive acts recorded. Serotonin levels in the striatum (a brain region associated with aggressive behavior) are also measured once per day for each animal. The resulting data are shown in Table 7.4. The number of aggressive acts for each animal is the average for the 6 months, given on a per day basis. Serotonin levels are also average values over the 6-month period. Analyze the data so you can tell how aggressive a monkey will be by its serotonin level.

6. A statistics professor conducts an experiment to compare the effectiveness of two methods of teaching his course. Method I is the usual way he teaches the course: lectures, homework assignments, and a final exam. Method II is the same as method I, except that students receiving method II get 1 additional hour per week in which they solve illustrative problems under the guidance of the professor. Since the professor is also interested in how the methods affect students of differing mathematical abilities, volunteers for the experiment are subdivided according to mathematical ability into superior, average, and poor groups. Five students from each group are randomly assigned to method I and 5 students from each group to method II. At the end of the course, all 30 students take the same final exam. The following final exam scores resulted:

7. On the basis of her newly developed technique, a student believes she can reduce the amount of time schizophrenics spend in an institution. As director of training at a nearby institution, you agree to let her try her method on 20 schizophrenics, randomly sampled from your institution. The mean duration that schizophrenics stay at your institution is 85 weeks with a standard deviation of 15 weeks. The scores are normally distributed. The results of the experiment show that the patients treated by the student stay a mean duration of 78 weeks with a standard deviation of 20 weeks.

8. A political scientist believes that, in recent years, the ethnic composition of the city in which he lives has changed. The most current figures (collected a few years ago) show that the inhabitants were 53% Norwegian, 32% Swedish, 8% Irish, 5% German, and 2% Italian. (Note that nationalities with percentages under 2% have not been included.) To test his belief, a random sample of 750 inhabitants is taken, and the results are shown in the following table:

- 9
- A sleep researcher conducts an experiment to
 - determine whether sleep loss affects the ability to

maintain sustained attention. Fifteen individuals are randomly divided into the following three groups of 5 subjects each: group 1, which gets the normal amount of sleep (7–8 hours); group 2, which is sleep-deprived for 24 hours; and group 3, which is sleep-deprived for 48 hours. All three groups are tested on the same auditory vigilance task. Subjects are presented with half-second tones spaced at irregular intervals over a 1-hour duration. Occasionally, one of the tones is slightly shorter than the rest. The subject's task is to detect the shorter tones. The following percentages of correct detections were observed:

10

An educator has constructed a test for mechanical aptitude. He wants to determine how reliable the test is over two administrations spaced by 1 month. A study is conducted in which 10 students are given two administrations of the test, with the second administration being given 1 month after the first. The data are given in the table.

11

A physical education professor believes that exercise can slow down the aging process. For the past 10 years, he has been conducting an exercise class for 14 individuals who are currently 50 years old. Normally, as one ages, maximum oxygen consumption decreases. The national norm for maximum oxygen consumption in 50-year-old individuals is 30 milliliters per kilogram per minute with a standard deviation of 8.6. The mean of the 14 individuals is 40 milliliters per kilogram per minute. What do you conclude?

12

To motivate citizens to conserve gasoline, the government is considering mounting a nationwide conservation campaign. However, before doing so on a national level, it decides to conduct an experiment to evaluate the effectiveness of the campaign. For the experiment, the conservation campaign is conducted in a small but representative geographical area. Twelve families are randomly selected from the area, and the amount of gasoline they use is monitored for 1 month prior to the advertising campaign and for 1 month following the campaign. The following data are collected:

13. The women's studies program believes that the amount of smoking by women has increased in recent years. A complete census taken 2 years ago of women living in a neighboring city showed that the mean number of cigarettes smoked daily by the women was 5.4 with a standard deviation of 2.5. To assess her belief, the professor determined the daily smoking rate of a random sample of 200 women currently living in that city. The data show that the number of cigarettes smoked daily by the 200 women has a mean of 6.1 and a standard deviation of 2.7.

14. A neurosurgeon believes that lesions in a particular area of the brain, called the thalamus, will decrease pain perception. If so, this could be important in the treatment of terminal illness that is accompanied by intense pain. As a first attempt to test this hypothesis, he conducts an experiment in which 16 rats are randomly divided into two groups of 8 each. Animals in the experimental group receive a small lesion in the part of the thalamus thought to be involved with pain perception. Animals in the control group receive a comparable lesion in a brain area believed to be unrelated to pain. Two weeks after surgery each animal is given a brief electrical shock to the paws. The shock is administered in an ascending series, beginning with a very low intensity level and increasing until the animal first flinches. In this manner, the pain threshold to electric shock is determined for each rat. The following data are obtained. Each score represents the current level (milliamperes) at which flinching is first observed. The higher the current level, the higher is the pain threshold. Note that one animal died during surgery and was not replaced.

15. A college professor wants to determine the best way to present an important topic to his class. He has the following three choices: (1) he can lecture, (2) he can lecture plus assign supplementary reading, or (3) he can show a film and assign supplementary reading. He decides to do an experiment to evaluate the three options. He solicits 27 volunteers from his class and randomly assigns 9 to each of three conditions. In condition 1, he lectures to the students. In condition 2, he lectures plus assigns supplementary reading. In condition 3, the students see a film on the topic plus receive the same supplementary reading as the students in condition 2. The students are subsequently tested on the material. The following scores (percentage correct) were obtained:

16. A clinical psychologist is interested in the effect that anxiety has on the ability of individuals to learn new material. She is also interested in whether the effect of anxiety depends on the difficulty of the new material. An experiment is conducted in which there are three levels of anxiety (high, medium, and low) and three levels of difficulty (high, medium, and low) for the material which is to be learned. Out of a pool of volunteers, 15 low-anxious, 15 medium-anxious, and 15 high-anxious subjects are selected and randomly assigned 5 each to the three difficulty levels. Each subject is given $\frac{1}{2}$ hour to learn the new material, after which the subjects are tested to determine the amount learned.

17. A group of researchers has devised a stress questionnaire consisting of 15 life events. They are interested in determining whether there is cross-cultural agreement on the relative amount of adjustment each event entails. The questionnaire is given to 300 Americans and 300 Italians. Each individual is instructed to use the event of "marriage" as the standard and to judge each of the other life events in relation to the adjustment required in marriage. Marriage is arbitrarily given a value of 50 points. If an event is judged to require greater adjustment than marriage, the event should receive more than 50 points. How many more points depends on how much more adjustment is required. After each subject within each culture has assigned points to the 15 life events, the points for each event are averaged. The results are shown in the table that follows.

APPENDIX K

Data about Karen

Table K1

Karen's Individual PSYC/STAT 104, Summer 2000 Participation Profile

	Week										
	1	2	3	4	5	6	7	8	9	10	Post
1 st Class Mondays 6:00 p.m.		Class 2 Pre- Tests	Class 4 EXAM #1 June12	Class 6	Class 8	Class 10 no class meeting	Class 12	Class 14 EXAM #3 July 17	Class 16	Class 18 Minitab Present, Post- Tests	Comp (extra) EXAM
2 nd Class Wednesdays 6:00 p.m.	Class 1	Class 3	Class 5 Minitab Partner: Catherine K absent	Class 7	Class 9 EXAM #2 June28	Class 11	Class 13	Class 15	Class 17 EXAM #4 July26	Class 19 EXAM #5 Aug. 2	
Study Group Wednesdays 4:30 p.m.					Study Gp 4 K partial						
Drop-In			June 12 K ½ hr					July 17 K 3hrs	July 24 K ¾hr	August 2 K cancelled	
Karen's Individual Sessions			June 12		June 26		July 10	July 17	July 24	July 31 K cancelled	
Meet with Instructor Extra								Stud Gp. before EXAM w/ Jill 5- 6 p m K partial			

Table K2

Karen's Progress in Tests in Relation to Mathematics Counseling Interventions							
	Exam #1 [20% of Final Grade]	Exam #2 [20% of Final Grade]	Exam #3 [20% of Final Grade]	Exam #4 [20% of Final Grade]	MINITAB projects [10% of Final Grade]	Exam #5 [10% of Final Grade]	Optional Exam (to replace lower grade)
General Strategies: average homework 6-7 hours per week, work by self (voluntary) or with experts (me/Ann) in class, organize, access Individual Mathematics Counseling, and Drop-In at the Learning Assistance Center							
Before	6/12 <i>Individual Math Counseling</i> just before exam, very anxious, angry	6/26 <i>Individual Math Counseling</i> 2 days before: discussed Exam #1, strategy: error analysis, course strategy => Karen gain control, deal with math depression	7/10 <i>Individual Math Counseling</i> 7/17 Drop-In: 3hrs + <i>Individual Math Counseling</i> : 1hr just before:— decision flow chart + unlabeled problems => formula sheet; math depression lifting	7/20 My Supervision meeting => have her assess her own change, a new metaphor? 7/24 <i>Individual Math Counseling</i>	7/10 went with Ann to Comp Lab for help on Mod 1		I offered meeting but Karen didn't want one.
Test Results	MC:-12,S:-1,Calc:-25 Total: 62%	MC:-14+2,S:-3,Calc:-11; Total: 74%	MC:-12,S:-1, Calc: -2 Total: 85 + 6%	MC:-12,S:-0,Calc:-0 Total: 88%	Mod 1: 100%; Present with Catherine Mod 3 100%	96%	MC: 12/40 (30%) Calc:45/60 (75%) Total: 57%
Analysis	Unhelpful Formula Sheet; literal symbols (N, Σ) issues statistical concepts issues Decimals fuzzy Preparation issues	Formula Sheet issues?; ALARM: Q6— decimals, operations issues and literal symbols ; Language issues-MC and Math Computation and S issues; Preparation improving	Math Computation “good enough” but compared ‘apples’ with ‘cheese’; Now has symbols in hand; Still language/ concept MC issues; Preparation much improved	Karen now felt she had it in hand (except for issues with MC)			
Post Strategies	Individual Math Counseling: focus: mirror her embryo mathematics self; develop JK and Ann as secure bases	Individual Math Counseling: focus: mirror her developing mathematics self; develop JK and Ann as secure bases	Individual Counseling: focus: provide bearable frustration; promote growth of realistic self-esteem, secure teacher attachments	Cancelled 7/31 individual appointment, didn't come to drop-in before Exam #5			Will come to Learning Center for finite mathematics next summer “if Jillian is there”

KAREN's Survey Profile Summary

MATHEMATICS FEELINGS

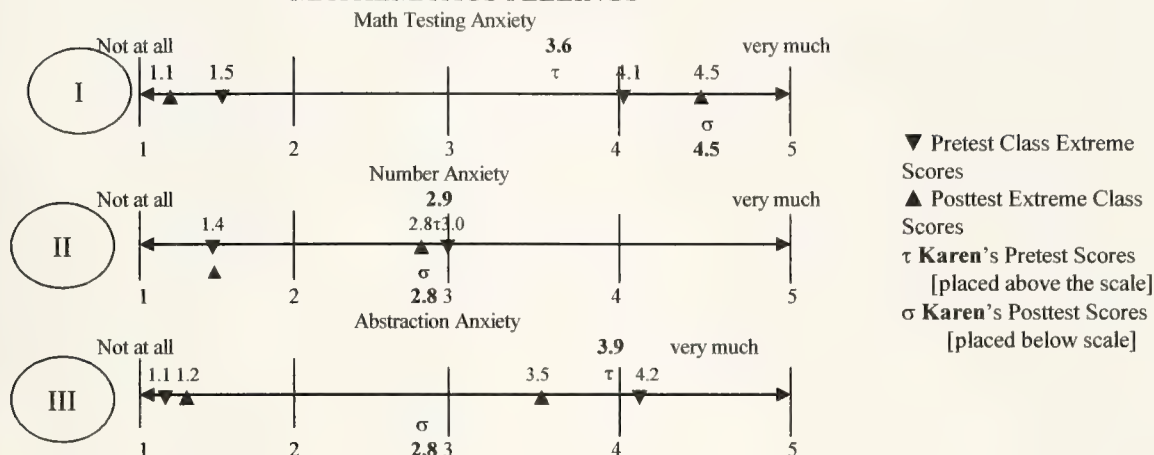


Figure K1. Karen's pre and post scores on the pre Mathematics Feelings Survey in relation to class extreme scores.

Mathematics Beliefs Scales

KAREN's Survey Profile Summary MATHEMATICS BELIEFS SURVEY

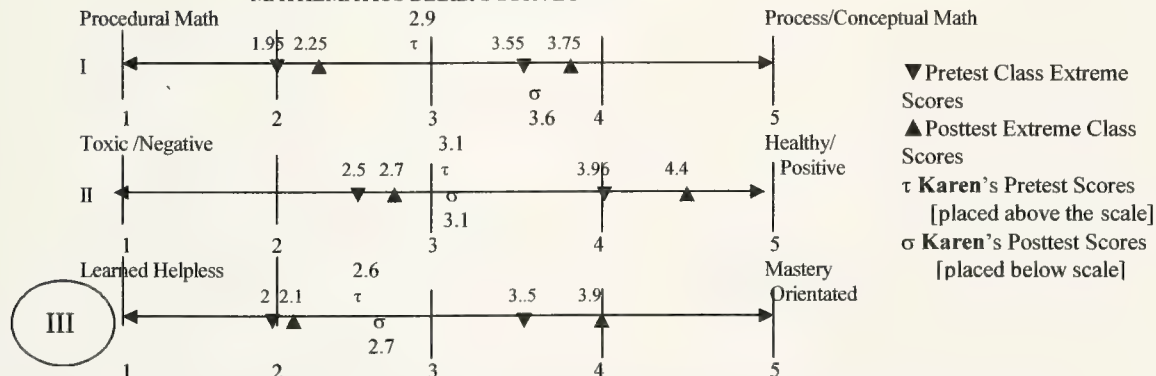
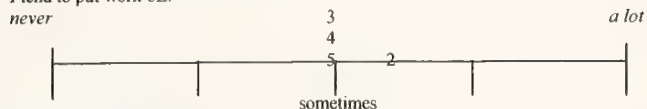


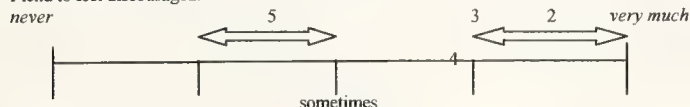
Figure K2. Karen's pre- and post-summary scores on the *Mathematics Beliefs Survey* in relation to class range scores.

JMK Mathematics Affect Scales

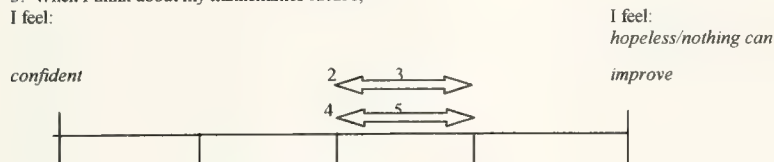
1. When I think about **doing** mathematics,
I tend to put work off:



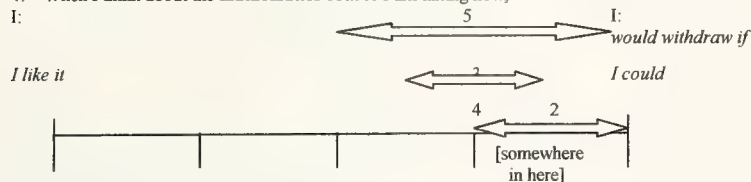
2. If I think about how I **experience** my problems with mathematics,
I tend to feel discouraged:



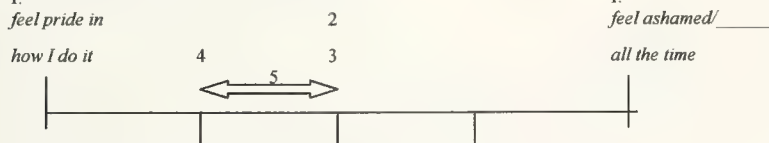
3. When I think about my mathematics **future**,
I feel:



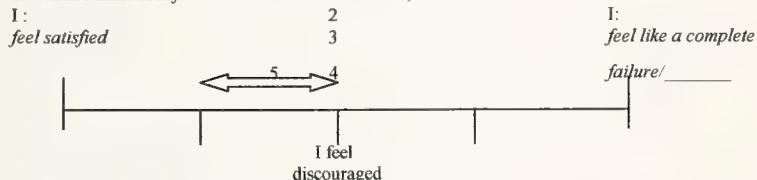
4. When I think about the **mathematics course** I am taking now,
I:



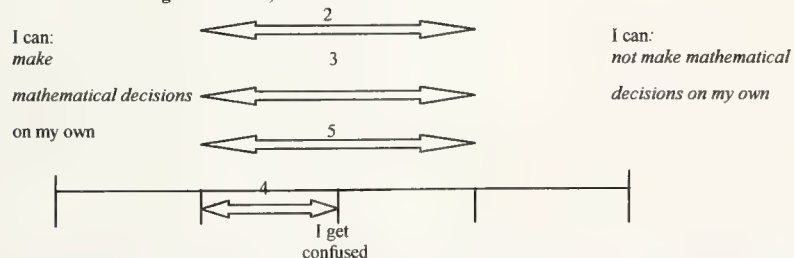
5. When I think about **how** I do mathematics,
I:



6. When I think of my **mathematical achievements**,



7. **While** I am doing mathematics,



2 Session #2, June 26, 2000

3 Session #3, July 10, 2000

4 Session #4, July 17, 2000

5 Session #5, July 24, 2000

↔ Karen's indication of her ranges of response.

[] Karen's written comments

Figure K3. Karen's responses on the JMK Mathematics Affect Scales¹ (in Mathematics Counseling Sessions 2 through 5)

Table K3

Karen's *JMK Mathematics Affect Scales* numerical responses.

JMK Mathematics Affect		Item	Item	Item	Item	Item	Item	Item	Average	
Scale		1	2	3	4	5	6	7		
Karen	JMK2:76/26	0.4	0.12	0.5	0.12	0.5	0.5	0.5	0.38	aftE1:62%
	JMK3:7/10	0.5	0.25	0.37	0.25	0.5	0.5	0.5	0.41	aftE2:74%
	JMK4:7/17	0.5	0.25	0.5	0.25	0.75	0.5	0.5	0.46	befE3:91%
	JMK5:7/24	0.5	0.62	0.37	0.3	0.62	0.63	0.5	0.51	befE4:88%
		0.48	0.31	0.43	0.23	0.59	0.53	0.53	0.44	

¹ See Appendix B for a discussion of the development and rationale for the use of these scales and a copy of the survey.

Appendix L

Data about Jamie

Table L1

Jamie's Individual PSYC/STAT 104, Summer 2000 Participation Profile

	Week										
	1	2	3	4	5	6	7	8	9	10	Post
1 st Class Mondays 6:00 p.m.		Class 2	Class 4 EXAM 1 June 12	Class 6	Class 8	Class 10 no class meeting	Class 12	Class 14 EXAM 3 July 17	Class 16	Class 18 Minitab Post- Tests	Comp (extra) EXAM
2 nd Class Wednesda ys 6:00 p.m. Study Group Wednesdays 4:30 p.m.	Class 1	Class 3	Class 5 Minitab Partner: Robin	Class 7	Class 9 EXAM 2 June 28	Class 11	Class 13	Class 15	Class 17 EXAM 4 July 26	Class 19 EXAM 5 Aug. 2	
Drop-In		Study Gp 1			Study Gp 4		No Show				
Individual Session				June 20 Jillian Initiate		July 3 Jillian Initiate	July 11 Jamie initiate		July 26 Jamie initiate		Aug. 6 Jamie initiate
Meet with Instructor											
Extra								S. Gp before EXAM w/Ann 4-6 p.m. w/Jill 5- 6 p.m.			

Table L2

Jamie's Progress in Tests in Relation to Mathematics Counseling Interventions							
	Test #1 June 12 [20% of Final Grade]	Test #2 June 28 [20% of Final Grade]	Test #3 July 17 [20% of Final Grade]	Test #4 July 26 [20% of Final Grade]	MINITAB projects July 31 [10% of Final Grade]	Test#5 August 2 [10% of Final Grade]	Optional Test August 7 (to replace lower grade)
General Strategies: average homework 5 hours per week, work by self (involuntary) or with Ann in class (leaves questions unasked, answers unspoken), listen, problem solve, Individual Mathematics Counseling—Jamie initiates last 3 sessions, attend 4 of 9 Study Groups (positive if speaks, negative if not)							
Before	6/7 <u>Study Group 1</u> : I asked her a question, J responded correctly	6/20 <i>Individual Math Counseling</i> : analyze Exam#1; metaphor: inside in storm. 6/28 <u>Study Group 4</u> : J watched, listened	7/3 <i>Individual Math Counseling</i> Did <i>The Algebra Test</i> 7/11 <i>Individual Math Counseling</i> before t-tests covered in class 7/17 <u>Extra Study Group</u> with Ann: J watched and listened	My Supervision meeting => commend her, give her a bouquet, have her write a new metaphor. 7/26 <i>Individual Math Counseling</i> : new metaphor Partly sunny day I can come out in 7/26 <u>Study Group 8</u> : J watched, listened		7/26 <u>Study Group 9</u> : I asked J a question, Jamie responded incorrectly	8/6 <i>Individual Math Counseling</i> Meeting at Starbucks—analyze all Tests...see her own competence see changes from pre to post test surveys e.g., reduced anxiety
Test Results	MC:-2,S:-1,C:-2 Total: 95%	MC:-4,S:-5,C:-17; Total: 74%	MC:-14,S:-0,C-2 Total: 84+ 6%	MC:-12,S:-0,C-12 Total: 76%	Mod 1: 100%; Present with Robin Mod 100%	100%	MC: 23/40 (58%) Calc: 48/60 (80%) Total: 71%. Too low to be used
Analysis	Former knowledge plus good problem solving even with panic, sound number sense	More anxious than before Test #1 because of 95% on Test #1 and family pressure; Study Group confusing; Formula Sheet issues?; Language issues on Math Computation, and problem solving didn't "work"; MC "good enough"; symbols a problem	Math Computation "good enough"; Now has symbols in hand; language/strategy MC issues	Math Computation: one analysis not understood—illogical use of literal symbols cf., numbers, one careless fix; two logical conclusions for incorrect calc—no credit; MC still an issue			
Post Strategies	6/20 <i>Individual Math Counseling</i> : focus—commend problem solving	7/3 <i>Individual Math Counseling</i> : focus—Exam Analysis Counseling	<i>Individual Math Counseling</i> : focus—				Probably will not go to a learning center for help in future; will access course related group help

Refer to the following information for questions 11 and 12.

An economics test was given and the following scores were recorded:

Individual	Score	$(X - \bar{X})^2$
A	12	
B	12	
C	7	
D	10	
E	9	
F	12	
G	13	
H	8	
I	9	
J	8	

11. Calculate the standard deviation of the distribution. (8 points)

$$\begin{aligned}
 &= \sqrt{\frac{\sum (X - \bar{X})^2}{N - 1}} \\
 &= \sqrt{\frac{40}{10 - 1}} \\
 &= \sqrt{4.44} \\
 &= 2.11
 \end{aligned}$$

12. What does this specific standard deviation tell you about the test scores for the sample? (3 points)

It tells you that these particular test scores were ~~close~~ close to the mean. Most of them ~~were~~ being standard deviations away. ~~It~~ It shows that this distribution was all spread out and there were extreme scores. A standard deviation of 2.11 tells you that 68% ~~of~~ of the scores were w/in 2.11 pts that is not very extreme.

When Jamie realized her z in question 13 was unlikely, she went back to question 11, to the s . She had found that she had divided the sum of squared deviations by $100 - 1$, that is, $\sum X - 1$, instead of by the correct $10 - 1$, that is, $n - 1$. When she corrected herself, her incorrect $s = .63$ changed to $s = 2.11$.

Figure L1. Jamie's responses on Questions 11 and 12 on Exam #1. Note her self-corrections in question 11.

13. Calculate the z score for individual G. (8 points)

$$\begin{aligned}
 z &= \frac{X - \bar{X}}{s} \\
 &= \frac{13 - 10}{2.11} \\
 &= \frac{3}{2.11} \\
 &= 1.42
 \end{aligned}$$

14. What does the z score tell you specifically about individual G? (3 points)

The z score tells you that individual G got a test score way ~~above~~ above the mean. (Almost 2 standard deviations away) This individual is an example of why the standard deviation is relatively large.

use chart
this individual or 92.2% of other participants scored was 92.2% above mean worse than G

When Jamie substituted her new $s = 2.11$ for the incorrect $s = .63$ in the z formula, the incorrect z of 4.76 became a more reasonable $z = 1.42$.

Figure L2. Jamie's responses on Questions 13 and 14 on Exam #1. Note her self-corrections in question 13.

Jamie's Responses on the Algebra Test

Jamie's level 4 score meant that she was able to treat letters appropriately as *specific unknowns* in some cases, as *generalized numbers* in some cases, and as *variables*

7. (d)

$$A = \cancel{10e} \\ (e + 2)5$$

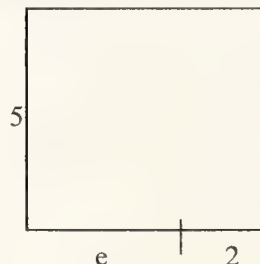


Figure L3. Jamie's solution to question 7 (d) Find the area of the figure, *Algebra Test* (Sokolowski; see Appendix D). Note Jamie's initial error that she scratched out and replaced with the correct properly coordinated area solution.

in others. Jamie's use of letter symbols:

1. as *specific unknowns* in some cases (as in Q.14: ...if $r = s + t$ and $r + s + t = 30$, [what is] r ?: **$r = 15$** (Jamie's correct response in bold),
2. as *generalized numbers* in some cases (as in Q.18: When [is this] true...? $L + M + N = L + P + N$ Always Never Sometimes, when: **$M = P$** (Jamie's correct response in bold), and
3. as *variables* in others (as in 5. (c) If $e + f = 8$ then $e + f + g =$: **$8 + g$** (Jamie's correct response in bold). (Appendix C, *Algebra Test*)

In addition, she was able to resolve ambiguity by coordinating two operations. For example, to determine the area of a rectangular figure she corrected her initial impulse to incorrectly use only one operation, multiplication, to get **$10e$** , to the coordination of addition and multiplication, to obtain **$(e + 2)5$** (see Figure L3).

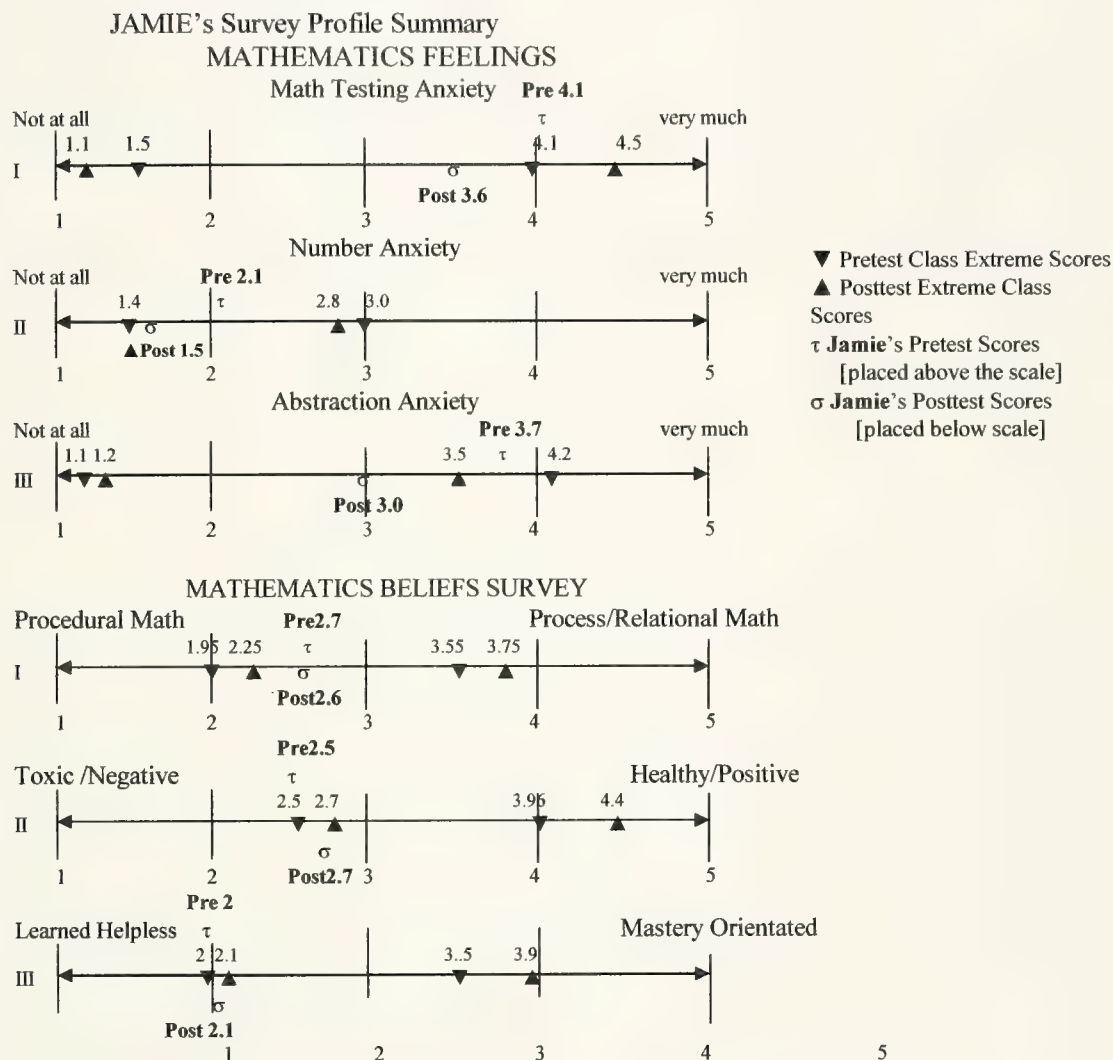


Figure L4. Jamie's mathematics pre- and post-feelings and beliefs *Survey Profile Summary* in relation to class range pre-and post-scores

Session 1. We discussed briefly Jamie's anxiety average scores on the *Mathematics Feelings* pretests that I had plotted with the class extreme scores on her *Survey Profile Summary* (see Figure L4). Jamie's *Number Anxiety* was low (2.1), close to the middle of the class range, but her *Abstraction Anxiety* was high. She was not surprised by her high *Math Testing Anxiety* score (the highest in the class at 4.1).

JMK Mathematics Affect Scales

1. When I think about **doing** mathematics,

I tend to put work off:



1 Session 1, June 20, 2000

2 Session 2, July 3, 2000

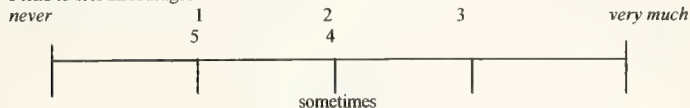
3 Session 3, July 11, 2000

4 Session 4, July 26, 2000

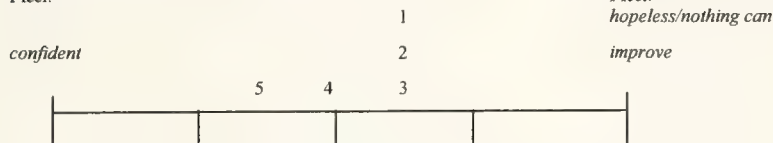
5 Session 5, August 6, 2000

2. If I think about how I **experience** my problems with mathematics,

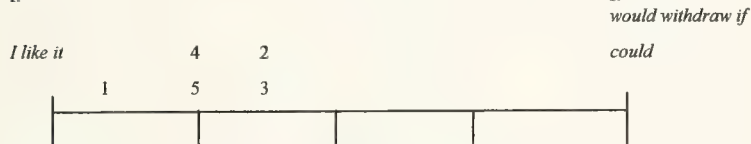
I tend to feel discouraged:

3. When I think about my mathematics **future**,

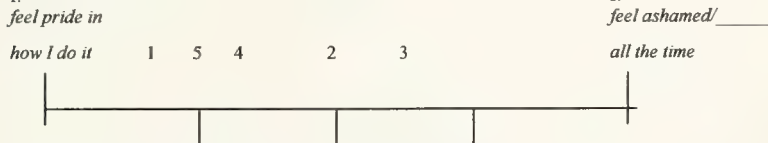
I feel:

4. When I think about the **mathematics** course I am taking now,

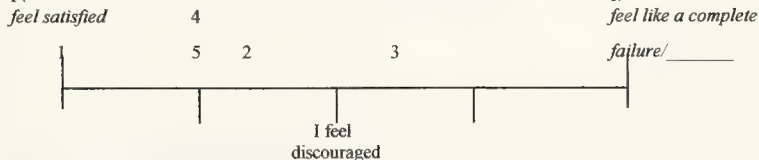
I:

5. When I think about **how** I do mathematics,

I:

6. When I think of my **mathematical** achievements,

I:

7. **While** I am **doing** mathematics,

I can:

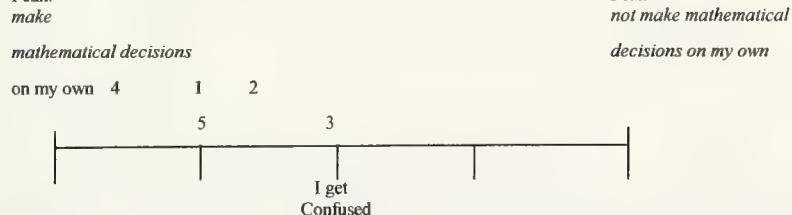


Figure L5. Jamie's responses on the JMK Mathematics Affect Scales, Sessions 1-5

Table L3. Numerical scores and averages of Jamie's *JMK Scale* responses (see Figure K5)

JMK Mathematics Affect		Item	Item	Item	Item	Item	Item	Item	Average	
Scale		1	2	3	4	5	6	7		
"Jamie"	JMK1:6/20	0.87	0.75	0.37	0.9	0.82	1	0.75	0.78	aftE1:95
	JMK2:7/3	0.75	0.5	0.37	0.63	0.5	0.7	0.5	0.564286	aftE2:74
	JMK3:7/11	0.75	0.25	0.37	0.63	0.37	0.4	0.5	0.467143	befE3:90
	JMK4:7/26	0.75	0.5	0.5	0.75	0.7	0.75	0.75	0.671429	befE4:76
	JMK5:8/6	0.87	0.75	0.63	0.75	0.75	0.75	0.75	0.75	befEC:71
		0.80	0.55	0.45	0.732	0.63	0.72	0.65	0.646571	

APPENDIX M

Data about MULDER

Table M1.

Mulder's Individual PSYC/STAT 104, Summer 2000 Participation Profile

	Week										
	1	2	3	4	5	6	7	8	9	10	Post
1 st Class		Class	Class	Class	Class	Class	Class	Class	Class	Class	
Mondays		2	4	6	8	10	12	14	16	18	
6:00 p.m.		Pre- Tests	EXAM #1 June 1 2 M late			no class meeting		EXAM #3 July 17		MINTAB; Post- Tests	
2 nd Class	Class	Class	Class	Class	Class	Class	Class	Class	Class	Class	
Wednesdays	1	3	5	7	9	11	13	15	17	19	
6:00 p.m.			Minitab Partner: Pierre		EXAM #2 June 28				EXAM #4 July 26	EXAM #5 Aug. 2	
Study Group Wednesdays 4:30 p.m.									Study Gp 8		
Drop-In										July 31 Minitab paper; Aug. 3 MATH 120 Finite	
Individual Session				June 21 Stats only	June 29 Finite math & Stats	July 6 Stats only	July 12 M cancelled	July 17 Stats only	July 25 Stats only		
Meet with Instructor											
Extra								S. Gp before EXAM w/Ann 4-6 p.m. w/Jill 5- 6 p.m.			

Table M2

Mulder's Progress in Tests in Relation to Mathematics Counseling Interventions

	Test #1 6/12/00 [20% of Final Grade]	Test #2 6/28/00 [20% of Final Grade]	Test #3 7/17/00 [20% of Final Grade]	Test #4 7/26/00 [20% of Final Grade]	MINITAB projects 7/31/00 [10% of Final Grade]	Test#5 8/2/00 [10% of Final Grade]	Optional Test (to replace lower grade)
General Strategies: average homework 3 hours per week, work with others in class, visualize/memorize access Individual Mathematics Counseling, and Drop-In at the Learning Assistance Center							
Before		6/21 First <i>Individual Math Counseling</i> ; analysis of Exam#1-lack of prep, lack of knowing what to expect	7/6 <i>Individual Math Counseling</i> : Finite Math 7/17 9:00a.m <i>Individual Math Counseling</i> focused on Exam #3; focus on symbol links; choosing and doing hypothesis test	My Supervision meeting=> trial MC resistance Test at 7/25 <i>Individual Math Counseling</i>	7/31 dropped in to have me read his and Pierre's MINITAB project write up		
Test Results	MC:-18,S:-1,Calc:-18 Total: 63	MC:-14,S:-5,Calc:-0 Total: 81	MC:-18,S:-0,Calc:-6 Total: 76 + 5	MC:-8+2,S:-0,Calc:-3 Total: 91	Mod : 100%; Present with Pierre Mod 92%	94%	
Analysis	Poor Formula Sheet; MC issues; lack of study	Has Math Computation more in hand; Verbal-MC, S issues	Math Computation still OK; Now has symbols in hand; STILL big MC issues	Mulder has mastered the last hurdle: MC!			
Post Strategies	Individual Math Counseling: focus—overall approach; Formula Sheet	6/29 <i>Individual Math Counseling</i> : [half on finite math: simplex method] focus—verbal connections especially symbols	<i>Individual Math Counseling</i> : focus MC, resistance			Drop-in help with Finite Word Problems	Probably won't go to a Learning Center when taking math in future

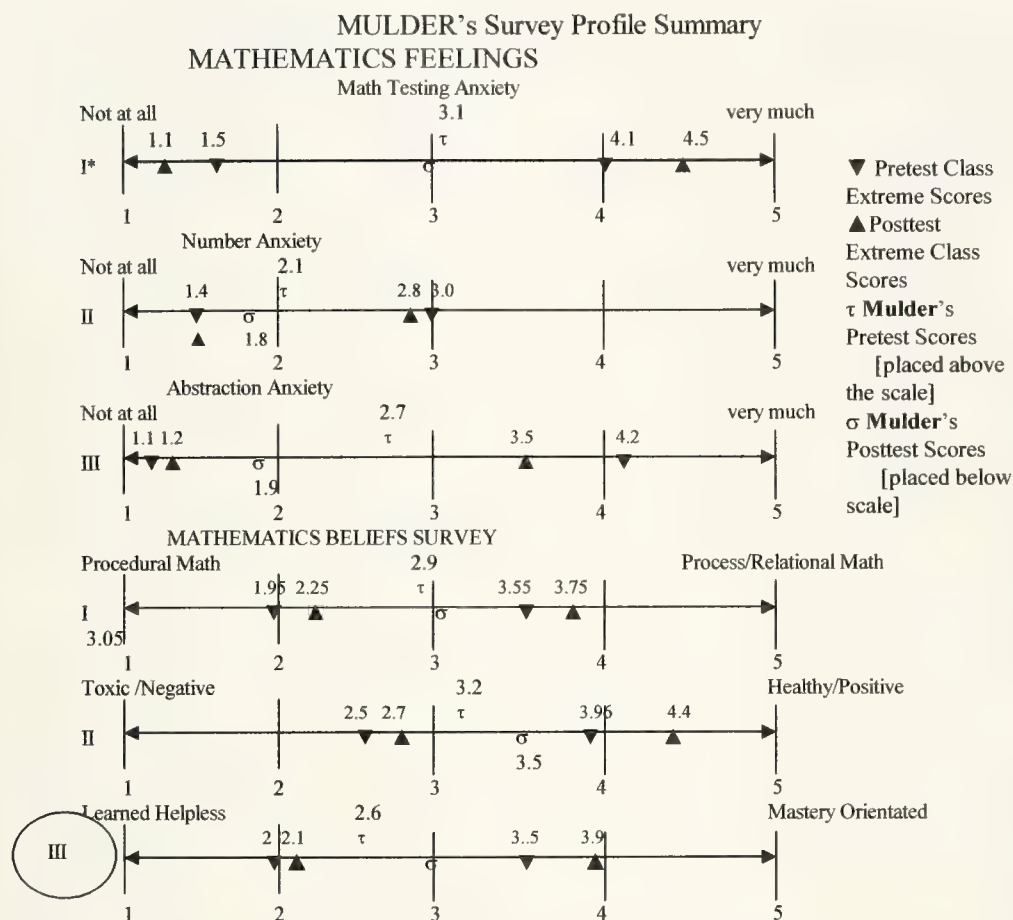


Figure M1. Mulder's responses to the pre- and post-Mathematics Feelings and Mathematics Beliefs surveys in relation to class extreme scores.

JMK Mathematics Affect Scales

1 Session #1, June 21 2000

3 Session #3, July 6, 2000

5 Session #5, July 25, 2000

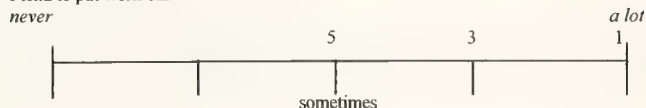
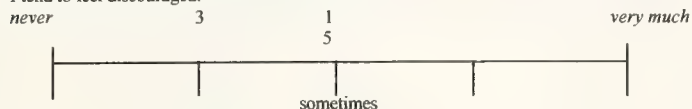
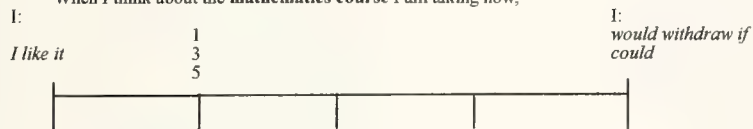
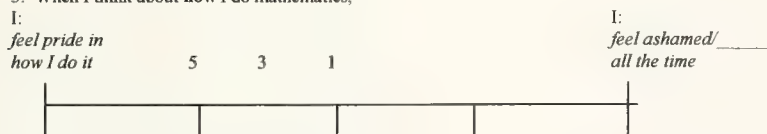
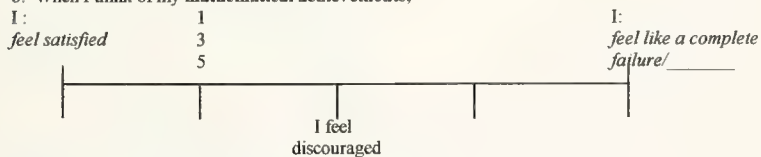
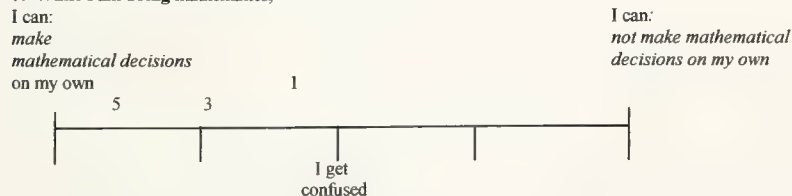
1. When I think about **doing** mathematics,
I tend to put work off:2. If I think about how I **experience** my problems with mathematics,
I tend to feel discouraged:3. When I think about my mathematics **future**,
I feel:4. When I think about the **mathematics course** I am taking now,5. When I think about **how** I do mathematics,6. When I think of my **mathematical achievements**,7. **While I am doing** mathematics,Figure M2. Mulder's responses on the *JMK Mathematics Affect Scales* for Sessions 1, 3, and 5.

Table M3.

Numerical Scores and Averages of Mulder's *JMK Affect Scale* Responses

JMK Mathematics		Item	Item	Item	Item	Item	Item	Item	Average
Affect Scale		1	2	3	4	5	6	7	
Mulder	JMK1:6/21	0.0	0.5	0.75	0.75	0.5	0.75	0.60	0.55 aftE1:63 ^a
	JMK3:7/6	0.25	0.75	0.75	0.75	0.63	0.75	0.60	0.64 aftE2:81befE3:81 ^b
	JMK5:7/25	0.5	0.5	0.75	0.75	0.75	0.75	0.75	0.68 befE4:91 ^c
	Average:	0.34	0.58	0.75	0.75	0.62	0.75	0.68	0.64

Notes: ^a after Exam #1 where he earned 63%; ^b after Exam #2 where he earned 81% and before Exam #3 where he earned 81% (with extra credit); ^c after Exam #4 where he earned 91%.

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